2.1 Introduction

- A gear box is a mechanical device used for transmitting a power from the prime mover to the machine with change in torque and speed.
- A machine tool is expected to perform various operations such as turning, facing, milling, boring etc. The tools used for these operations work the best for a given cutting speed, feed and depth of cut.
- Depending on the diameter of the job being machined, different spindle speeds would be necessary to determine optimum cutting speeds and feed rates.
- The different possible spindle speeds for different diameter of the job would have to be generated by a multi-speed gear box. A multi-speed gear box can be used for stepped regulation of the rpm.





2.1.1 Types of Gear Boxes

Generally there are three types of gear boxes used commercially.

- I. Sliding Mesh Gear Box
- II. Constant Mesh Gear Box
- III. Synchromesh Mesh Gear Box

2.2 DETERMINATION OF VARIABLE SPEED RANGE

- The ideal speed range in a machine tool would be the one where there would be infinite number of speed ratios. This would allow an optimum operation of a machine for any job diameter. Such a speed regulation would be called as a *stepless regulation*.
- Higher the speed ratios available, higher would be the cost of the gear box. Hence, from the cost and utility perspective, optimum gear ratios should be available to get the different speed ranges between the maximum and minimum speed. These intermediate discrete values can be obtained by different laws.
- Let N_1 and N_2 be the minimum and maximum spindle speeds to be achieved in z number of intermediate steps. These steps can be arranged by the following laws:
 - I. Arithmetic Progression (AP)
 - II. Geometric Progression (GP)
 - III. Harmonic Progression (HP)

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• From the above, Geometric Progression (G.P.) series is preferred over others as it gives minimum loss of economic cutting speed in the whole rpm range as well as it gives better gear box design.

2.2.1 Geometric Progression (G.P.) series

- In a geometric progression, the gear ratio between two consecutive speed ranges remains the same. It gives constant gear ratio at every stage and is based on a preferred number of series.
- Geometric progression ratio,

$$\phi = \left(\frac{N_{\max}}{N_{\min}}\right)^{\frac{1}{z-1}}$$

where, ϕ = Common ratio

N_{max} =Maximum speed,

N_{min} = Minimum speed,

z= Number of speed steps required

- To decide on the geometric progression ratio φ, the preferred number series are used. Series of numbers have been standardised in order that the various sizes of a series can be determined in an orderly fashion and these are called as preferred numbers.
- The preferred numbers are classified into different series, which has its own series factor and are determined as follows:

R40	\rightarrow	ф 40	=	⁴⁰ √10	=	1.06
R20	\rightarrow	\$ 20	=	²⁰ √10	- 	1.12
R 10	\rightarrow	\$ 10	= '	¹⁰ √10	a , ∎,°	1.26
R(20/3)	\rightarrow	ф20/3	æ	$\frac{20}{\sqrt{3}}$	-	1.41
R5	\rightarrow	φ5	=	√10	=	1.58
R4	\rightarrow	\$ 4	=	∜10	-	1.78
R(20/6)	\rightarrow	\$ 20/6	=	$\frac{20}{6\sqrt{10}}$	=	2.00

 A series is established by multiplying the first number with the series factor to get the second size. On similar lines the third size will be determined by multiplying the second size by the same series factor. The procedure is continued until the complete series is built up. Necessary rounding as per international standards may be done to generate various numbers.

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2.2.2 Some important parameters in designing of multi-speed gear box

For machine tools, the cutting speed is generally expressed as,

$$V = \frac{\pi DN}{1000}$$
, (m/min)
where, D = Job diameter, mm
N = required spindle speed, rpm

$$N = \frac{1000 \cdot V}{\pi D}$$

Maximum Spindle Speed(N_{max})

$$N_{\rm max} = \frac{1000 \cdot V_{\rm max}}{\pi D_{\rm min}}$$

Minimum Spindle Speed(N_{min}) \geq

$$N_{\min} = \frac{1000 \cdot V_{\min}}{\pi D_{\max}}$$

Speed Range Ratio (R_N)

$$R_{N} = \frac{N_{\max}}{N_{\min}}$$

 \succ Cutting Speed Range Ratio (R_{ν})

$$R_{v} = \frac{v_{\text{max}}}{v_{\text{min}}}$$

 \succ Diameter Range Ratio (R_v)

$$R_d = \frac{d_{\max}}{d_{\min}}$$

Selection of No. of Speed Steps (z)

$$\phi = \left(\frac{N_{\max}}{N_{\min}}\right)^{\frac{1}{z-1}}$$
$$\therefore \frac{N_{\max}}{N_{\min}} = \phi^{(z-1)}$$
Also, $R_N = \frac{N_{\max}}{N_{\min}}$
$$\therefore \frac{N_{\max}}{N_{\min}} = R_N = \phi^{(z-1)}$$
$$\therefore R_N = \phi^{(z-1)}$$
$$\therefore \log R_N = (z-1)\log \phi$$
$$\therefore \frac{\log R_N}{\log \phi} = (z-1)$$
$$\therefore z = \frac{\log R_N}{\log \phi} + 1$$

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$$\therefore z = \frac{\log R_N + \log \phi}{\log \phi}$$
$$\therefore z = \frac{\log (R_N \phi)}{\log \phi}$$

The value of z can be determined by knowing value of RN and ϕ . The number of speed steps is then rounded off to the nearest integer number which can be split into the multiples of 2 and 3.

2.3 STRUCTURE FORMULA

- The structural formula can be defined as an expression which gives the distribution of the number of transmission stages and the difference between the adjacent speed steps in each of these stages.
- Let the total number of speed steps be made in a number of stages or transmissions.
- Let, p1, p2 ... pn = Number of speed steps in the first, second and nth stage of transmission.
- This would mean that after first stage there would be p1 number of speed steps, whereas at the end of stage two, there would be p1 x p2 steps. Thus for the complete z speed steps,

$$z = p_1 x p_2 x p_3 x....p_n$$

where, n = number of stages of the gear box

- We now consider the case where we analyse the difference between two adjacent speeds 'x' in a given stage, x is referred to as the characteristic of that transmission stage.
- As explained earlier, p in general represents the number of speed steps available in a stage. Consider any two adjacent speeds available in this stage. If the difference between these two speeds is 1, then x = 1, whereas if the value is 2, then x = 2. Thus for each stage, there would be a particular value of x.
- As the rpm of the output shaft is in GP, there must be one transmission stage which would give x = 1. This group is called as the main transmission group and has a progression ratio of $\phi x = \phi 1 = \phi$. The next transmission group would then have x2 = p1 and the progression ratio of $\phi^{x_2} = \phi^{P_1}$, whereas the next transmission group would have x3 = P1 P2 with a progression ratio of $\phi^{x_2} = \phi^{P_1}$. Thus, the values of x1, x2, x3.....xn would be:

• This expression is represented in the form of a structural formula given as:

$$z = p_1(x_1) p_2(x_2) p_3(x_3) \dots P_n(x_n)$$

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2.4 STRUCTURE DIAGRAM

- The structural formula can be represented in the form of special graphs called as • structure diagrams which describes the kinematic structure of the gear box.
- To understand the method of representation of structure diagrams, consider Fig. 2.2. • Consider a 4 speed gear box.



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 $z = 4 = p_1 \times p_2 = 2^2$ $p_1 = p_2 = 2$ n = 2 $x_1 = 1$ $x_2 = p_1 = 2$

There would be two possible structural equations :

z = 2(1)2(2)and z = 2(2) 2(1)

- The structure diagrams to explain the relations with structure formulae for both • these cases are shown in Fig. 2.2 (a) and (b).
- The structure diagram shown in Fig. 2.2 (a) has lines crossing each other and is called as a Crossed Diagram, whereas Fig. 2.2 (b) represents an Open Structure Diagram.
- The general rules for drawing structure diagrams are listed below.
 - 1. If the number of stages in a gear box are 'n', draw (n + l) vertical lines spaced at a convenient distance.
 - 2. The first vertical line represents the input shaft, where as the last one would represent the output shaft. The other vertical lines represent the intermediate shafts which form the transmission groups or stages of the gear box.
 - 3. Z horizontal lines are next drawn to represent total number of speeds steps for the gear box. The spacing between the horizontal lines is taken as $\log \phi$.
 - 4. Using the structure formula,

 $z = p_1(x_1) p_2(x_2) p_3(x_3) \dots P_n(x_n)$

Plot the values of $p_1(x_1)$, $p_2(x_2)$, $p_3(x_3)$, etc. as shown earlier in Fig. 2.2.

- The structure diagram allows the designer to conceptualise the flow of transmission in the gear box and gives the following information:
 - 1. The number of shafts in the gear box
 - 2. The number of gears on each shaft. The approximate number of gears would be found by the following equation:

$$G=2\times(p_1+p_2+p_3+\ldots p_n)$$

- 3. The order of changing transmissions at each stage to get the required spindle speed
- 4. The transmission range and the characteristic of each group
- Fig. 2.3 shows the kinematic arrangement of a four speed gear box based on the two structures shown in Fig. 2.2 (a) and (b). Both these structures would have the same arrangement, but different sizes of gears as their transmission ranges are different.

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Fig. 2.3 Arrangement for a 4 Speed Gear Box

• Fig. 2.4 gives the possible arrangements which would provide the required four speeds. It should be noted that the gears which are fixed are marked with a cross (x), whereas sliding gears are shown with a two way arrow.



Fig .2.4 Flow of Transmission to get the 4 Speeds

- In Fig. 2.4, the gears shown with dotted lines rotate idly without transmitting any power.
- The typical characteristics of a structure diagram are:
 - 1. Each line must connect another follow-up line till the final output shaft is reached.
 - 2. At any stage the arrow must terminate at one and only point. However at any input there can be more than one arrow. Generally this number is limited to a maximum of three.
 - 3. On the final shaft, all the points must be connected by the arrows.
 - 4. No arrow should go beyond the bounds of the speeds required speeds at the final output shaft.
- Consider some of the structure diagrams shown in Fig. 2.5 for a 4 speed arrangement. These diagrams are invalid as they violate the principles laid down earlier.

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(c) Violates Laws 3 and 4

Fig. 2.5 Invalid Structure Diagrams

Table 2.1 Preferred Structure Formulae

Number of	Number of	Preferred Structure	Approximate
Speed Steps, z	Stages, n	Formula,	Number of gears
		P ₁ (x ₁), p ₂ (x ₂)	2 x (p ₁ + p ₂)
4	2	2(1) 2(2)	2 x (2 + 2) = 8
6	2	2(1) 3(2)	2 x (2 + 3) = 10
9	2	3(1) 3(3)	2 x (3 + 3) = 12
12	3	2(1) 3(2) 2(6)	2 x (2 +3 + 2) = 14

2.5 RAY AND SPEED DIAGRAM

- The structure diagram establishes the transmission flow in a gear box. It does not give information related to the individual speeds of each gear, nor does it help in establishing the number of teeth on each gear. Once the structure diagram is finalised, the gear box arrangement is fixed.
- To determine the speeds of the various gears, the ray diagram and the speed diagram are drawn. These diagrams allow the designer to read the speeds of all the shafts, gears and finally the spindle speeds.
- The steps for drawing the ray and speed diagram for a gear box are listed below:
 - 1. If the number of steps in a gear box is z, and the number of stages of the gear box is n, draw (n + 2) vertical lines at a convenient spacing. The one extra line as compared to the structure diagram would represent the motor shaft.
 - 2. If the input speed from the motor is less than the maximum speed, draw z horizontal lines, whereas if the input speed from the motor is more than the maximum speed, draw as many horizontal lines as necessary to locate the motor speed on the diagram. The spacing between the horizontal lines is taken as log ϕ , just as was the case in structure diagram. Thus,

taken as log φ , just as was the case in structure diagram. Thus,				
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If $N_m < N_{max}$, z number of Horizontal Lines and,

- If $N_m > Nmax$, z number of Horizontal Lines + additional horizontal lines to locate N_m on the chart.
- 3. Using the value of GP ratio ϕ , mark the spindle speeds on the vertical shaft. Mark the various spindle speeds starting with minimum speed at the lowermost line.
- 4. Draw the rays depicting transmission between the last shaft and the shaft preceding it. The rays are drawn for the lowest rpm of the last shaft.

Comparison Structure Diagram		Speed Diagram
Parameter		
Vertical Lines	The structure diagram contains	The speed diagram contains one
	(N + 1) vertical lines for input	vertical line for electric motor
	shaft, intermediate shafts and	shaft and (N + 1) vertical lines for
	output shaft.	input shaft, intermediate shafts
		and output shafts.
Horizontal Line	The structure diagram contains	The speed diagram contains the
	the number of horizontal lines	number of horizontal lines
	equal to the number of spindle	required for locating all spindle
	speed steps.	speeds as well as electric motor
		speed.
Spindle Speeds (n)	The structure diagram does not	The speed diagram gives all
	give the spindle speeds.	spindle speeds.
Range Ratio of	The structure diagram does not	The speed diagram gives the
Spindle Speeds (R_n)	give the range ratio of spindle	range ratio of spindle speeds.
	speeds.	
Electric Motor Speed	The structure diagram does not	The speed diagram gives the
	give the electric motor speed.	electric motor speed.

Table 2.2 Comparison between Structure Diagram and Speed (Ray) Diagram:

2.6 RULES AND GUIDELINES FOR GEAR BOX LAYOUT

- Once, the structure and the speed diagrams are finalised, a rough layout of the gears should be made.
- This layout would then be used in determining the number of teeth on each gear, on the basis of the speeds calculated from the speed diagram.
- The gears, shafts and other components can then be designed from strength considerations.
- While developing the gear layout, the following rules should be observed :

I. Minimum Number of Teeth on Gears:

The minimum number of teeth on gears to avoid undercutting or interference should be determined. For a 20° pressure angle, full depth involute profile, the minimum number of teeth to avoid interference is taken as about 18. Hence, for any gear of this kind on the gear box, $Z_{\min} \geq 18$.

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II. Sum of Number of Teeth on Gear Pairs for Parallel Shafts should be the same:

If the gear pairs on parallel shafts have the same module, then the sum of the number of gears on each gear pair should be the same. Consider the gear arrangement shown in Fig. 2.6.



Fig. 2.6 Mating Gears on Parallel Shafts

The modules of all the gears are the same. The centre distance in such a case would be 'a'.

$$a = \frac{mZ_1}{2} + \frac{mZ_1'}{2} = \frac{mZ_2}{2} + \frac{mZ_2'}{2} = \frac{mZ_3}{2} + \frac{mZ_3'}{2}$$

$$\therefore a = \frac{m(Z_1 + Z_1')}{2} = \frac{m(Z_2 + Z_2')}{2} = \frac{m(Z_3 + Z_3')}{2}$$

$$\therefore Z_1 + Z_1' = Z_2 + Z_2' = Z_3 + Z_3' = \frac{2a}{m} = C$$

$$Also, \frac{Z_1'}{Z_1} = i_1, \quad \frac{Z_2'}{Z_2} = i_2, \quad \frac{Z_3'}{Z_3} = i_3,$$

where, i_1 , i_2 and i_3 are gear ratios, which are known values as the speeds of individual gears on engagement is known. Thus, the maximum gear ratio (speed increase or reduction) can be read and for that gear pair, the pinion may be chosen with 18 teeth. The rest of the gear teeth can then be calculated keeping in mind the above equations.

III. Check on Percentage Deviation of Actual and Theoretical speeds:

While calculating, the number of teeth as suggested above, it is possible that the number of teeth calculated may not be integers. In such cases rounding off is done. This rounding off changes the gear ratio values, which in turn would change the values of speed obtained. This deviation between actual speeds obtained and the

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theoretically calculated values can be plotted in a graph called as deviation diagram as shown in Fig. 2.7.



Fig. 2.7 Deviation Diagram

Thus, If N_{act} and N_{th} are the actual and theoretical speeds respectively, the deviation in speed ΔN would be calculated as:

$$\Delta N = N_{act} - N_{th}$$

These deviations can be positive or negative. For best results, the sum of all such deviations should be approximately zero.

$$\sum \Delta N \approx 0$$

The percentage speed deviation can then be calculated as:

$$\%\Delta N = \frac{N_{act} - N_{th}}{N_{th}} \times 100 = \frac{\Delta N}{N_{th}} \times 100$$

The percentage speed deviation at all times should be less than $\pm 10 (\phi - 1)$.

IV. The Spacing between Two Adjacent Gears:

The spacing between two adjacent gears on a shaft should be such that one gear pair gets completely disengaged, before the next begins to mesh. Consider the arrangement shown in Fig. 2.8 (i) and (ii).

In arrangement (i), it can be seen that both the gear pairs are simultaneously meshing which would lead to gear failures. In arrangement (ii), the spacing is greater than two times the face width (b) and hence, the first gear is completely disengaged, before the second gear can start meshing. Thus, the spacing between two adjacent gears should be:





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2.7 Procedure for Designing Multi – Speed Gear Box

The following procedure is adopted in designing of multi-speed gear box.

1. The data Input:

Minimum diameter of workpiece to be machined ' D_{min} ' and corresponding maximum cutting speed ' V_{max} '.

Maximum diameter of workpiece to be machined ' D_{max} ' and corresponding minimum cutting speed ' V_{min} '.

2. Calculate maximum and minimum spindle speeds (N_{max} and N_{min}):

The maximum and minimum spindle speeds are calculated by using equations:

$$N_{\max} = \frac{1000 \cdot V_{\max}}{\pi D_{\min}} \qquad \qquad N_{\min} = \frac{1000 \cdot V_{\min}}{\pi D_{\max}}$$

3. Calculate range ratio of spindle speeds (R_N):

Calculate the range ratio of spindle speeds by using equation:

$$R_N = \frac{N_{\text{max}}}{N_{\text{min}}}$$

4. Select geometric progression ratio (φ):

Select the geometric progression ratio '¢'.

5. Calculate the total number of spindle speed steps (z):

Calculate the total number of spindle speed steps by using equation:

$$z = \frac{\log(R_N\phi)}{\log\phi}$$

6. List all the spindle speeds from N_{min} to N_{max} :

List all the spindle speeds N_1 , N_2 , N_3 ... N_z .

7. Decide number of stages (transmission groups) in gear box (r):

Estimate the number of stages (transmission groups) in gear box.

8. Write structural formula and construct corresponding structure diagram:

Write the structural formula $z = p_1(x_1) p_2(x_2) p_3(x_3) \dots P_n(x_n)$ and draw the corresponding structure diagram.

9. Select electric motor speed:

Knowing the speed of input shaft, select the standard electric motor speed.

10. Construct speed diagram:

Construct speed diagram from the structure diagram.

11. Construct the kinematic (gearing) diagram for the gear box:

The gearing diagram is constructed by calculating the number of teeth on different gears.

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Example 2.1

Draw the structure diagrams and gear box layout for the following equations.

- a) 2(1) 2(2)
- b) 3(1) 2(3)
- c) 2(1) 2(2) 2(4)
- d) 3(1) 2(3) 2(6)

Solution:

```
a) 2(1) 2(2)
```





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b) 3(1) 2(3)



Fig. 2.12 Gear Box Layout

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c) 2(1) 2(2) 2(4)



Fig. 2.13 Structure Diagram





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d) 3(1) 2(3) 2(6)



Fig. 2.16 Gear Box Layout

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Example 2.2

A 2 x 2 drive is required to be designed for transmitting speeds starting from 400 rpm with a geometric progression ratio of 1.4. Draw a suitable structure and speed diagram. Also draw the layout of the gear box and determine the number of teeth on each gear. Solution:

The design would be for a 2 stage, 4 speed gear box.

The fours speeds on the output shaft can be calculated as under:

$$N_{1} = 400 \text{ rpm}$$

$$N_{2} = N_{1} \phi = 400 \text{ x } 1.4 = 560 \text{ rpm}$$

$$N_{3} = N_{2} \phi = 560 \text{ x } 1.4 = 784 \text{ rpm}$$

$$N_{4} = N_{3} \phi = 784 \text{ x } 1.4 = 1097.6 \text{ rpm}$$
Stage 1 Stage 2
I
II
II
II
II
2(1) 2(2)

Fig. 2.17 Structure Diagram

We now draw the speed diagram for the same. The diagram is drawn, keeping in mind that the shaft preceding the output shaft III has the maximum speed. This would ensure lesser torque and hence a smaller and compact design. In the present case, the motor shaft speed is not known.



Fig. 2.18 Speed Diagram

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Fig. 2.19 Gear Box Layout

It can be seen that at stage I, the input speed is 1097.6 rpm. This would be the speeds for gears *a* and *b*. The output speeds from this stage as seen from the speed diagram would be $N_3 = 784$ rpm and $N_4 = 1097.6$ rpm, which would be the speeds of gears *a'* and *b'* respectively. The gear ratio can then be calculated as under:

$$\frac{Z_a'}{Z_a} = \frac{N_a}{N_a'} = \frac{1097.6}{784} = 1.4$$
$$\frac{Z_b'}{Z_b} = \frac{N_b}{N_b'} = \frac{1097.6}{1097.6} = 1$$

For the two gear pairs a - a' and b - b' to lie on the parallel shafts, the sum of the number of teeth on both should be the same. The largest gear ratio in this case would be for gears a and a'. Hence the number of teeth on these gears should be determined. The pinion in this case would be gear a. Let the number of teeth on gear a be Z_a .

$$Z_a '=1.4Z_a$$

$$Z_b = Z_b '$$

$$Z_b + Z_b '= Z_a + Z_a '= Z_a + 1.4Z_a = 2.4Z_a$$

$$\therefore 2Z_b = 2.4Z_a$$

$$\therefore Z_b = 1.2Z_a$$

To determine the integer value of number of teeth on gears of stage I, tabulate as under:

Z_a	$Z_{a}' = 1.4 Z_{a}$	$Z_a + Z_a'$	$Z_{b} = 1.2 Z_{a}$	$Z_{b}' = 1.2 Z_{b}$	$Z_b + Z_b'$
18	25.2 ≈ 25	43	21.6 ≈22	22	44
19	26.6 ≈27	46	22.8 ≈23	23	46
20	28	48	24	24	48
	$Z_a = 20$				
$Z_a'=28$					
	$Z_{b} = Z_{b}' = 24$				

The same logic used earlier can now be used to determine the number of teeth in stage 2. It can be seen that at stage 2, the input speeds are 784 rpm and 1097 .6 rpm. These would be the speeds for gears c and d.

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$$\frac{Z_c'}{Z_c} = \frac{N_c}{N_c'} = \frac{1097.6}{1097.6} = \frac{784}{784} = 1$$
$$\frac{Z_d'}{Z_d} = \frac{N_d}{N_d'} = \frac{784}{400} = \frac{1097.6}{560} = 1.96$$

The largest gear ratio in this case would be for gears d and d'. Hence the number of teeth on these gears should be determined. The pinion in this case would be gear d. Let the number of teeth on gear d be Z_d .

$$Z_{c} = Z_{c} '$$

$$Z_{d} '=1.96Z_{d}$$

$$Z_{c} + Z_{c} '= Z_{d} + Z_{d} '= Z_{d} + 1.96Z_{d} = 2.96Z_{d}$$

$$\therefore 2Z_{c} = 2.96Z_{d}$$

$$\therefore Z_{c} = 1.48Z_{d}$$

To determine the integer value of number of teeth on gears of stage 2, tabulate as under. It can be seen that in this case to get exact gear ratio, a large size pinion may be required. To prevent that, some approximations may be taken. This would change the actual speeds obtained at the spindle end from the theoretically calculated speeds.

Zd	$Z_{d}' = 1.96 Z_{d}$	$Z_d + Z_d'$	$Z_{c} = 1.48 Z_{d}$	$Z_c' = Z_c$	$Z_c + Z_c'$
18	35.28 ≈ 35	53	26.64 ≈27	27	54
19	37.24 ≈37	56	28.12 ≈28	28	56
20	39.2 ≈ 39	59	29.6 ≈30	30	60
	$Z_{d} = 19$				
	Z _d '=37				
	$Z_{c} = Z_{c}' = 28$	3			

Example 2.3

A nine speed gear box is to be connected to a motor running at 720 rpm through a belt drive. The gear box is to have a minimum speed of 31.5 rpm and a maximum speed of 500 rpm. Using standard spindle speeds,

- a) Draw the structure and speed diagram for the arrangement.
- b) Draw the gear box layout
- c) Select suitable standard pulley diameters for connecting the motor to the gear box shaft.

The standard pulley diameters are based on R20 series with a diameter starting from 80 mm.

Solution:

The design would be for a 2 stage, 9 speed gear box.

$$\begin{split} N_{M} &= 720 \text{ rpm} \\ N_{1} &= N_{min} = 31.5 \text{ rpm} \\ N_{z} &= N_{max} = 500 \text{ rpm} \end{split}$$

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$$\phi = \left(\frac{N_{\text{max}}}{N_{\text{min}}}\right)^{\frac{1}{z-1}} = \left(\frac{500}{31.5}\right)^{\frac{1}{9-1}} = 1.4128$$

The nine speeds on the output shaft can be calculated as under:



Fig. 2.21 Speed Diagram

It is given that a belt drive is used to connect the motor with the input shaft of the gear box. The motor speed was $N_M = 720$ rpm, whereas the input shaft speed from the speed diagram is $N_I = 500$ rpm. Thus, if the pulley connected to the input shaft has a diameter D_M and that connected of the shaft I of the gear box has a diameter D_I , then the pulley diameters would be in the ratio,

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$$\frac{D_I}{D_M} = \frac{N_M}{N_I} = \frac{720}{500} = 1.44$$

: $D_I = 1.44 D_M$

The pulleys are in R20 series starting with a minimum diameter of 80 mm. Hence standard sized pulleys would be found by first determining the GP ratio.

$$\phi_p = \sqrt[20]{10} = 1.125$$

Standard Pulley diameters would be

= 80 mm

$80 \times 1.125 = 90 \text{ mm}$
$90 \times 1.125 \approx 100 \text{ mm}$
$100 \times 1.125 \approx 112 \text{ mm}$
112 × 1.125 ≈ 125 mm
$125 \times 1.125 \approx 140 \text{ mm}$
$140 \times 1.125 \approx 160 \text{ mm}$
$160 \times 1.125 \approx 180 \text{ mm}$
$180 \times 1.125 \approx 200 \text{ mm}$

Assuming a minimum pulley diameter D_M , the input shaft pulley diameter should be calculated till the desired standard pulley is available. The procedure as shown:

D _M	$D_{I} = 1.44 D_{M}$
80	115.2
90	129.6
100	144
112	161.28
125	180 🗸

Thus the pulley diameter on the motor would be 125 mm and that on the input shaft of the gear box will be 180 mm. This would ensure that the input shaft speed would be 500 rpm.



Fig. 2.22 Gear Box Layout

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Example 2.4

A three stage gear box with twelve speeds is to be designed based on R10 series with a minimum spindle speed of 125 rpm. The electric motor is connected to the gear box through a belt drive and runs at 1440 rpm and transmits power of 5 kW. Using standard spindle speeds,

- a) Draw the structure and speed diagram for the arrangement.
- b) Determine the ratio of the belt pulley diameters.
- c) Draw the gear box layout.

Solution:

The design would be for a 3 stage, 12 speed gear box.

$$\phi = \sqrt[10]{10} = 1.26$$

N₁ = N_{min} = 125 rpm
N_M = 1440 rpm
kW = 5

The twelve standard spindle speeds on the output shaft can be calculated as under:

N ₁ = 125 rpm		
$N_2 = N_1 \varphi$	= 125 x 1.26	= 160 rpm
$N_3 = N_2 \varphi$	= 160 x 1.26	= 200 rpm
$N_4 = N_3 \phi$	= 200 x 1.26	= 250 rpm
$N_5 = N_4 \phi$	= 250 x 1.26	= 315 rpm
$N_6 = N_5 \phi$	= 315 x 1.26	= 400 rpm
N ₇ = N ₆ φ	= 400 x 1.26	= 500 rpm
N ₈ = N ₇ φ	= 500 x 1.26	= 630 rpm
N ₉ = N ₈ φ	= 630 x 1.26	= 800 rpm
N ₁₀ = N ₉ φ	= 800 x 1.26	= 1000 rpm
N ₁₁ = N ₁₀ φ	= 1000 x 1.26	= 1250 rpm
$N_{12} = N_{11} \phi$	= 1250 x 1.26	= 1600 rpm
Stage	1 Stage 2 Stage 3	3
I [1 V 7
	/	,
		*
		*
	+///	*
	\star	7
	KXX	4
		J



Fig. 2.23 Structure Diagram

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Fig. 2.24 Speed Diagram

It is given that a belt drive is used to connect the motor with the input shaft of the gear box. The motor speed was $N_M = 1440$ rpm, whereas the input shaft speed from the speed diagram is $N_I = 1600$ rpm. Thus, if the pulley connected to the input shaft has a diameter D_M and that connected of the shaft I of the gear box has a diameter D_I , then the pulley diameters would be in the ratio,

$$\frac{D_{I}}{D_{M}} = \frac{N_{M}}{N_{I}} = \frac{1440}{1600} = 0.9$$

$$\therefore D_{I} = 0.9D_{M}$$
Input \Box I Shaft \Box I Stage 1
II Shaft \Box I Stage 2
III Shaft \Box I Stage 2
III Shaft \Box I Stage 3
Output Shaft

Fig. 2.25 Gear Box Layout

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