## A

## Laboratory Manual for

# Dynamics of Machinery (3151911) 

B.E. Semester 5
(Mechanical)


Directorate of Technical Education
Gandhinagar, Gujarat

# L. E. College - Morbi MECHANICAL ENGINEERING DEPARTMENT 

## CERTIFICATE

This is to certify that Mr./Miss $\qquad$ of Mechanical Branch, Sem-V, Enrollment No.
has satisfactorily completed his/her term work for the subject Dynamics of Machinery (3151911) during odd term 2023-24.

Date:

Sign of Faculty
Head of the Department

## Preface

Main motto of any laboratory/practical/field work is for enhancing required skills as well as creating ability amongst students to solve real time problems by developing relevant competencies in the psychomotor domain. By keeping in view, GTU has designed competency focused outcome-based curriculum for engineering degree programs where sufficient weightage is given to practical work. It shows the importance of enhancement of skills amongst the students and it pays attention to utilize every second of time allotted for practical amongst students, instructors and faculty members to achieve relevant outcomes by performing the experiments rather than having merely study type experiments. It is essential for effective implementation of competency focused outcome-based curriculum that every practical is keenly designed to serve as a tool to develop and enhance relevant competency required by the various industries among every student. These psychomotor skills are very difficult to develop through traditional chalk and board content delivery methods in the classroom. Accordingly, this lab manual is designed to focus on the industry-defined relevant outcomes, rather than the old practice of conducting practical to prove concepts and theories.

By using this lab manual students can go through the relevant theory and procedure in advance before the actual performance which creates an interest and students can have basic ideas prior to performance. This in turn enhances predetermined outcomes amongst students. Each experiment in this manual begins with competency, industry relevant skills, course outcomes as well as practical outcomes (objectives). The students will also achieve safety and necessary precautions to be taken while performing practical.

This manual also provides guidelines to faculty members to facilitate student centric lab activities through each experiment by arranging and managing necessary resources in order that the students follow the procedures with required safety and necessary precautions to achieve the outcomes. It also gives an idea of how students will be assessed by providing rubrics.
Dynamics of machinery is the course designed to give fundamental knowledge of behavior of machines under the influence of external-time varying loads. It provides a platform for the students to correlate theoretical concepts with the practical applications thereof, by performing experiments. They will learn the basics of static and dynamic balancing, and their effects on dynamic response on structures resulting in mechanical vibrations. Students will get insight into mathematical modeling of real-life complex systems and their analysis for natural-damped and undamped vibrations, forced-damped and undamped vibrations. Moreover, they will also learn about the behavior of rotating shafts and validate theoretically evaluated critical speeds of shafts. They will also be able to understand gyroscopic effects in automobiles, ships, air-planes.
Utmost care has been taken while preparing this lab manual however there are always chances of improvement. Therefore, we welcome constructive suggestions for improvement and removal of errors if any.

## Practical - Course Outcome matrix

Course Outcomes (COs):

| $\mathbf{S r} \mathbf{~ N o}$ | CO statement |
| :--- | :--- |
| CO-1 | Summarize dynamic forces and turning moments in mechanisms. |
| CO-2 | Minimize unbalance in mechanical systems by means of static and dynamic balancing. |
| CO-3 | Analyze gyroscopic effects in airplanes, ships and automobiles. |
| CO-4 | Demonstrate longitudinal vibrations, transverse vibrations and torsional vibrations in <br> single degree of freedom systems. |
| CO-5 | Determine critical speed of the shaft. |


| Sr. | Objective(s) of Experiment | $\mathrm{CO1}$ | CO2 | CO3 | CO4 | C05 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | To study the balancing of several masses rotating in a single plane-Static Balancing, analytically and experimentally. |  | $\checkmark$ |  |  |  |
| 2. | To study the balancing of several masses rotating in different planes - Dynamic Balancing, analytically and experimentally. |  | $\checkmark$ |  |  |  |
| 3. | To study balancing of reciprocating masses analytically and experimentally. |  | $\checkmark$ |  |  |  |
| 4. | To understand and analyze the gyroscopic effects using the motorized gyroscope. |  |  | $\checkmark$ |  |  |
| 5. | To demonstrate longitudinal vibration of helical spring and to determine natural frequency of vibration oscillation analytically and experimentally. |  |  |  | $\checkmark$ |  |
| 6. | To study the natural frequency of undamped torsional vibration for a single rotor shaft system and to demonstrate undamped torsional vibration. |  |  |  | $\checkmark$ |  |
| 7. | To study the natural frequency of undamped torsional vibration for a two-rotor shaft system and to demonstrate undamped torsional vibration. |  |  |  | $\checkmark$ |  |


| 8. | To study free damped vibrations of a single degree <br> of freedom system. and to demonstrate free damped <br> vibrations. |  |  |  | $\sqrt{ }$ |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 9. | To study the dynamic forces and turning moments in <br> mechanisms analytically. | $\sqrt{ }$ |  |  |  |  |
| 10. | To study and analyze the single DOF forced <br> vibration system at various damping ratios and <br> frequency ratios. |  |  |  | $\sqrt{ }$ |  |
| 11. | To determine critical speed of shaft theoretically and <br> experimentally. |  |  |  |  | $\sqrt{ }$ |

## Industry Relevant Skills

The following industry-relevant competencies are expected to be developed in the students by undertaking the practical work of this laboratory.

1. Field balancing of electric motor rotors, rotary (single stage/multi-stage) pump shafts, rope drums, and reciprocating HP \& LP boiler dosing pumps, IC engines etc.
2. Identification of major causes of mechanical vibrations and to take remedial steps or corrective actions to keep machinery safe within their operating range.

## Guidelines for Teachers

1. Teachers should provide the guideline with the demonstration of practicality to the students with all features.
2. Teacher shall explain basic concepts/theory related to the experiment to the students before starting of each practical.
3. Involve all the students in the performance of each experiment.
4. Teachers are expected to share the skills and competencies to be developed in the students and ensure that the respective skills and competencies are developed in the students after the completion of the experimentation.
5. Teachers should give opportunities to students for hands-on experience after the demonstration.
6. Teachers may provide additional knowledge and skills to the students even though not covered in the manual but are expected from the students by the concerned industry.
7. Give practical assignments and assess the performance of students based on tasks assigned to check whether it is as per the instructions or not.
8. Teacher is expected to refer to the complete curriculum of the course and follow the guidelines for implementation.

## Instructions for Students

1. Students are expected to carefully listen to all the theory classes delivered by the faculty members and understand the Course Outcomes (COs), content of the course, teaching and examination scheme, skill set to be developed, etc.
2. Students shall organize the work in the group and make a record of all observations.
3. Students shall develop maintenance skills as expected by industries.
4. Students shall attempt to develop related hands-on skills and build confidence.
5. Students shall develop the habits of evolving more ideas, innovations, skills, etc. apart from those included in the scope of the manual.
6. Students shall refer to technical magazines and data books.
7. Students should develop a habit of submitting the experimentation work as per the schedule and $\mathrm{s} / \mathrm{he}$ should be well prepared for the same.

## Common Safety Instructions

The department is always intended to ensure the safe operation of the laboratories. Students are expected to conduct experiments in a safe manner respecting the physical well-being of their fellow students and themselves. Students should read and understand all contents which are highlighted in the laboratories in the department.

## GENERAL LABORATORY RULES:

[] BE PREPARED. Read and fully comprehend the lab procedure as set forth in the lab manual before you begin any experiment. If you do not understand the procedure, see your instructor/faculty.
[] THINK SAFETY. Work deliberately and carefully. No horseplay
[] ALL LABORATORY STUDENTS MUST BE SUPERVISED Never work alone
[] KNOW THE HAZARDS OF ANY MATERIALS OR MACHINERY YOU ARE WORKING WITH. The laboratory manual and/or instructor will review specific safety issues on individual experiments before you perform any tests.
[] ALL STUDENTS MUST WEAR APPROPRIATE SAFETY EQUIPMENT. Safety equipment must be utilized based on specific experiment requirements.
[] ALL STUDENTS MUST WEAR APPROPRIATE LABORATORY ATTIRE. No open toed shoes; no loose-fitting clothing; Jewelry should be removed; long hair should be tied back
[] NO FOOD OR BEVERAGE IN THE LABORATORY.
[] KNOW EMERGENCY PROCEDURES. Make note of fire escape routes and emergency phone locations.
[] REPORT ANY PERCEIVED SAFETY HAZARDS. Immediately report any spills, equipment malfunctions, injuries or other perceived safety hazards to your Instructor / TA / or staff member.
[] KEEP YOUR WORK AREA CLEAN.
FAILURE TO CONFORM WITH ANY OF THE ABOVE RULES MAY RESULT IN NOT BEING ALLOWED TO PARTICIPATE IN THE LABORATORY EXPERIMENT.

Rubric for Dynamics of Machinery Lab Experiment

| Sr. <br> No. | Criteria | Excellent $>80 \%$ | $\begin{gathered} \text { Very Good }>70 \\ \% \end{gathered}$ | Good $>\mathbf{6 0 \%}$ | Fair $>40 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 4 | 3 | 2 | 1 |
| 1 | C1- <br> Interpretation and understanding objective of experiment. <br> 25\% Marks | Interpret and understand experiments accurately. | Interpret and understand experiments with few errors. | Interpret practical but difficult to understand experiments. | Difficulties in Interpreting practical and understanding experiments. |
| 2 | C2-Clearity in Practical concepts 25\% Marks | Students can thoroughly understand the concept of practical and clarity in the practical. | Students can understand the concept of practical and clarity in the practical. | Students can partially understand the concept of practical and partially understanding the practical. | Poor understanding of concept and lack of attitude for clarity in practical |
| 3 | C3-Ethical conduct and Teamwork in Lab session 25\% Marks | Always shows ethical conduct \& attitude of working in a team. | Shows ethical conduct \& attitude of working in a team many times. | Shows ethical conduct \& attitude of working in a team sometimes. | Shows ethical conduct sometimes but does not perform any duties of the assigned role. |
| 4 | C4-Timely completion and quality of lab report <br> 25\% Marks | All captions and readings are accurate with neat diagrams/graphs and submits in time. | All captions and readings are accurate with neat diagrams/graphs but do not submit in time. | Few <br> captions/readings /diagrams/graphs need correction and do not submit in time. | Many captions/readings /diagrams/graphs need correction and do not submit in time. |

## Index

(Progressive Assessment Sheet)

| $\begin{array}{\|l} \text { Sr. } \\ \text { No. } \end{array}$ | Objective(s) of Experiment | Page No. | Date of Performance | Date of Submission | Marks | Sign. | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | To study the balancing of several masses rotating in a single planeStatic Balancing, analytically and experimentally. |  |  |  |  |  |  |
| 2. | To study the balancing of several masses rotating in different planes - Dynamic Balancing, analytically and experimentally. |  |  |  |  |  |  |
| 3. | To study balancing of reciprocating masses analytically and experimentally. |  |  |  |  |  |  |
| 4. | To understand and analyze the gyroscopic effects using the motorized gyroscope. |  |  |  |  |  |  |
| 5. | To demonstrate longitudinal vibration of helical spring and to determine natural frequency of vibration oscillation analytically and experimentally. |  |  |  |  |  |  |
| 6. | To study the natural frequency of undamped torsional vibration for a single rotor shaft system and to demonstrate undamped torsional vibration. |  |  |  |  |  |  |
| 7. | To study the natural frequency of undamped torsional vibration for a two-rotor shaft system and to demonstrate undamped torsional vibration. |  |  |  |  |  |  |
| 8. | To study free damped vibrations of a single degree of freedom system. and to demonstrate free damped vibrations. |  |  |  |  |  |  |
| 9. | To study the dynamic forces and turning moments in mechanisms analytically. |  |  |  |  |  |  |


| 10. | To study and analyze the single <br> DOF forced vibration system at <br> various damping ratios and <br> frequency ratios. |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 11. | To determine critical speed of <br> shaft theoretically and <br> experimentally. |  |  |  |  |  |

## Experiment No: 1

## To study the balancing of several masses rotating in a single plane-Static Balancing, analytically and experimentally.

## Date:

Competency and Practical Skills: Students will be able to understand the fundamentals of static balancing and will be able to analyze the analytical, and graphical results with experiment/simulation results.
Relevant CO: 2

## Objectives:

a) Understand and verify the fundamental laws of static balancing.
b) Calculate the mass and angular position required for balancing the unbalanced force.
c) Analyze the analytical results with the simulation results.
d) Evaluate how a change in mass and position can affect the balance of the rotating body.

## Equipment/Instruments:

Dynamic balancing apparatus having variable speed drive motor with controller,
Known balancing masses (eg. $50 \mathrm{~g}, 100 \mathrm{~g}, 150 \mathrm{~g}, 200 \mathrm{~g}$ etc.),
High precision digital weighing scale and
Bolts-nuts and Spanner set.

## Theory:

The high speed of engines and other machines is a common phenomenon nowadays. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not correctly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members but also produce unpleasant and even dangerous vibrations.

## Balancing of Rotating Masses:

We have already discussed, that whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal force of both masses is made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called the balancing of rotating masses.

The following cases are essential from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing a single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different planes.

## Static Balancing:

Consider a disturbing mass $m_{1}$ attached to a shaft rotating at $\omega \mathrm{rad} / \mathrm{s}$ as shown in Figure 1. Let $r_{1}$ be the radius of rotation of the mass $m_{1}$ (i.e., the distance between the axis of rotation of the shaft and the centre of gravity of the mass $m_{1}$ ).
We know that the centrifugal force exerted by the mass $m_{1}$ on the shaft,

$$
\begin{equation*}
F_{c 1}=m_{1} * \omega * r_{1} \tag{1}
\end{equation*}
$$



Figure 1 Balancing of a single rotating mass by a single mass rotating in the same plane.
This centrifugal force acts radially outwards and thus produces a bending moment on the shaft. In order to counteract the effect of this force, a balancing mass $\left(m_{2}\right)$ may be attached in the same plane of rotation as that of disturbing mass $\left(m_{1}\right)$ such that the centrifugal forces due to the two masses are equal and opposite.

Let, $r_{2}=$ Radius of rotation of the balancing mass $m_{2}$ (i.e., distance between the axis of rotation of the shaft and the centre of gravity of mass $m_{2}$ ).

Centrifugal force due to mass $m_{2}$,

$$
\begin{equation*}
F_{c 2}=m_{2} * \omega * r_{2} \tag{2}
\end{equation*}
$$

Equating equations (1) and (2),

$$
\begin{equation*}
m_{1} * \omega * r_{1}=m_{2} * \omega * r_{2} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
m_{1} * r_{1}=m_{2} * r_{2} \tag{4}
\end{equation*}
$$

## Part A

## Balancing of Several Masses Rotating in Same Plane: -

Balancing of several masses can be done by two methods:

1. Analytical method
2. Graphical method

Consider any number of masses (say four) of magnitude $m_{1}, m_{2}, m_{3}$ and $m_{4}$ at distances of $r_{1}, r_{2}, r_{3}$ and $r_{4}$ from the axis of the rotating shaft. Let $\theta_{1}, \theta_{2}, \theta_{3}$ and $\theta_{4}$ be the angles of these masses with the horizontal line $O X$, as shown in Figure 2. Let these masses rotate about an axis through $O$ and perpendicular to the plane of paper, with a constant angular velocity of $\omega$ $\mathrm{rad} / \mathrm{sec}$. The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below:


Figure 2 Space diagram.

## Analytical Method: -

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below:
a) First of all, find out the centrifugal force (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.
b) Resolve the centrifugal forces horizontally and vertically and find their sums, i.e., $\sum H$ and $\sum V$.
c) We know that sum of horizontal components of the centrifugal forces,

$$
\begin{equation*}
\sum H=m_{1} * r_{1} * \cos \theta_{1}+m_{2} * r_{2} * \cos \theta_{2}+\ldots \ldots \tag{5}
\end{equation*}
$$

d) The sum of vertical components of the centrifugal forces,

$$
\begin{equation*}
\sum V=m_{1} * r_{1} * \sin \theta_{1}+m_{2} * r_{2} * \sin \theta_{2}+\ldots \ldots \tag{6}
\end{equation*}
$$

e) Magnitude of the resultant centrifugal force

$$
\begin{equation*}
F_{c}=\sqrt{\left(\sum H\right)^{2}+\left(\sum V\right)^{2}} \tag{7}
\end{equation*}
$$

f) If $\theta$ is the angle, which the resultant force makes with the horizontal, then

$$
\begin{equation*}
\tan \theta=\frac{\sum V}{\sum H} \tag{8}
\end{equation*}
$$

g) The balancing force is then equal to the resultant force, but in the opposite direction.
h) Now find out the magnitude of the balancing mass, such that

$$
\begin{equation*}
F_{c}=m * r \tag{9}
\end{equation*}
$$

where, $m=$ Balancing mass, and $r=$ Its radius of rotation

## Graphical Method: -

The magnitude and position of the balancing mass may also be obtained graphically as discussed below:
a) First of all, draw the space diagram with the positions of the several masses, as shown in Figure 2
b) Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
c) Now draw the vector diagram with the obtained centrifugal forces (or the product of the Masses and their radii of rotation), such that $\boldsymbol{a b}$ represents the centrifugal force exerted by the mass $m_{1}$ (or $m_{1} r_{1}$ ) in magnitude and direction to some suitable scale. Similarly, bc, $\boldsymbol{c d}$ and $\boldsymbol{d e}$ to represent centrifugal forces of other masses $m_{2}, m_{3}$ and $m_{4}$ (or $m_{2} r_{2}, m_{3} r_{3}$ and $m_{4} r_{4}$ )
d) Now, as per polygon law of forces, the closing side ae represents the resultant force in Magnitude and direction, as shown in Figure 3.
e) The balancing force is, then, equal to the resultant force, but in the opposite direction.
f) Now find out the magnitude of the balancing mass ( $m$ ) at a given radius of rotation (r) such that,
$m \cdot \omega^{2} \cdot r=$ Resultant centrifugal force
$m . r=$ Resultant of $m_{1} r_{1}, m_{2} r_{2}, m_{3} r_{3}$ and $m_{4} r_{4}$


Figure 3 Vector diagram.

Set up diagram: Draw below a line sketch of the experimental setup.

## Safety and necessary Precautions:

1. Keep safe distance from the rotating discs and motor-pulley-belt sets. Do not start the electric motor before ensuring tight assembly of the balancing masses to be attached to the disc/s.
2. Observe the behavior of the system by keeping close watch over the effect of unbalanced masses. There may be detrimental vibrations due to unknowingly large unbalanced masses, if attached mischievously.
3. Stop the apparatus immediately if found some mal-functioning of the system. Re-run the setup after carefully inspecting and overcoming the problems, if any.

## Procedure: -

[] First of all, arrange the set up.
[] From the given data calculate the required balancing mass at a particular radius for the vibrating system.
[] Check out the system under normal conditions.
[] Attach masses at a given radius and at a given angular position from the data.
[] Observe the unbalanced force causing the system to vibrate.
[] Now attach the balancing mass that we have calculated from observation data
[] Again, observe the system and difference of vibration of setup before and after attaching the balancing mass.

## Observation Table: -

| Sr. No. | Mass <br> $m(k g)$ | Radius of rotation <br> $\boldsymbol{r}(\mathbf{c m})$ | Angular position <br> $\boldsymbol{\theta}$ (degrees) | $\boldsymbol{m} * \boldsymbol{r}$ <br> $(\mathrm{~kg}-\mathrm{cm})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |

## Calculation: -

## Result Table: -

| Sr. <br> No | Balance mass <br> $(\mathbf{k g})$ | Balance mass angular position <br> $(\theta)$ | Balance mass radial position <br> $(\mathbf{r})$ |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

## Conclusion:

## Part B - Virtual Lab simulation

(V-Lab- link - https://dom-nitk.vlabs.ac.in/exp/multiple-mass-in-single-plane/)

## Experimental setup and procedure

[1 In the simulation window, the front view and side view of a rotating shaft with two unbalanced masses in different planes are displayed.
[7 There are pointers given on the right side of the screen for choosing the values of input variables, viz., two unbalanced masses on the shaft $\left(m_{1}, m_{2}\right)$, their radii of rotation $\left(r_{1}, r_{2}\right)$, relative angular position (thetal) and distance between their planes $\left(l_{1}\right)$ as shown in Figure 4. The variables can be set by moving the slider left or right. After setting these variables, it is required to move on to the next pane by clicking on the navigation button at the bottom right corner.


Figure 4 Simulation Window.
[0 After moving on to the next page, the radii and positions of the balancing masses have to be entered by adjusting the pointer as shown in Figure 5. After setting the values, it is required by the user to calculate the values of balancing masses and their angular position.
[7 Click on submit to submit the results and press the play/pause button to pause the simulation as displayed in Figure 5
[7] Compare analytically calculated results and the error in the result with the simulation results, displayed at the bottom of the simulator page as shown in Figure 6.


Figure 5 Balance mass and its positions.

## Balancing of Multiple Mass in Single Plane



Figure 6 Error in the result

Observation table:

| Sr. <br> No | $\mathbf{m}_{\mathbf{1}}$ | $\mathbf{m}_{\mathbf{2}}$ | $\mathbf{r}_{\mathbf{1}}$ | $\mathbf{r}_{2}$ | $\theta_{1}$ | $\mathbf{1}_{1}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

Calculations: -

## Result Table: -

| Sr. <br> No | Balance <br> mass <br> $(\mathrm{kg})$ | Balance mass <br> angular position <br> $(\theta)$ | Balance mass <br> radial position <br> $(\mathbf{r})$ | Error in mass <br> $(\%)$ | Error in <br> position <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |

## Conclusion:

$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Tutorial problems:

1. Four masses A, B, C, and D revolve at equal radii and are equally spaced along a shaft. The mass $B$ is (Last three digits of your enrolment number $x 5$ ) kg and the radii of C and D make angles of $90^{\circ}$ and $240^{\circ}$ respectively with the radius of B. Find the magnitude of the masses A, C, and D and the angular position of A so that the system may be completely balanced.

## Quiz: Answer the following questions

## (Give a neat diagram/sketch wherever necessary).

1. Why is balancing necessary for rotors of high speed engines?
2. How is balancing done practically for rotors of pump shafts and motor shafts?
3. Explain the method of wheel balancing of a passenger car.
4. What are soft bearing and hard bearing balancing machines? Explain in detail.

## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan.
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

## Rubric-wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

## Experiment No: 2

## To study the balancing of several masses rotating in different planes - Dynamic Balancing, analytically and experimentally.

## Date:

Competency and Practical Skills: Students will be able to understand the fundamentals of dynamic balancing and will be able to compare the analytical, and graphical results with experiment/simulation results.
Relevant CO: 2

## Objectives:

a) Understand and verify the fundamental laws of dynamic balancing.
b) Calculate the mass and angular position required for balancing the unbalanced force.
c) Analyze the analytical results with the simulation results.
d) Evaluate how a change in mass and position can improve the balance of the rotating body.

## Equipment/Instruments:

Dynamic balancing apparatus having variable speed drive motor with controller,
Known balancing masses (eg. $50 \mathrm{~g}, 100 \mathrm{~g}, 150 \mathrm{~g}, 200 \mathrm{~g}$ etc.),
High precision digital weighing scale and
Bolts-nuts and Spanner set.

## Theory:

When several masses revolve in different planes, they may be transferred to a reference plane (briefly written as R. P.), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitudes equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied:

1. The forces in the reference plane must balance i.e., the resultant force must be zero.
2. The couples about the reference plane must balance, i.e. the resultant couple must be zero.

Let us now consider four masses $m_{1}, m_{2}, m_{3}$ and $m_{4}$ revolving in planes $1,2,3$, and 4 respectively as shown in Figure 7 (a). The relative angular positions of these masses are shown in the end view as displayed in Figure $7(b)$. The magnitude of the balancing masses $m_{l}$ and $m_{m}$ in planes $\boldsymbol{L}$ and $\boldsymbol{M}$ may be obtained as discussed below:
a) Take one of the planes, say $L$ as the reference plane (R.P.). The distances of all the other planes to the left of the reference plane may be regarded as negative, and those to the Right as positive.
b) Tabulate the data as shown in Table 1. The planes are tabulated in the same order in which they occur, reading from left to right.
c) A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple $C_{1}$ introduced by transferring $m_{1}$ to the reference plane through $\boldsymbol{O}$ is proportional to $m_{1} \cdot r_{1} \cdot l_{1}$ and acts in a plane through $O m_{1}$ and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to as shown by in Figure 7 (c).
d) The vector $O m_{1}$ as shown by $O C_{1}$ in Figure 7 (c). Similarly, the vectors $O C_{2}, O C_{3}$ and $O C_{4}$ are drawn perpendicular to $\mathrm{Om}_{2}, \mathrm{Om}_{3}$ and $\mathrm{Om}_{4}$ respectively and in the plane of the paper.

Table 1 Detail of several masses with couple

| Plane | Mass <br> $(\mathrm{m})$ | Radius <br> $(\mathrm{r})$ | (Cent. force $\left.\div \omega^{2}\right)$ <br> (m.r) | Distance from <br> Plane L <br> $(\mathrm{l})$ | $\left(\right.$ Couple $\left.\div \omega^{2}\right)$ <br> $(\mathrm{m} \cdot \mathrm{r} \cdot \mathrm{l})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $m_{1}$ | $r_{1}$ | $m_{1} \cdot r_{1}$ | $-l_{1}$ | $m_{1} \cdot r_{1} l_{1}$ |
| L (R.P.) | $m_{L}$ | $r_{L}$ | $m_{L} \cdot r_{L}$ | 0 | 0 |
| 2 | $m_{2}$ | $r_{2}$ | $m_{2} \cdot r_{2}$ | $l_{2}$ | $m_{2} \cdot r_{2} \cdot l_{2}$ |
| 3 | $m_{3}$ | $r_{3}$ | $m_{3} \cdot r_{3}$ | $l_{3}$ | $m_{3} \cdot r_{3} \cdot l_{3}$ |
| M | $m_{M}$ | $r_{M}$ | $m_{M} \cdot r_{M}$ | $l_{M}$ | $m_{M} \cdot r_{M} \cdot l_{M}$ |
| 4 | $m_{4}$ | $r_{4}$ | $m_{4} \cdot r_{4}$ | $l_{4}$ | $m_{4} \cdot r_{4} \cdot l_{4}$ |

- ve $\longleftarrow$ R.P. $\longrightarrow+$ ve

(a) Position of planes of the masses.

(c) Couple vector.

(d) Couple vectors turned counter clockwise through a right angle.

Figure 7 Balancing of several masses rotating in different planes.
e) The couple vectors as discussed above are turned counterclockwise through a right angle for convenience of drawing as shown in Figure $7(d)$. We see that their relative position remains unaffected. Now the vectors $O C_{2}, O C_{3}$ and $O C_{4}$ are parallel and in the same direction as $O m_{2}, O m_{3}$ and $O m_{4}$ while the vector $O C_{1}$ is parallel to $O m_{4}$ but in opposite directions. Hence the couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the Reference plane.
f) Now draw the couple polygons as shown in Figure 7 (e). The vector d'o' represents the balanced couple. Since the balanced couple $C_{M}$ is proportional to $m_{M} \cdot r_{M} \cdot l_{m}$ therefore,

$$
\begin{equation*}
C_{M}=m_{M} \cdot r_{M} \cdot l_{m}=\text { vector } d^{\prime} o^{\prime} \text { or } m_{M}=\frac{\text { vector } d^{\prime} o^{\prime}}{r_{M} \cdot l_{m}} \tag{10}
\end{equation*}
$$

From this expression, the value of the balancing mass $m_{M}$ in the plane $\boldsymbol{M}$ may be obtained, the inclination $\emptyset$ of this mass may be measured from Figure 7 (b).
g) Now draw the force polygon as shown in Figure 7 (f). The vector $e o$ (in the direction from $e$ to $o$ ) represents the balanced force. Since the balanced force is proportional to therefore,

$$
\begin{equation*}
m_{L} \cdot r_{L}=\text { vector eo or } m_{L}=\frac{\text { vector eo }}{r_{L}} \tag{11}
\end{equation*}
$$

From this expression, the value of the balancing mass $m$ in the plane $L$ may be obtained and the angle of inclination $\alpha$ of this mass with the horizontal may be measured from Figure 7 (b).

## Part A

## Procedure:

The following procedure should be followed to verify the balancing conditions can be achieved by adding balancing mass.
[] Start the motor and see that the system rotates. Wait until the initial vibrations stop. Now check the unloaded system is in a balanced condition of vibration and oscillation.
[] Stop the motor and load the discs A and B and D at different points with suitable radius and weight to get the unbalanced condition.
[] Switch on the motor and observe the system is out of balance, switch off the motor
[] Place the balancing weights in places as calculated
[] Now start the motor and observe that the rotor remains horizontal hence there are no vibrations and oscillations.

Set up diagram: Draw below a line sketch of the experimental setup.

## Observations: -

Distance between the two pulleys $=$ $\qquad$
Observation Table: -

| Planes | Mass <br> m (kg) | Radius of rotation $r$ (cm) | Angular position <br> $\theta$ (degree) | Distance of planes from RP x (cm) | $\begin{gathered} \mathbf{m r s i n} \boldsymbol{\theta} \\ (\mathbf{k g}- \\ \mathbf{c m}) \end{gathered}$ | mrcos $\theta$ <br> (kg - <br> cm) | $\begin{gathered} \mathbf{m} * r^{*} \times{ }^{*} \sin \theta \\ \left(\mathbf{k g}-\mathbf{c m}^{2}\right) \end{gathered}$ | $\begin{gathered} \mathbf{m}^{*} \mathbf{r}^{*} \mathbf{x}^{*} \cos \theta \\ \left(\mathbf{k g}-\mathbf{c m}^{2}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  |  |  |  |  |
| B |  |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |  |
| D |  |  |  |  |  |  |  |  |

## Calculations:

| Plane | Balanced Mass <br> $(\theta)$ | Balance mass <br> radial position <br> $(\mathbf{r})$ | Balance mass <br> angular position <br> $(\%)$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Conclusion:

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Part B - Virtual Lab simulation <br> (V-Lab- link - https://dom-nitk.vlabs.ac.in/exp/muliple-mass-in-multiple-plane/)

## Procedure:

[] In the simulation window, the front view and side view of a rotating shaft with two unbalanced masses in different planes as displayed in Figure 8.
[] There are pointers given on the right side of the screen for choosing the values of input variables, viz., two unbalanced masses on the shaft ( $m_{1}, m_{2}$ ), their radii of rotation $\left(r_{1}, r_{2}\right)$, relative angular position (theta1) and distance between their planes $\left(l_{1}\right)$. The variables can be set by moving the slider left or right. After setting these variables, it is required to move on to the next pane by clicking on the navigation button at the bottom right corner.


Figure 8 Simulation Window
[] After moving on to the next page, the radii and positions of the balancing masses have to be entered by adjusting the pointer as shown in Figure 9. After setting the values, it is required by the user to calculate the values of balancing masses and their angular position.
[] Once the values of masses and their angular position from the previous step are calculated, the user needs to navigate to the next pane, enter the values in the boxes provided, and balancing masses with their angular positions is displayed instantly as shown in Figure 10.
[] Click on the submit button to submit the results and press the play/pause button to pause the simulation, and the direction change button to reverse the direction of rotating masses. Refer to Figure 11.


Figure 9 Radii and positions of the balancing masses


Figure 10 Balancing masses with their angular positions


Figure 11 Information regarding different buttons.
[] Further, it can be noted that, the correct values of balancing masses and their angular positions as calculated by the system is also displayed at the bottom of the simulation window as shown in Figure 12.


Figure 12 Result display window.

## Calculations: -

## Result Table: -

| Plane | Balanced Mass <br> $(\theta)$ | Balance mass <br> radial position <br> $(\mathbf{r})$ | Balance mass <br> angular position <br> $(\%)$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Conclusion: -

## Tutorial problems:

1. A rotating shaft carries four masses $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D which are radially attached to it. The mass centres are $30 \mathrm{~mm}, 38 \mathrm{~mm}, 40 \mathrm{~mm}$ and 35 mm respectively from the axis of rotation. The masses A, C and D are (Last three digits of your enrolment number +5) kg, (Last three digits of your enrolment number +10 ) kg and (Last three digits of your enrolment number $+15) \mathrm{kg}$ respectively. The axial distances between the planes of rotation of A and B are 400 mm and between B and C is 500 mm . The masses A and C are at right angles to each other. Find for a complete balance, 1. the angles between the masses B and D from mass A, 2. the axial distance between the planes of rotation of C and $\mathrm{D}, 3$. the magnitude of mass B.
2. Four masses A, B, C and D are completely balanced. Masses C and D make angles of $90^{\circ}$ and $210^{\circ}$ respectively with B in the same sense. The planes containing B and C are 300 mm apart. Masses A, B, C and D can be assumed to be concentrated at radii of 360, 480, 240 and 300 mm respectively. The masses are as follows
$B=$ Last three digits of your enrolment number +5 Kg ,
C $=$ Last three digits of your enrolment number +10 kg
$\mathrm{D}=$ Last three digits of your enrolment number +15
Determine the (i) mass A and its angular position (ii) positions of planes A and D A rotor has the following properties:

| Plane | Mass <br> $\mathrm{m}(\mathrm{Kg})$ | Radius <br> $(\mathrm{m})$ | $\theta$ | 1 <br> $(\mathrm{~m})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Last three digits of your <br> enrolment number +5 | 0.1 | 0 | --- |
| 2 | Last three digits of your <br> enrolment number + 10 | 0.12 | 60 | 0.16 |
| 3 | Last three digits of your <br> enrolment number +15 | 0.14 | 135 | 0.32 |
| 4 | Last three digits of your <br> enrolment number + 20 | 0.12 | 270 | 0.56 |

Sample: Enrolment no: 190190119005

| Plane | Mass <br> $\mathrm{m}(\mathrm{Kg})$ | Radius <br> $(\mathrm{m})$ | $\theta$ | 1 <br> $(\mathrm{~m})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $005+5=10 \mathrm{Kg}$ | 0.1 | 0 | --- |
| 2 | $005+10=15 \mathrm{Kg}$ | 0.12 | 60 | 16 |
| 3 | $005+15=20 \mathrm{Kg}$ | 0.14 | 135 | 0.32 |
| 4 | $005+20=25 \mathrm{Kg}$ | 0.12 | 270 | 0.56 |

If the shaft is balanced by two counter masses located at 100 mm radii and revolving in planes midway of planes 1 and 2 , and midway of 3 and 4 , determine the magnitude of the masses and their respective angular positions.

Quiz: Answer the following questions (Give neat diagram/sketch wherever necessary).

1. Two masses in different planes are necessary to rectify the dynamic unbalance. Comment.

## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan.
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | 2 | 3 | 4 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

## Experiment No: 3

## To study balancing of reciprocating masses analytically and experimentally.

## Date:

Competency and Practical Skills: Students will be able to understand the fundamentals of reciprocating mass balancing and will be able to analyze the analytical results with experiment results.

## Relevant CO: 2

## Objectives:

a) Understand and verify the fundamental laws of balancing of reciprocating mass.
b) Calculate the mass and angular position required for balancing the unbalanced force.
c) Analyze the analytical results with the experimental results.

## Equipment/Instruments:

Dynamic balancing apparatus having variable speed drive motor with controller,
Known balancing masses (eg. $50 \mathrm{~g}, 100 \mathrm{~g}, 150 \mathrm{~g}, 200 \mathrm{~g}$ etc.),
High precision digital weighing scale and
Bolts-nuts and Spanner set.

## Theory:

The force rewarded to accelerate mass $m$ is

$$
\begin{gather*}
F=m \times r \times \omega^{2}\left[\cos \theta+\frac{\cos 2 \theta}{n}\right]  \tag{12}\\
F=m \times r \times \omega^{2} \times \cos \theta+m \times r \times \omega^{2} \times \frac{\cos 2 \theta}{n} \tag{13}
\end{gather*}
$$

Where, $m \times r \times \omega^{2} \times \cos \theta=$ primary accelerating force

$$
\begin{equation*}
m \times r \times \omega^{2} \times \frac{\cos 2 \theta}{n}=\text { secondary accelerating force } \tag{14}
\end{equation*}
$$

As n is, usually, much greater than unity, the secondary force is small compared with the primary force and can be safely neglected for slow speed engines.


Figure 13 Reciprocating engine mechanism.

In the slider crank mechanism shown in Figure 13, the mass at B of the reciprocating parts is accelerating. Thus, a primary force is required to accurate the reciprocating mass. This force acts in a direction from $B$ to $A$. the reaction of this force tends to move the frame in a direction from A to B. To prevent this; an attempt has to be made to balance this reaction. The usual approach is by addition of a relating counter mass at radius directly opposite the crank, which however, provides only a partial balance. This counter mass is in addition to the mass used to balance the rotating unbalance due to the mass at the crank pin.

The horizontal component of the centrifugal force due to the balancing mass is $m \times r \times \omega^{2} \times \cos \theta$ in the line of stroke. This neutralizes the unbalanced reciprocating force. But the rotating mass also has a component $m \times r \times \omega^{2} \times \sin \theta$ perpendicular to the line of stroke, which remains unbalanced. The unbalanced force is zero at the end of the stroke when $\theta=0^{\circ}$ or $180^{\circ}$ and maximum at the middle when, $\theta=90^{\circ}$. The magnitude of the unbalance force remains the same i.e. equal to $m \times r \times \omega^{2}$. Thus, instead of sliding to and fro on its mounting, the mechanism tends to jump up and down.
To minimize the effect of the unbalanced force, a compromise is, usually, made i.e. $2 / 3$ of reciprocating mass is balanced (or a value between $1 / 2$ and $3 / 4$ ). It ' C ' is the traction of the reciprocating mass thus balanced, then

Primary force balanced by the mass $=C \times m \times r \times \omega^{2} \times \cos \theta$
Primary force unbalanced by the mass $=(1-C) \times m \times r \times \omega^{2} \times \cos \theta$
Vertical component of centrifugal force $=C \times m \times r \times \omega^{2} \times \sin \theta$
Which remains unbalanced
Resultant unbalanced force at any instant

$$
=\sqrt{\left((1-C) \times m \times r \times \omega^{2} \times \cos \theta\right)^{2}+\left(C \times m \times r \times \omega^{2} \times \sin \theta\right)^{2}}
$$

The resultant unbalanced force is maximum when $\mathrm{C}=1 / 2$
If $m_{c r}$ is the mass at the crank pin and C is the fraction of the reciprocating mass at piston $m_{p}$ to be balanced, the total mass at the crank pin man be considered as $\left(m_{c r}+C . m_{p}\right)$ which is to be completely balanced.

## Procedure: -



Figure 14 Experimental Setup layout
The following procedure should be followed to verify_the partial balancing conditions can be achieved by adding balancing mass.
[] Start the motor and see that the system rotates. Wait until the initial vibrations stop. Now check the unloaded system is in a balanced condition of vibration and oscillation.
[] Stop the motor and load the disc B and D at different points with suitable radius and weight to get an unbalanced condition.
[⿴囗 Switch on the motor and observe the system is out of balance, switch off the motor
[] Place the balancing weights in places as calculated.
[] Now start the motor and observe that the rotor remains horizontal hence there are no vibrations and oscillations.

## Observations: -

## Data for inside cylinder locomotive

1) Equivalent rotating mass per cylinder ( kg ) =
2) Equivalent reciprocating mass per cylinder $(\mathrm{kg})=$ $\qquad$
3) Fraction of the reciprocating mass at piston = $\qquad$
4) Angle between two cranks (degree) = $\qquad$
5) Distance between centre lines of wheels (m) = $\qquad$
6) Stroke length (m) = $\qquad$
7) Distance between centre line of cylinder (m) =
8) Radius of balanced mass (m) = $\qquad$

Observation Table: -

| Masses | Mass <br> $\mathbf{m}(\mathbf{k g})$ | Radius of <br> rotation <br> $\mathbf{r}(\mathbf{c m})$ | Angular position <br> $\boldsymbol{\theta}$ (degrees) | $\mathbf{m * r}$ <br> $(\mathbf{k g}-\mathbf{c m})$ |
| :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  |
| B |  |  |  |  |
| C |  |  |  |  |
| D |  |  |  |  |

## Calculations:

## Result Table: -

| Plane | Balanced Mass <br> $(\theta)$ | Balance mass <br> radial position <br> $(\mathbf{r})$ | Balance mass <br> angular position <br> $(\%)$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Conclusion: -

## Tutorial problems:

1. The cylinder axes of a V-engine are at right angles to each other. The weight of each piston is 2 kg and of each connecting rod is 2.8 kg . The weight of the rotating parts like crank webs and the crank pin is 1.8 kg . The connecting rod is 400 mm long and its centre of mass is 100 mm from the crankpin centre. The stroke of the piston is 160 mm . Show that the engine can be balanced for the revolving and the primary force by a revolving counter mass. Also, find the magnitude and the position if its centre of mass from the crankshaft centre is 100 mm . What is the value of the resultant secondary force if the speed is (last three digits of your enrolment number) rpm?
2. The stroke of each piston of a six-cylinder two-stroke inline engine is 320 mm and the connecting rod is 800 mm long. The cylinder centre lines are spaced at 500 mm . The cranks
are at $60^{\circ}$ apart and the firing order is 1-4-5-2-3-6. The reciprocating mass per cylinder is 100 kg and the rotating parts are 50 kg per crank. Determine the out-of-balance forces and couples about the mid plane if the engine rotates at (last three digits of your enrolment number) rpm.

## Quiz: Answer the following questions

(Give a neat diagram/sketch wherever necessary).

1. Why are reciprocating masses partially balanced?
2. In the context of balancing reciprocating masses, what is the significance of firing order in a multi-cylinder inline engine?
3. Explain the method of direct and reverse cranks to determine the unbalance forces in radial engines.
4. Discuss the method of Balancing of v - engines and determine the expression for magnitude and direction of resultant primary force.
5. Explain method of balancing of radial engines.
6. Discuss in details the balancing of V-engines

## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan.
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

## Experiment No: 4

## Aim: To understand and analyze the gyroscopic effects using the motorized gyroscope.

## Date:

Competency and Practical Skills: Students will be able to understand the fundamentals of gyroscopic effects and will be able to analyze the analytical results with experiment results.

## Relevant CO: 3

## Objectives:

a) Understand and verify the fundamentals of the gyroscopic effect.
b) Calculate the gyroscopic couple and its direction.
c) Analyze the analytical results with the experimental results.

## Equipment/Instruments:

Motorized gyroscope, known masses, stop watch, digital stroboscope etc.

## Theory:

Gyroscope is a body that spins about an axis and is free to rotate in other directions under the action of external forces. Examples are Locomotives, automobiles, and airplanes making a turn. In certain cases, the gyroscopic forces are undesirable whereas in other cases the gyroscopic effect may be utilized in developing desirable.

## Gyroscopic effect:



Figure 15 Detail of axis in gyroscopic effect.
To a body revolving or spinning about an axis by say ' $O X$ ' \& if a couple represented by a vector 'OY' perpendicular to 'OX' is applied, then the body tries to process about an axis 'OZ' which is perpendicular both to 'OX' \& 'OY'. Thus, the plane of spin, plane of precession, and plane of the gyroscopic couple are mutually perpendicular. The above-combined effect is known as a gyroscopic effect.

The motor is coupled to a disc rotor, which is balanced. The disc shaft rotates about the ' XX ' axis in two ball bearings housed in frame no. 1. This frame can swing about the ' YY ' axis bearing provided in the yoke type frame no. 2. In a steady position, frame no. 1 is balanced. The yoke frame is free to rotate about the vertical axis ' ZZ '. Thus, freedom of rotation about three perpendicular axes is given to the rotor.


Figure 16 Experimental Setup Layout

## Safety and Necessary Precautions:

1. When the speed of rotor spin is changed, some time is required to obtain the constant speed due to the rotor inertia. Hence, it is advised to wait until the rotor spin reaches constant speed.
2. The mass applied should not exceed 2 kg .
3. Observe the steady speed before measuring

## Procedure: -

## Rule no. 1:

"The spinning body exerts a torque or couple in such a direction which tends to make the axis of spin coincides with that of the precession".
To study rule no. 1 following procedure may be adopted:
[] Balance the initial horizontal position of the rotor.
[] Start the motor by increasing the voltage with a dimmer and wait until it attains constant speed.
(] Process the yoke frame about the vertical axis by applying the necessary force by hand to the same (in the clockwise sense seen from above).
(⿴囗 It will be observed that the rotor frame swings about the horizontal axis ' YY ' motor side is seen coming upward \& the weight pan side going downwards.
[] Rotate the vertical yoke axis in the anticlockwise direction seen from above and
(3) Observe that the rotor frame swings in the opposite sense as compared to that in the previous case following the rule.

## Rule no. 2:

'The spinning body processes in such a way as to make the axis of spin coincide with that of the couple applied, through a $90^{\circ}$ turn axis'.
To study rule no. 2 following procedure may be adopted:
[6] Balance the rotor position on the horizontal frame.
[] Start the motor by increasing the voltage with the autotransformer and wait till the disc attains constant speed.
[] Add weight ( $0.5 \mathrm{~kg}, 1 \mathrm{~kg}$, etc.) in the weight pan and start the stopwatch to note the time in seconds required to precession, through $90^{\circ}$.
[] The vertical yoke processes about the 'OZ' axis as per rule no. 2 .
[] Speed may be measured by the tachometer.

## Observations: -

1) Mass of rotor assembly: $\qquad$ kg
2) Rotor diameter: $\qquad$ m
3) Rotor thickness: $\qquad$ m
4) Distance of belt of weight pan from disc center: $\qquad$ m
5) Motor Phase and rpm: $\qquad$ rpm, single/three

## Observation Table: -

| Sr. <br> No | Speed <br> $\mathbf{N}(\mathbf{r p m})$ | Mass in the pan <br> $\mathbf{m}(\mathbf{k g})$ | Change in angle <br> $\mathbf{d} \varphi$ (degrees) | Change in time <br> $\mathbf{d t}$ (seconds) |
| :--- | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

## Calculation: -

(1) Angular velocity of the precession of disc, $\omega=\frac{2 \pi N}{60} \mathrm{rad} / \mathrm{s}$, where $\mathrm{N}=$ speed of disc
[] Angular velocity of the precession of yoke about a vertical axis,

$$
\omega_{p}=\frac{d \varnothing}{d t} \frac{\pi}{180} \mathrm{rad} / \mathrm{s}
$$

[] I $=$ Moment of inertia of disc $=m \frac{D^{2}}{8} \mathrm{~kg}-\mathrm{cm} / \mathrm{s}^{2}$,
Where, $\mathrm{g}=$ gravitational acceleration $=9.81 \mathrm{~m} / \mathrm{s}^{2}$
[] $T_{\text {actual }}=I \times \omega \times \omega_{p} \quad \mathrm{~N}-\mathrm{m}$
(] $\quad T_{\text {theoretical }}=W \times L \quad \mathrm{~N}-\mathrm{m}$, Where $\mathrm{W}=$ Weight on pan \& $\mathrm{L}=$ Distance of bolt center from the disc center in cm
(a) Graphs: Plot $T_{\text {actual }} V_{s} \omega$ and $T_{\text {theoretical }} V_{s} \omega$. Comment on the results.

## Result Table: -

| Sr. <br> No. | Angular velocity of <br> the precession of disc <br> $\omega(\mathbf{r a d} / \mathbf{s})$ | Angular velocity of <br> the precession of yoke <br> $\omega_{p}(\mathbf{r a d} / \mathbf{s})$ | $T_{\text {actual }}$ <br> (N-m) | $T_{\text {theoretical }}$ <br> (N-m) |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

## Conclusion:

$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Tutorial problems:

1. The moment of inertia of an airplane air screw is $20 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ and the speed of rotation is 1400 rpm counterclockwise when viewed from the front. The speed of the flight is $160 \mathrm{~km} / \mathrm{hr}$. Calculate the gyroscopic reaction of the air screw on the airplane when it makes a righthand turn on a path of a 200 m radius.
2. The turbine of the rotor of a ship has a mass of 3000 kg . \& radius of gyration of 0.4 m , and clockwise speed of $2500 \mathrm{r} . \mathrm{p} . \mathrm{m}$. when looking from the stern. Determine gyroscopic couple and its effect when (i) The ship steers to the left on a curve of 100 m radius at a speed of $36 \mathrm{~km} / \mathrm{hr}$. and (ii) When the ship is pitching in S.H.M., the bow falls with its maximum velocity. The period of pitching is 40 Sec . and the total angular displacement between the bow extreme positions of pitching is $12^{\circ}$.
3. A car of total mass 2200 kg has a track width 1.5 m . Each wheel having an effective diameter 0.66 m and the mass moment of inertia $2.4 \mathrm{~kg} \mathrm{~m}^{2}$. The mass moment of inertia of rotating parts of the engine is $1.2 \mathrm{~kg} \mathrm{~m}^{2}$. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The gear ratio of the engine to the rear wheel is 3 . The centre of mass of the car is 0.55 m above the road level. If the car is rounding a curve of 80 m radius at a speed of $100 \mathrm{~km} / \mathrm{h}$, determine the load distribution on the inner and outer wheels.
4. A two-wheeler motor vehicle and its rider weigh 225 kg and their combined centre of gravity is 600 mm above the ground level, when the vehicle is upright. Each road wheel is of 600 mm diameter and has a moment of inertia of $1 \mathrm{kgm}^{2}$. The rotating parts of the engine have a moment of inertia of 0.175 kgm 2 . The engine rotates at 5.5 times the speed of the road wheels and in the same sense. Determine the angle of heel necessary, when the vehicle is rounding a curve of 30 m radius at a speed of $55 \mathrm{~km} / \mathrm{hr}$.

## Quiz: Answer the following questions

## (Give a neat diagram/sketch wherever necessary).

1. Explain the following terms with neat sketch, i) axis of spin, ii) angular velocity of spin, iii) axis of precession, iv) angular velocity of precession.
2. Explain the following terms with neat sketch
3. i) Active Gyroscopic Couple
4. ii) Reactive Gyroscopic Couple
5. Explain the effect of gyroscopic couple and centrifugal couple on the reaction of the four wheels of a vehicle negotiating a curve.
6. Explain the effect of gyroscopic couples on airplanes.
7. Explain effect of gyroscopic couple on Ship during Pitching.
8. Explain in what way the gyroscopic couple affects the motion of an aircraft while taking a turn.

## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

## Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

## Experiment No: 5

To demonstrate longitudinal vibration of helical spring and to determine natural frequency of vibration oscillation analytically and experimentally.

## Date:

Competency and Practical Skills: Students will be able to demonstrate the longitudinal vibration and to analyze the natural frequency of vibration analytically, and experimentally.

## Relevant CO: 4

## Objectives:

a) Understand and demonstrate the longitudinal vibration of helical springs.
b) Calculate the natural frequency of longitudinal vibration analytically and experimentally.
c) Analyze the experimental results with the analytical results.

## Equipment/Instruments:

Vibration apparatus, spring mass system, known masses, digital balance, stop watch.

## Theory:



Figure 17 Spring mass system - Free body diagram.
Following equation is obtained under static equilibrium condition as shown in Figure 17,

$$
\begin{array}{r}
m g=k \Delta s t \\
\text { Restoring force }=m g-k(\Delta s t+x)=k x \\
\text { Accelerating force }=m \ddot{\mathrm{x}} \quad . . \tag{18}
\end{array}
$$

Differential equation of motion for a single degree freedom spring mass system

$$
\begin{equation*}
m \ddot{\mathrm{x}}+k x=0 \tag{19}
\end{equation*}
$$

$$
\begin{gather*}
x+\frac{k x}{m}=0  \tag{20}\\
\frac{k}{m}=\omega_{n}^{2}  \tag{21}\\
\omega_{n}=\sqrt{\frac{k}{m}}  \tag{22}\\
f_{n}=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \tag{23}
\end{gather*}
$$

Set up diagram: Draw below a line sketch of the experimental setup.

## Procedure: -

[] Measure the free length of spring
(2] Put some weight on a weight pan and note down the stretched length of spring.
[] Take the difference of stretched length and free length of spring which is deflection of spring.
[] Repeat steps 2 and 3 for different values of weight and note down reading in Observation Table 1.
[3 Now take the weight of any value and put on a weight pan.
[] Pull the weight pan down and release it.
[] Note down the time taken for 10 numbers of oscillations.
[] Find natural frequency analytically and compare it experimentally.

## Observation Table 1: -

| Sr. | Mass | Stretch <br> No. <br> $\mathbf{m}(\mathbf{k g})$ <br> Spring <br> $(\mathbf{c m})$ | Deflection <br> $\boldsymbol{\delta}(\mathbf{c m})$ | Stiffness <br> $\mathbf{K}=\mathbf{m} / \boldsymbol{\delta}$ <br> $(\mathbf{k g} / \mathbf{c m})$ | Mean <br> Stiffness <br> $(\mathbf{k g} / \mathbf{c m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ |  |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  |  |
| $\mathbf{3}$ |  |  |  |  |  |

Observation Table 2:

| $\begin{gathered} \text { Sr. } \\ \text { No. } \end{gathered}$ | $\begin{gathered} \text { Mass } \\ \mathrm{m}(\mathrm{~kg}) \end{gathered}$ | Deflectio n $\boldsymbol{\delta}$ (m) | $\begin{gathered} \text { Time For } \\ 10 \\ \text { Oscillations } \\ \text { (Second) } \end{gathered}$ | Time required for one Oscillation (Second) | Average <br> Time ' $T$ ' <br> (Second) | Frequency ' $\mathbf{f}$ ' (Hz) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |

## Calculations:

Spring Deflection $\delta=$ $\qquad$ m

Weight Attached, $\mathrm{W}=\mathrm{mg}=$ $\qquad$ N
Where,
$\mathrm{m}=$ Mass of Pan $\qquad$ Kg + Added Mass $\qquad$
Stiffness of Spring, $\mathrm{k}=\mathrm{W} / \delta \mathrm{N} / \mathrm{m}$
Frequency of Oscillation, $f_{n}=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \mathrm{Cps}$

## Result Table:

| Sr. <br> No. | Theoretical natural <br> frequency (fn) | Experimental natural <br> frequency (fn) | \% Error |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

## Conclusion:

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

1. Determine the equivalent spring stiffness and the natural frequency of the following vibrating systems for the following conditions:
a) Mass is suspended to a spring
b) Mass is suspended at the bottom of two springs in series
c) Mass is fixed in between two springs
d) Mass is fixed to the midpoint of a spring
e) Mass is fixed to a point on a bar joining free ends of two springs.

Take: $S_{1}=5 \mathrm{~N} / \mathrm{mm}$,
$\mathrm{m}=0.5^{*}$ (Last two digit of your enrollment number +20 ) in kg $\mathrm{S}_{2}=8 \mathrm{~N} / \mathrm{mm}, \mathrm{a}=20 \mathrm{~mm}$ and $\mathrm{b}=12 \mathrm{~mm}$

(a)

(b)

(c)

(d)

(e)
2. Determine the natural frequency of the following vibrating system.


Quiz: Answer the following questions

## (Give a neat diagram/sketch wherever necessary).

1. Why does the amplitude of free vibration gradually diminish in practical systems?
2. Why is it important to find the natural frequency of a vibrating system?
3. What effect does a decrease in mass have on the frequency of a system?
4. What effect does a decrease in the stiffness of the system have on the natural period?
5. Define the terms i) Longitudinal vibrations, ii) lateral vibrations and iii) Torsional vibration.

## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan
[] Theory of Machines : Kinematics and Dynamics, Sadhu Singh, Pearson
[⿴囗 Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

## Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Marks |  |  |  |  |  |

## Experiment No: 6

## To study the natural frequency of undamped torsional vibration for a single rotor shaft system and to demonstrate undamped torsional vibration.

## Date:

Competency and Practical Skills: Students will be able to demonstrate the undamped torsional vibration for a single rotor shaft system and will be able to analyze and compare the analytical and experimental results of natural frequencies.

## Relevant CO: 4

## Objectives:

a) Demonstrate the undamped torsional vibration for a single rotor shaft system.
b) Calculate the natural frequency of undamped torsional vibration for a single rotor shaft system analytically and experimentally.
c) Analyze the experimental results with the analytical results.

## Equipment/Instruments:

Vibration apparatus, stop watch.

## Theory:

When the particles of the shaft or disc move in a circle about the axis of the shaft, then the vibrations are known as torsional vibrations. The shaft is twisted and untwisted alternatively and the torsional shear stresses are induced in the shaft. Since there is no damping in the system these are undamped vibrations. Also, there is no external force is acting on the body after giving an initial angular displacement then the body is said to be under free or natural vibrations. Hence the given system is an undamped free torsional vibratory system.

## Set up diagram:



Figure 18 Experimental Setup layout

## Procedure: -

[] Fix the rotor at the threaded spindle fitted in the bearing over the upright of the frame.
[] Fix the gripping chuck over the spindle.
[] Fix the stationary spindle bracket at suitable length from the rotor and fix another gripping chuck to the stationary spindle.
[] Thread the shaft through the spindle and tighten the chucks.
[] Twist the rotor through some angle and release.
(2] Note down the time required for 10 oscillations.
[] Repeat the procedure for different lengths of the shaft.

## Observations: -

(2) Shaft diameter $(\mathrm{d})=$ $\qquad$ (m)
[3 Diameter of disc, $(\mathrm{D})=$ $\qquad$ (m)
[] Mass of the disc, $(\mathrm{m})=$ $\qquad$ (kg)
[] Modulus of rigidity for shaft, $(\mathrm{C})=\underline{80 * 10^{9}}\left(\mathrm{~N} / \mathrm{m}^{2}\right)$
Observations table:

| Sr. <br> No. | Length of shaft L(m) | Time for 10 oscillations (sec) | Time required for one oscillation (sec) | Average Time 'T' (sec) | Experimental natural frequency 'f' (Hz) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |

## Calculations:

Theoretical natural frequency of undamped torsional vibration for a single rotor shaft system.

$$
\begin{equation*}
f_{n}=\frac{1}{2 \pi} \sqrt{\frac{C . J}{L . I}} \tag{24}
\end{equation*}
$$

Where,
$\mathrm{J}=$ Polar moment of inertia of shaft $=(\pi / 32) \mathrm{x} \mathrm{d}^{4}$
$\mathrm{I}=$ Mass moment of inertia of disc. $=\frac{m D^{2}}{8}$

## Result Table:

| Sr. <br> No. | Theoretical natural <br> frequency (fn) | Experimental natural <br> frequency (fn) | \% Error |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

## Conclusion:

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Tutorial problems:

1. Determine the frequency of torsional vibrations of the disc shown in fig. if both the ends of the shaft are fixed and the diameter of the shaft is 40 mm . The disc has a mass of 96 kg and a radius of gyration of 0.4 m . Take modulus of rigidity for the shaft material as $85 \mathrm{GN} / \mathrm{m}^{2} .1_{1}=$ your height in meters and $\mathrm{l}_{2}=0.8 \mathrm{~m}$


## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Marks |  |  |  |  |  |

## Experiment No: 7

## To study the natural frequency of undamped torsional vibration for a two-rotor shaft system and to demonstrate undamped torsional vibration.

## Date:

Competency and Practical Skills: Students will be able to demonstrate the undamped torsional vibration for a two-rotor shaft system and will be able to analyze the analytical, and experiment results of natural frequency.
Relevant CO: 4

## Objectives:

a) Demonstrate the undamped torsional vibration for a two-rotor shaft system.
b) Calculate the natural frequency of undamped torsional vibration for a two-rotor shaft system analytically and experimentally.
c) Analyze the experimental results with the analytical results.

## Equipment/Instruments:

Vibration apparatus, stop watch.

## Theory:

The system which requires two coordinates independently to describe its motion completely is called a two degree of freedom system. The system having two degrees of freedom has two natural frequencies. The two-rotor system consists of a shaft having torsional stiffness K and two rotors having their inertias as $I_{a}$ and $I_{b}$ at its two ends. Torsional vibration occurs only when the rotor A and rotor B rotates in the opposite direction. If the rotor A and rotor B rotates in the same direction then it has zero frequency. When the rotors rotate in the opposite direction then the amplitude of vibration at the two ends will be in the opposite direction and there exists a point on the shaft having zero amplitude called node point.
In order to locate the node point, let the distance of the small rotor from the node point be La and hence distance of the bigger rotor is ( $\mathrm{L}-\mathrm{La}$ ).
Therefore,

$$
\begin{gather*}
I_{a} \times L_{a}=I_{b}\left(L-L_{a}\right)=I_{b} \times L-I_{b} \times L_{a}  \tag{25}\\
\text { Or } I_{b} \times L=\left(I_{a} \times L_{a}\right)+\left(I_{b} \times L_{a}\right)=L_{a}\left(I_{a}+\right.  \tag{26}\\
\left.I_{b}\right) \\
I_{b} \times L=L_{a}\left(I_{a}+I_{b}\right) \tag{27}
\end{gather*}
$$

Therefore,

$$
\begin{equation*}
L_{a}=\frac{I_{b} \times L}{\left(I_{a}+I_{b}\right)} \tag{28}
\end{equation*}
$$

## Set up diagram:



Figure 19 Experimental Setup layout

## Procedure: -

[3. Fix two discs to the shaft and fit the shaft in bearings.
[] Turn the disc in an angular position in the opposite direction by hand and release.
[] Note down the time required for a particular number of oscillations.
[] Fit the cross arm to one of the discs say A and again note down the time.
[] Repeat the procedure with different equal masses attached to the ends of the cross arm and note down the time.

## Observations: -

(6) Shaft diameter $(\mathrm{d})=$ $\qquad$ (m)
(6) Diameter of disc A, $\left(D_{a}\right)=$ $\qquad$ (m)
(1) Diameter of disc B, $\left(D_{b}\right)=$ $\qquad$ (m)
[6 Mass of the disc $\mathrm{A},\left(m_{a}\right)=$ $\qquad$ (kg)
(2) Mass of the disc B, $\left(m_{b}\right)=$ $\qquad$ (kg)
[⿴囗 Modulus of rigidity for shaft, $(\mathrm{C})=\underline{80 * 10}{ }^{9}\left(\mathrm{~N} / \mathrm{m}^{2}\right)$

| Sr. | Length of shaft <br> between the <br> rotor <br> No. | Time for 10 <br> oscillations <br> $($ sec $)$ | Time required <br> for one <br> oscillation <br> (sec) | Average <br> time <br> ' $\mathbf{T}$ '(sec) | Experimental <br> natural <br> frequency <br> 'f'(Hz) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |

## Calculations:

Theoretical natural frequency of undamped torsional vibration for a two-rotor shaft system.

$$
\begin{equation*}
f_{n}=\frac{1}{2 \pi} \sqrt{\frac{C . J}{L_{a} \cdot I_{a}}}=\frac{1}{2 \pi} \sqrt{\frac{C . J}{L_{b} \cdot I_{b}}} \tag{29}
\end{equation*}
$$

Where,
$\mathrm{J}=$ Polar moment of inertia of shaft $=(\pi / 32) \mathrm{xd}^{4}$
$I_{a}=$ Mass moment of inertia of disc A. $=\frac{m D_{a}^{2}}{8}$
$I_{b}=$ Mass moment of inertia of disc B. $=\frac{m D_{b}^{2}}{8}$
$L_{a}=$ Distance of the disc A from node point.
$L_{b}=$ Distance of disc B from node point.

## Result Table:

| Sr. <br> No. | Theoretical natural <br> frequency (fn) | Experimental natural <br> frequency (fn) | \% Error |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |

## Conclusion:

$\qquad$
$\qquad$
$\qquad$

## Tutorial problems:

1. The shaft shown in fig. carries two masses. The mass A is 300 kg with a radius of gyration of 0.75 m and the mass B is 500 kg with a radius of gyration of 0.9 m . Determine the frequency of the torsional vibrations. It is desired to have the node at the mid-section of the shaft of 120 mm diameter by changing the diameter of the section having a 90 mm diameter. What will be the new diameter?


## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Marks |  |  |  |  |  |

## Experiment No: 8

## To study free damped vibrations of a single degree of freedom system and to demonstrate free damped vibrations.

## Date:

Competency and Practical Skills: Students will be able to demonstrate the free damped vibrations and be able to calculate the damping factor ( $\xi$ ) and damping coefficient of free damped vibrations experimentally.

## Relevant CO: 4

## Objectives:

a) Demonstrate the free damped vibrations.
b) Draw Logarithmic decrement damping factor ( $\xi$ ) and damping coefficient of free damped vibrations experimentally.

## Equipment/Instruments:

Vibration apparatus, stop watch.

## Theory:

Logarithmic decrement is defined as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position. If $x_{1}$ and $x_{2}$ are successive values of the amplitude on the same side of the mean position, then logarithmic decrement is given by,

$$
\begin{equation*}
\delta=\frac{x_{1}}{x_{2}} \tag{30}
\end{equation*}
$$

Figure 20 shows the experimental setup, which consists of a disc immersed in oil whose damping is to be found. The disc is suspended with the help of a wire. The disc can then be set to motion and with the help of the drum and pencil arrangement, the amplitude versus linear traverse can be recorded.

## Set up diagram:



Figure 20 Experimental Setup layout

## Procedure: -

[0 Fix the shaft at a bracket fitted near the top beam of the frame.
[1] Attach the rotor and damping drum to the bottom of the shaft.
[] Attach the descender to the frame.
[6] Put the damping liquid into the damping reservoir.
[- Set the pen holder at a suitable position. Fix the paper over the recording drum.
[] Fix the guide bush at a suitable position.
[0] Oscillate the rotor carefully so that lateral oscillation should not appear.
[1 Lift the descender and gently press the pen over the paper.
[- Repeat the procedure by changing the depth of immersion and damping fluid.

Observations table:

| Sr. <br> No | Oil depth <br> (m) | Successive Amplitude |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $x_{1}(\mathbf{m m})$ | $x_{2}(\mathbf{m m})$ | $x_{3}(\mathbf{m m})$ | $x_{4}(\mathbf{m m})$ | $x_{5}(\mathbf{m m})$ |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

## Calculations:

Damping factor $\zeta=\frac{\delta}{2 . \Pi}$

## Conclusion:

## Tutorial problems:

1. A vibrating system consists of a mass equivalent to 55 kg , a spring with a stiffness of 30 $\mathrm{kN} / \mathrm{m}$ and a damper. The damping provided is only $20 \%$ of the critical value. Determine the, (i) damping factor, (ii) critical damping coefficient, (iii) natural frequency of damped vibrations, (iv) logarithmic decrement and (v) ratio of two consecutive amplitudes.
2. A machine mounted on springs and fitted with a dashpot has a mass of 60 kg . There are three springs, each of stiffness $12 \mathrm{~N} / \mathrm{mm}$. The amplitude of vibrations reduces from 45 to 8 mm in two complete oscillations. Assuming that the damping force varies as the velocity, determine the (i) damping coefficient, (ii) ratio of frequencies of damped and undamped vibrations and (iii) Periodic time of damped vibrations.

## Quiz: Answer the following questions

## (Give a neat diagram/sketch wherever necessary).

1. Define damping. Explain different ways of providing the damping for reducing vibrations.
2. Is the frequency of a damped free vibration smaller or greater than the natural frequency of the system?
3. Define logarithmic decrement and derive an expression for it.
4. Derive expressions for equations of motion of under-damped and critically damped systems.
5. Derive an expression for the equation of motion of an overdamped system.
6. Draw \& explain the response curves for Over damped, Underdamped, and Critically damped systems.
7. Draw the vector diagrams showing the effect of damping on the system response, for all the three cases of question 8 above.

## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[1 Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

## Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

## Experiment No: 9

## To study the dynamic forces and turning moments in mechanisms analytically.

## Date:

Competency and Practical Skills: Students will be able to understand the fundamentals of the dynamic forces and turning moments in mechanisms analytically.

## Relevant CO: 1

## Objectives:

a) Calculate the velocity, acceleration and force acting on each link.
b) Dynamic force analysis of 4-bar mechanism and slider crank mechanism (Analytical Methods)
c) Design of flywheel for IC engine and Punch press.
d) Static force analysis of 4-bar mechanism and slider crank mechanism (Use Analytical Methods)

## Tutorial problems:

1. The connecting rod of a vertical reciprocating engine is 2 m long between centres and weighs 250 kg . The mass centre is 800 mm from the big end bearing. When suspended as a pendulum from the gudgeon pin axis, it makes 8 complete oscillations in 22 seconds. Calculate the radius of gyration of the rod about an axis through its mass centre. The crank is 400 mm long and rotates at 200 rpm . Find the inertia torque exerted on the crankshaft when the crank has turned through $40^{\circ}$ from the top dead centre and the piston is moving downwards.
2. The piston diameter of an internal combustion engine is 125 mm and the stroke is 220 mm . The connecting rod is 4.5 times the crank length and has a mass of 50 kg . The mass of the reciprocating parts is 30 kg . The centre of mass of the connecting rod is 170 mm from the crank-pin centre and the radius of gyration about an axis through the centre of mass is 148 mm . The engine runs at 320 rpm . Find the magnitude and the direction of the inertia force and the corresponding torque on the crankshaft when the angle turned by the crank is $140^{\circ}$ from the inner dead centre.

## Case Study

1. A Punching press executes 20 hoes of 20 mm diameter per minute in a FE410 plate of 15 mm thick. This causes the variation of speed in the flywheel attached to the press from 250 rpm to 225 rpm . The punching operation takes 1.5 seconds per hole. Assume that the frictional losses account for $15 \%$ of the work supplied for punching. Find:
i. Power needed to operate the punching press in kW .
ii. Mass of flywheel with radius of gyration of 0.5 m .
2. A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs $10000 \mathrm{~N}-\mathrm{m}$ of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute?
3. A four-link mechanism is subjected to the following external forces as shown in table 01 and figure 01 . Determine the shaft torque $T_{2}$ on the input link $A B$ for static equilibrium of the mechanism. $\angle \mathrm{BAD}=60^{\circ}$.


| Link | Length(mm) | Force | Magnitude (N) | Point of application of force (mm) |
| :---: | :---: | :---: | :---: | :---: |
| AB | 500 | $\mathrm{~F}_{2}$ | $80 \angle 73.5^{\circ}$ | 325 mm from A |
| BC | 660 | $\mathrm{~F}_{3}$ | $144 \angle 58^{\circ}$ | 297 mm form B |
| CD | 560 | $\mathrm{~F}_{4}$ | $60 \angle 42^{\circ}$ | 373 mm from D |
| AD | 1000 | - | Fixed link | - |

5. A slider-crank mechanism with the following dimensions is acted upon by a force $\mathrm{F}=2 \mathrm{kN}$ at B as shown in figure 02 . $\mathrm{OA}=100 \mathrm{~mm}, \mathrm{AB}=450 \mathrm{~mm}$. Determine the input torque T on the link OA for the static equilibrium of the mechanism for the given configuration.


## Quiz: Answer the following questions

## (Give a neat diagram/sketch wherever necessary).

1. What are the requirements of an equivalent dynamically system?
2. What do you mean by piston effort and crank effort? Derive the expression of turning moment at crankshaft in terms of piston effort and angle turn by the crank.

## Suggested Reference:

[] Theory of Machines, R. S. Rattan, Tata Mc-Graw Hill

## References used by the students:

Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

## To study and analyze the single DoF forced vibration system at various damping ratios and frequency ratios.

## Date:

Competency and Practical Skills: Students will be able to analyze the effect of various damping ratios and frequency ratios in a single DOF forced vibration system experimentally.

## Relevant CO: 4

## Objectives:

a) Demonstrate the single DOF forced vibration system.
b) Analyze the effect of various damping ratios and frequency ratios in a single DOF forced vibration system experimentally.

## Equipment/Instruments:

Vibration apparatus, stop watch.

## Theory:

Forced vibrations occur if any system is continuously driven or excited by an external force. A simple example is a child's swing that is pushed on each downswing. Vibrations caused in a rotating machinery due to a small unbalanced or disturbing mass may be considered to be another example of a forced vibration.

Life of components and their joints in an assembly under vibration greatly depends on amplitude of vibration. Hence, it is important to know the amplitude of vibration. Amplitude of vibration tends to be high particularly at and near resonance that occur when the excitation frequency is close to the natural frequency of a system. And therefore, the natural frequency of vibration must be separated considerably from the excitation frequency.
A Spring-Mass-Damper model is considered in this experiment for finding its response to a harmonic excitation. Resonance is characterized by a high value of amplitude of vibration. The Spring-Mass-Damper arrangement is an idealized model of a machine resting on a resilient pad or sheet. The machine, say a lathe-bed and its accessories, can be considered rigid representing the mass in the SDOF system, and the resilient pad, a spring and damper together. The concepts developed while studying a SDOF system are fundamental and useful for studying complex systems also.


Figure 21 Forced damped vibration of single degree of freedom system.

The equation of motion for single DOF forced vibration system as shown in Figure 21 may be written as

$$
\begin{equation*}
m \cdot \frac{d^{2} x}{d t^{2}}+c \cdot \frac{d x}{d t}+k \cdot x=F_{0} \cdot \sin \sin \omega t \tag{31}
\end{equation*}
$$

The solution of above equation may be written as

$$
\begin{equation*}
x=x_{c}+x_{p} \tag{32}
\end{equation*}
$$

$x_{c}$ is the complementary solution which is transient type and dies out over a period of time and it is given by,

$$
\begin{gather*}
x_{c}=M \cdot e^{\left(-\omega_{n} \cdot \zeta \cdot t\right)} \cdot \sin \left(\omega_{n} \cdot t+\Psi\right)  \tag{33}\\
x_{p}=\frac{F_{0} \cdot \sin \sin \left(\omega_{n} \cdot t-\phi\right)}{k \cdot \sqrt{\left(1-r^{2}\right)^{2}+(2 \cdot \zeta \cdot r)^{2}}} \tag{34}
\end{gather*}
$$

Magnification factor

$$
\begin{equation*}
\Omega=\frac{1}{\sqrt{\left(1-r^{2}\right)^{2}+(2 . \zeta . r)^{2}}} \tag{35}
\end{equation*}
$$

Setup diagram: Draw below a line sketch of the experimental setup.

## Safety and necessary Precautions:

[] Do not run the motor at low voltage.
[6] Do not increase the speed at once.
[] Damper is always in the perpendicular direction.
[] A motor bolt is properly tightly packed with weight.
[] A beam is properly tight in bearing with a bolt.

## Part - A

## Procedure: -

[] Connect the exciter Motor to the control panel.
[] Start the Motor and allow the system to vibrate.
[] Wait for 5 minutes for amplitude to build up for a particular forcing frequency.
[] Adjust the position of the strip chart recorder. Take the record of amplitude Vs. time on a strip chart recorder by starting the recorder motor.
[0 Take record by changing forcing frequency.
(2) Repeat the experiment for different damping.
[] Plot the graph of amplitude $\mathrm{v} / \mathrm{s}$. frequency for each damping.

## Observations table:

| Sr. <br> No | Stiffness <br> $(\mathbf{k})(\mathbf{N} / \mathbf{m})$ | Mass <br> $(\mathbf{m})(\mathbf{k g})$ | Damping <br> Ratio | Force <br> amplitude <br> (N) | Force <br> frequency <br> $(\mathbf{r a d} / \mathbf{s})$ | Remark |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

## Calculations:

## Part - B

## Procedure: -

[] Set a value of Stiffness and Mass for the system. Set a small value of damping ratio (less than half of 0.707). Notice the natural frequency of the system as shown in Figure 6 the graphs. You can set a value of operating frequency which will be marked in the graph.
[] Click on the right arrow to load the graph of magnification factor and phase angle versus frequency ratio.
[] Click on the left arrow to come back to the system display screen. Edit the damping ratio. Set to a value between 0.35 and 0.707 .
[] Click on the right arrow to load the graph of magnification factor and phase angle versus frequency ratio. Observe the change in Magnification factor curve as shown in Figure 7.
[] Once again click on the left arrow to come back to the system display screen. Edit the damping ratio. Set to a value higher than 0.707 .
(1) Click on the right arrow to load the graph of magnification factor and phase angle versus frequency ratio and notice the change in Magnification Factor values against frequency ratio.


Figure 22 Input variables for Forced damped vibration of SDF system.


Figure 23 Magnification factor and Phase angle for forced damped vibration of SDOF system.

Observation table:

| Sr. <br> No | Stiffness <br> $(\mathbf{k})(\mathbf{N} / \mathbf{m})$ | Mass <br> $(\mathbf{m})(\mathbf{k g})$ | Damping <br> Ratio | Force <br> amplitude <br> (N) | Force <br> frequency <br> $(\mathbf{r a d} / \mathbf{s})$ | Remark |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

## Calculations:

## Conclusion:

## Tutorial problems:

1. When a single cylinder engine of total mass 300 kg is placed on four springs, the springs are compressed by 2 mm . A dashpot offering 400 Newtons of damping force at relative velocity of $200 \mathrm{~mm} / \mathrm{sec}$ is attached to the engine to damp out the vibrations. The reciprocating mass of the engine is 20 kg and stroke of the piston is 130 mm . The engine is running at 1500 rpm . Find out amplitude of vibrations neglecting secondary unbalance.
Quiz: Answer the following questions (Give neat diagram/sketch wherever necessary).
2. Explain why a constant force on the vibrating mass has no effect on the steady-state vibration.
3. What will be the frequency of the applied force with respect to the natural frequency of the system if the magnification factor is less than unity?
4. Give a physical explanation of why the magnification factor is nearly equal to 1 for small values of frequency ratio (r) and is small for large values of frequency ratio (r).
5. Classify the vibration measuring instruments.
6. Clearly explain the working principle of vibrometer and accelerometer.
7. Explain the working principle of seismic instruments.
8. Explain in brief vibration isolation and isolation materials.

## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand

## References used by the students:

## Rubric wise marks obtained:

| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

## Experiment No: 11

## To determine critical speed of shaft theoretically and experimentally.

## Date:

Competency and Practical Skills: Students will be able to understand the fundamentals of critical speed of shaft and will be able to analyze the analytical results with experiment results.

## Relevant CO: 5

## Objectives:

a) Understand the fundamental critical speed of the shaft.
b) Analyze the analytical results with the experimental results.

## Equipment/Instruments:

Whirling of shaft apparatus, stroboscope.

## Theory:



Figure 24 Critical speed of the shaft

In actual practice, a rotating shaft carries different mountings and accessories in the form of gears, pulleys etc. When the gears or pulleys are put on the shaft, the centre of gravity of the pulley or gear does not coincide with the centre line of the bearing or with the axis of the shaft, when the shaft is stationary.

This means that the centre of gravity of the pulley or gear is at a certain distance from the axis of rotation and due to this, the shaft is subjected to centrifugal force. This force will bend the shaft, which will further increase the distance of centre of gravity of the pulley or gear from the axis of rotation. The bending of the shaft not only depends on the value of eccentricity (distance between centre of gravity of the pulley and axis of rotation) but also depends upon the speed at which the shaft rotates. The speed, at which the shaft runs so that the additional
deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.
Set up diagram: Draw below a line sketch of the experimental setup.

## Safety and necessary Precautions:

1) The speed of the shaft should be increased gradually.
2) If the speed of the shaft increases, it may lead to violent instability.

## Procedure: -

[] Fix the shaft to be tested in the fix ends.
[0 Supply the main power to the motor through dimmer-stat.
(3) Gradually increase the speed of the motor until the first mode of vibration is not arrived.
[] Study the first mode of vibration and note down the corresponding speed of the shaft with the help of a hand tachometer.
(3) Gradually increase the speed of the motor again, until the second mode of vibration is not arrived.
[7] Study the second mode of vibration and note down the corresponding speed of the shaft with the help of a hand tachometer.
[] Reduce the speed gradually and when the shaft stops rotating, cut-off the main power supply.
[] Repeat the experiment for the shaft of different diameter.

## Observations: -

(2) $\quad$ Shaft diameter $\left(d_{1}\right)=$ m
(2] $\quad$ Shaft diameter $\left(d_{2}\right)=$ $\qquad$ m
(2) Shaft diameter $\left(d_{3}\right)=$ $\qquad$ m
[] Length of the shaft $\left(L_{1}\right)=$ $\qquad$ m
[9 Length of the shaft $\left(L_{2}\right)=$ $\qquad$ m
[] Length of the shaft $\left(L_{3}\right)=$ $\qquad$ m
[⿴囗 Modulus of Elasticity $(\mathrm{E})=$ $\qquad$
[1] Modulus of Rigidity (G) = $\qquad$
(3) Density of the shaft material $(\rho)=$ $\qquad$

## Observations table:

(1) When both the ends are fixed:

| Sr. <br> No. | Theoretical Whirling speed, <br> Nt (rpm) | Actual Whirling speed, <br> Na (rpm) |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

(2) When both the ends are supported:

| Sr. <br> No. | Theoretical Whirling speed, <br> Nt (rpm) | Actual Whirling speed, <br> Na (rpm) |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

(3) When one end is fixed and one end is supported:

| Sr. <br> No. | Theoretical Whirling speed, <br> Nt (rpm) | Actual Whirling speed, <br> Na (rpm) |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

## Calculations:

1. Moment of Inertia of shaft, $I=\frac{\pi D^{4}}{64} m^{4}$
2. Mass of the shaft, $W=A . L . \rho$
3. Area of the shaft, $A=\frac{\pi D^{2}}{4} m^{2}$
4. When both the ends are fixed:

Static deflection of shaft due to mass of the shaft, $\delta_{S}=\frac{W L^{4}}{384 E I} m$
Frequency of transverse vibration, $f_{n}=\frac{0.4985}{\sqrt{\frac{\delta_{s}}{1.27}}} \mathrm{~Hz}$
Critical or whirling speed of shaft (in rps) is equal to the frequency of transverse vibration in Hz .

$$
\begin{array}{ll}
\mathrm{Nc}(\mathrm{rps}) & =\mathrm{fn}(\mathrm{~Hz}) \\
\mathrm{Nc}(\mathrm{rpm}) & =60 . \mathrm{fn}
\end{array}
$$

2. When both the ends are supported:

Static deflection of shaft due to mass of the shaft, $\delta_{s}=\frac{5 W L^{4}}{354 E I} m$
Frequency of transverse vibration, $f_{n}=\frac{0.4985}{\sqrt{\frac{\delta_{s}}{1.27}}} \mathrm{~Hz}$
Critical or whirling speed of shaft (in rps) is equal to the frequency of transverse vibration in Hz .

$$
\begin{array}{ll}
\mathrm{N}_{\mathrm{c}}(\mathrm{rps}) & =\mathrm{f}_{\mathrm{n}}(\mathrm{~Hz}) \\
\mathrm{N}_{\mathrm{c}}(\mathrm{rpm}) & =60 . \mathrm{f}_{\mathrm{n}}
\end{array}
$$

3. When one end is fixed and one end is supported:

Static deflection of shaft due to mass of the shaft, $\delta_{s}=\frac{W L^{4}}{185 E I} m$
Frequency of transverse vibration, $f_{n}=\frac{0.4985}{\sqrt{\frac{\delta_{s}}{1.27}}} \mathrm{~Hz}$
Critical or whirling speed of shaft (in rps) is equal to the frequency of transverse vibration in Hz .

$$
\begin{array}{ll}
\mathrm{N}_{\mathrm{c}}(\mathrm{rps}) & =\mathrm{fn}(\mathrm{~Hz}) \\
\mathrm{N}_{\mathrm{c}}(\mathrm{rpm}) & =60 . \mathrm{fn}
\end{array}
$$

## Conclusion:

## Tutorial problems:

1. A rotor having a mass of 5 kg is mounted midway on a simply supported shaft of diameter

10 mm and length 400 mm . Because of manufacturing tolerances, the CG of the rotor is 0.02 mm away from the geometric centre of the rotor. If the rotor rotates at 3000 rpm . find the amplitude of steady state vibrations and the dynamic force transmitted to the bearings. Neglect the effect of damping. Take $\mathrm{E}=2 \times 10^{\wedge} 11 \mathrm{~N} / \mathrm{m}^{2}$.
2. A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at $75 \%$ of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. $\mathrm{E}=200 \mathrm{GN} / \mathrm{m}^{2}$.
3. A vertical steel shaft 15 mm diameter is held in long bearings 1 meter apart and carries at its middle a disc of mass 15 kg . The eccentricity of the centre of gravity of the disc from the centre of the rotor is 0.30 mm . The modulus of elasticity for the shaft material is 200 $\mathrm{GN} / \mathrm{m}^{2}$ and the permissible stress is $70 \mathrm{MN} / \mathrm{m}^{2}$. Determine: 1 . The critical speed of the shaft and 2 . The range of speed over which it is unsafe to run the shaft. Neglect the mass of the shaft.
4. A shaft of diameter 40 mm and length 2.5 m has a mass of 15 kg per meter. It is simply supported and has 3 masses as shown in the figure below. Find the frequency of transverse vibration by Dunkerley's method taking E $=200 \mathrm{GPa}$.

5. Using Dunkerley's method; find the natural frequency of transverse vibration of the system shown below:


## Suggested Reference:

[] Theory of Machines, S. S. Rattan, Tata Mc-Graw Hill
[] Dynamics of Machinery, Farazdak Haideri, Nirali Prakashan
[] Theory of Machines, R. S. Khurmi, J. K. Gupta, S. Chand
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| Rubrics | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |

