CHATER-3 Flexural stresses OR Stresses in Beams

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Introduction:-

A bending moment bends a member. stresses introduced by bending moment are called Bending stresses. similarly a shear force will introduce stresses called Shear stresses

Pure Bending:-

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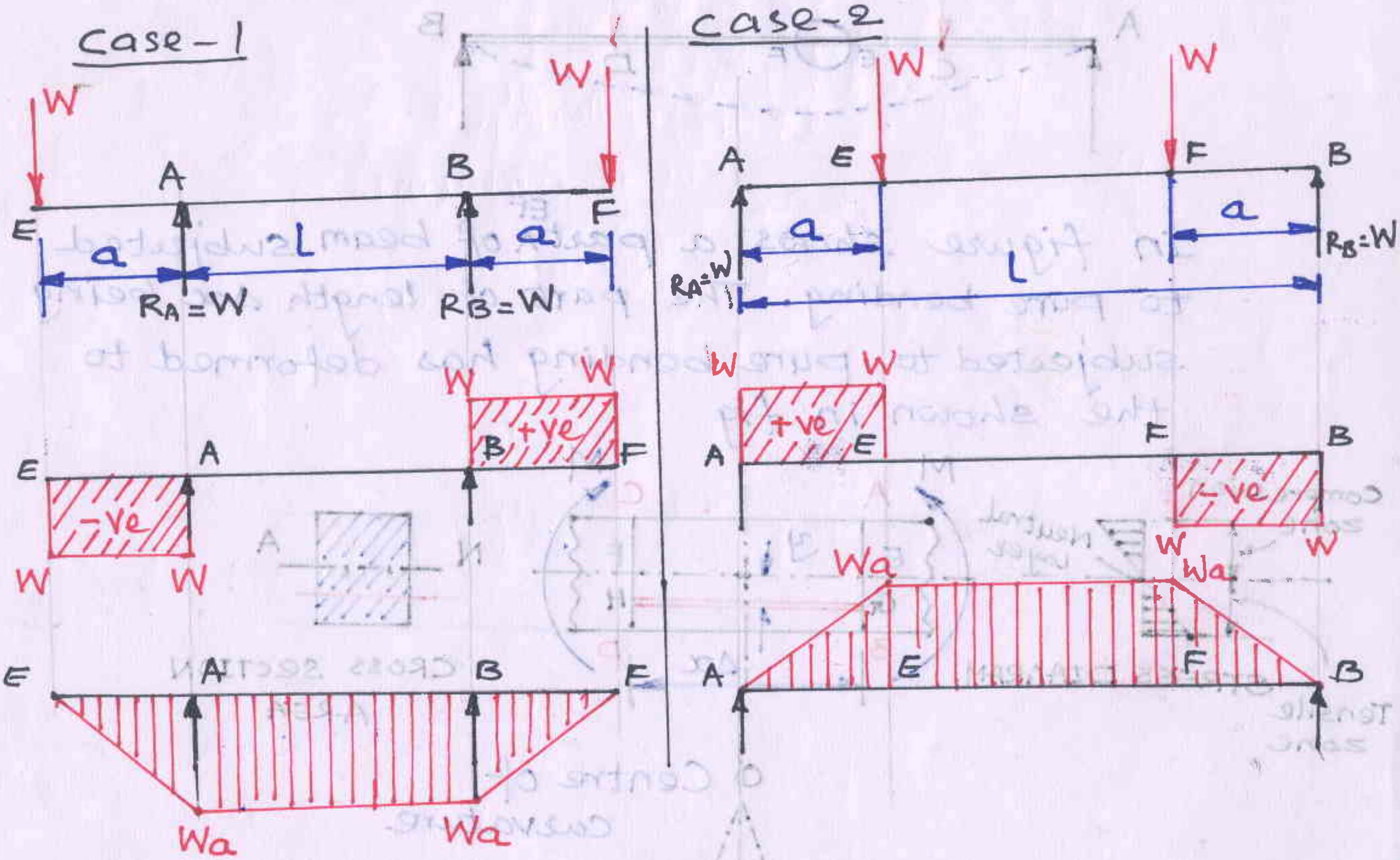


Both the Beams are supported at A and B

Case-1 Point Load W be applied at each end of Beam
It is easily seen that bet^n A & B the B.M. is constant and shear is zero

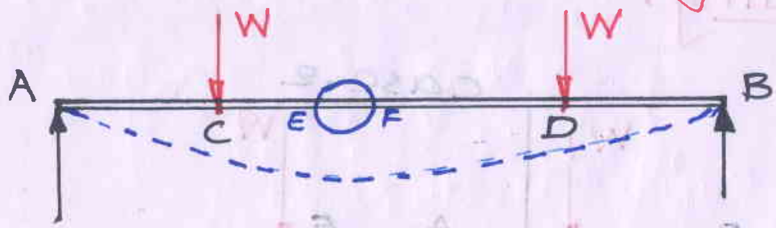
Case-2 It is easily seen that bet^n E & F the B.M. is constant and there is no shear force at all bet^n E & F

Case-1 :- Betⁿ. A & B the beam is absolutly free from shear but is subjected to a bending moment W.a

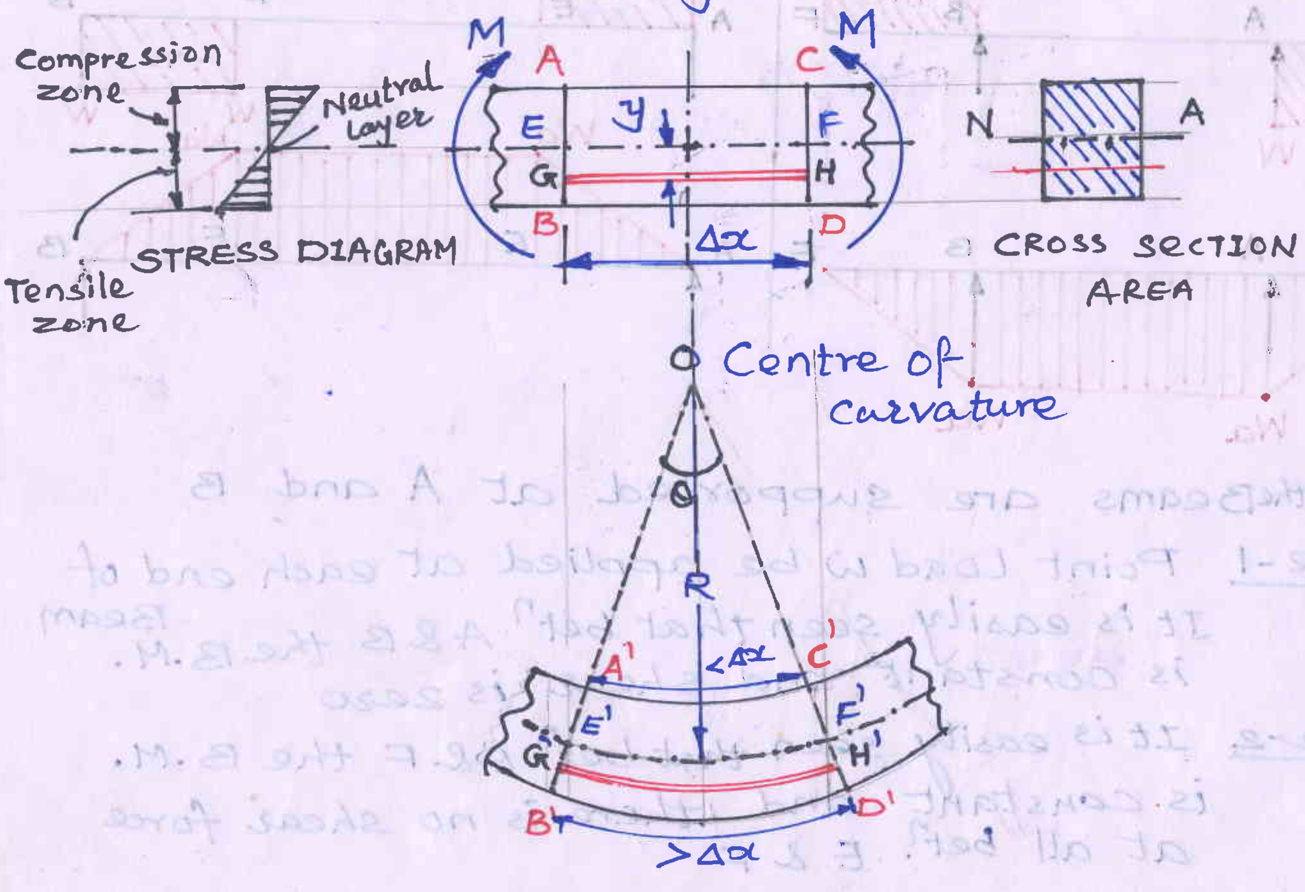
Case-2 Betⁿ. E & F the beam is absolutly free from shear but is subjected to a bending moment W.a

This condition of the beam is called pure bending or simple bending

Theory of simple Bending



In figure shows a part EF of beam subjected to pure bending. The part of length Δx being subjected to pure bending has deformed to the shown in fig



A fibre such as AC has deformed to the shape A'C'. This fibre has been shortened in its length. The fibre BD on the contrary has been elongated and has taken the shape B'D'. Similarly, the fibre GH has been elongated and has taken the shape G'H'. Hence, if the beam for the length Δx be taken to consist of large number of fibres, we find that all of them have changed their shape: some of them have been shortened while some of them are elongated.

At a level bet^n the top & bottom of beam there will be a layer of fibres which are neither shortened nor extended. Fibres in this layer are not stressed at all. This layer is called the neutral layer or the neutral surface. The line of intersection of the neutral surface on a cross-section is called the neutral axis

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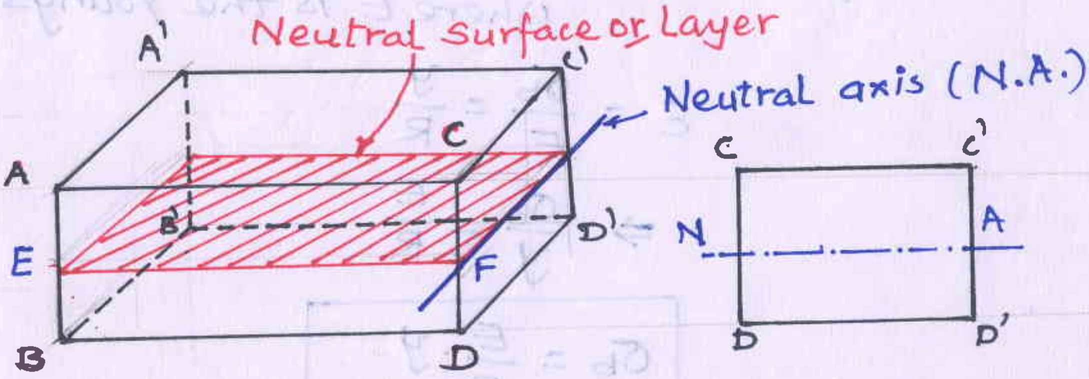

Consider the fibre GH distance y from the neutral layer

Original length of GH $= \Delta x$

After deformation this fibre will deform and take the position G'H', the new length of the fibre being

$(R+y)\theta$

The fibre EF in the neutral Layer takes the position $E'F'$ without undergoing any change in Length

$EF = E'F' = \Delta x$

$\Delta x = R\theta$

$\therefore$ change in length of the fibre GH

$= G'H' - GH$

$= (R+y)\theta - \Delta x$

$= (R+y)\theta - R\theta$

$= y\theta$

$\therefore$ strain of the fibre GH

$$\varepsilon = \frac{\text{change in length}}{\text{Original length}} = \frac{y\theta}{R\theta} = \frac{y}{R}$$

Suppose the stress intensity in the fibre is σ_b. We have strain of the fibre $= \frac{\sigma_b}{E}$

where E is the Young's Modulus.

$$\varepsilon = \frac{\sigma_b}{E} = \frac{y}{R}$$

$$\Rightarrow \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\boxed{\sigma_b = \frac{E}{R} \cdot y}$$

Hence the stress intensity in any fibre is proportional to the distance of the fibre from the neutral axis

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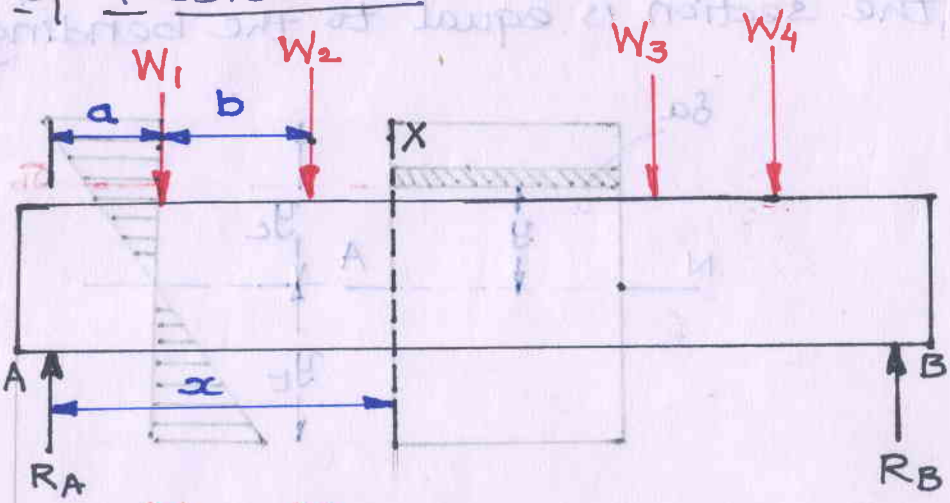
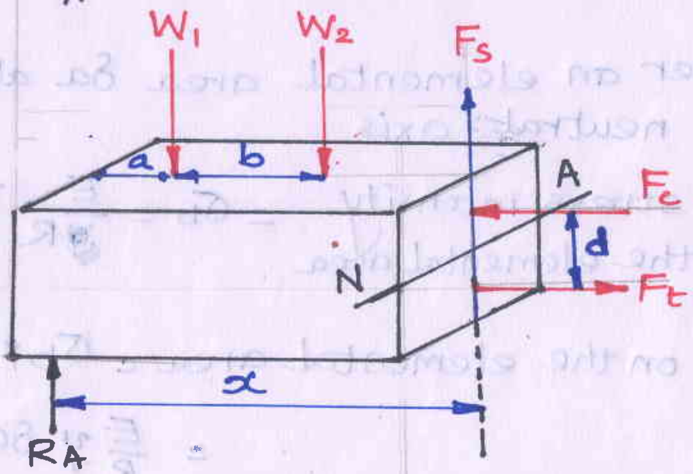
Moment of Resistance





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The forces keeping the part AX in equilibrium
This part is in equilibrium under the action of the following forces

i) Vertical reaction R_A
ii) Downward Loads W_1, W_2
iii) shear resistance F_s offered by the section X
iv) Compressive resistance F_c offered by the secⁿ X
v) Tensile resistance F_t offered by the section X

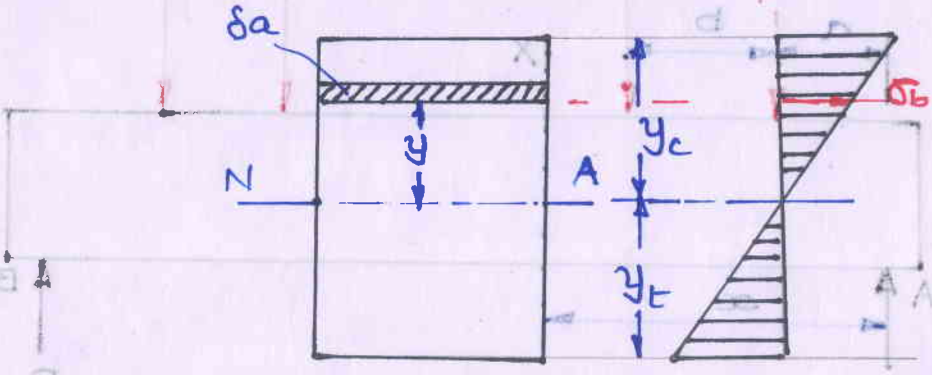
Since F_c & F_t are the only longitudinal forces on the beam we have $F_c = F_t$

The two equal and opposite resistances F_c & F_t will form a couple $F_c \times d$ or $F_t \times d$. This couple is called the moment of resistance

Taking moments of the forces on this part about the N.A. of the section X

$$F_c \times d = R_A \times x - W_1 \times (x - a) - W_2 \times (x - a - b)$$

Hence the moment of resistance offered by the section is equal to the bending moment.



Now consider an elemental area δa at a distance y from the neutral axis

The stress intensity on the elemental area $= \sigma_b = \frac{E}{R} \cdot y$

Thrust on the elemental area $= \sigma_b \times \delta a$

$= \frac{E}{R} y \cdot \delta a$

Moment of resistance $= \frac{E}{R} y^2 \delta a$

Total Moment of resistance offered by the beam secⁿ.

$$M = \frac{E}{R} \sum_{y_t}^{y_c} y^2 \delta a$$

But $\sum_{y_t}^{y_c} y^2 \delta a$ is the moment of inertia of beam section about the neutral axis

$$M = \frac{EI}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$ ($\because$ We know $\frac{\sigma_b}{y} = \frac{E}{R}$)

IE = Bending stiffness

Assumption in the theory of pure bending

1) The value of the Young's Modulus is the same for the beam material in tension as well as compression.

2) A transverse section of the beam which is a plane before bending will remain a plane after bending.

3) The material of the beam is homogeneous and isotropic (Isotropic means having the same elastic properties in all the direction).

4) The Elastic limit is not exceeded

5) The resultant pull or thrust on a transverse section of the beam is zero

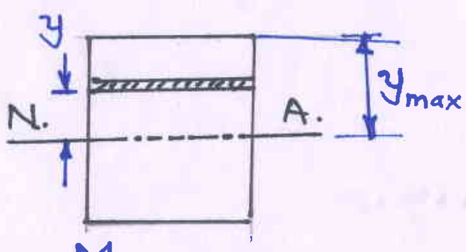
6) The transverse section of beam is symmetrical about an axis passing through the centroid of the section and parallel to the plane of bending

Section Modulus :-

The stress at any point on the section distance y from the neutral axis is given by

$$\boxed{\sigma_b = \frac{M}{I} \cdot y}$$

The maximum stress occurs at the greatest distance from the neutral axis



$$(\sigma_b)_{max} = \frac{M}{I} \cdot y_{max} \Rightarrow M = (\sigma_b)_{max} \times \frac{I}{y_{max}}$$

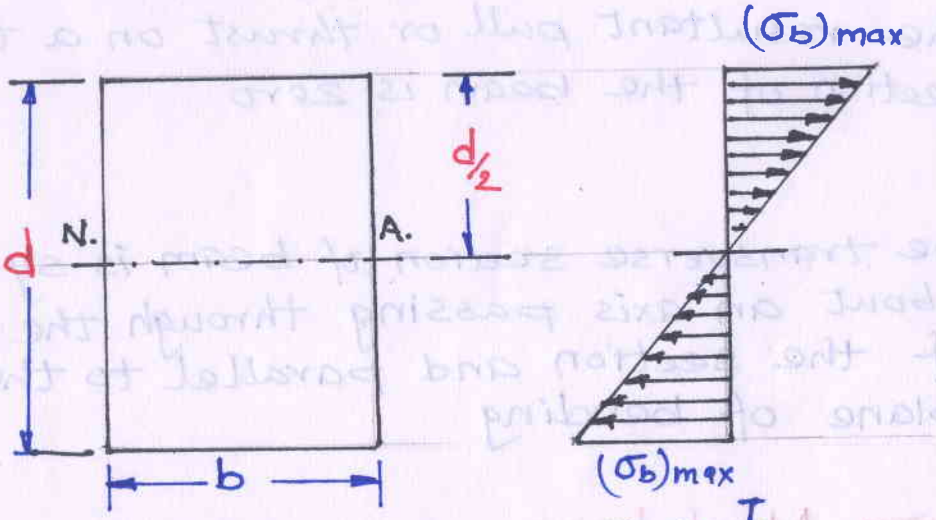
$$M = (\sigma_b)_{max} \times Z \quad \left(\text{where } Z = \frac{I}{y_{max}}\right)$$

$$Z = \frac{\text{Moment of inertia about the neutral axis}}{\text{Distance of the most distant point from the neutral axis}}$$

This ratio is called the section modulus.

Section modulus for various shapes of beam section.

i) Rectangular section;-



$$\text{Section modulus} = Z = \frac{I}{y_{max}}$$

$$I = \frac{bd^3}{12} \quad \& \quad y_{max} = \frac{d}{2}$$

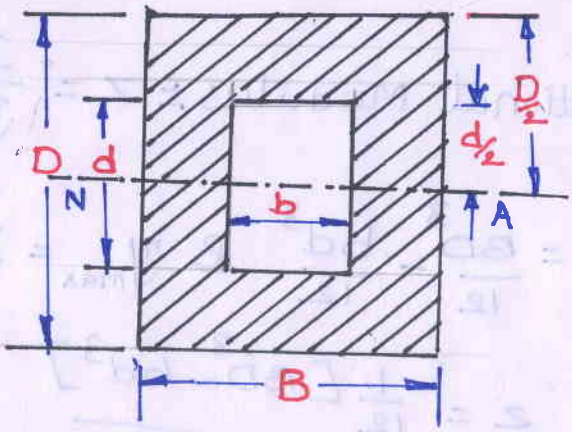
$$\therefore \boxed{Z = \frac{bd^2}{6}}$$

Let $(\sigma_b)_{max}$ be the max. stress offered by the beam secⁿ.

$$\text{Moment of resistance} = M = (\sigma_b)_{max} Z$$

$$= \frac{1}{6} (\sigma_b)_{max} bd^2$$

2) Hollow Rectangular section



Section modulus : $Z = \frac{I}{y_{max}}$

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} \quad \& \quad y_{max} = \frac{D}{2}$$

$$Z = \frac{\frac{1}{12}\left[BD^3 - bd^3\right]}{D/2}$$

$$= \frac{1}{6}\,\frac{\left[BD^3 - bd^3\right]}{D}$$

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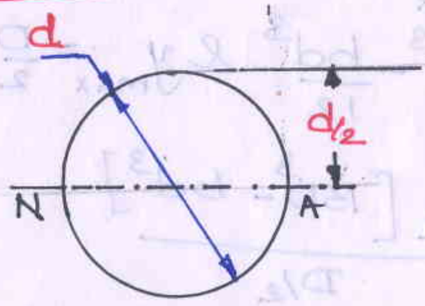
3) Circular Section :-



Section modulus = $Z = \frac{I}{y_{max}}$

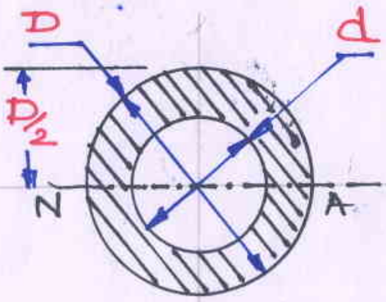
$$I = \frac{\pi d^4}{64} \quad \& \quad y_{max} = \frac{d}{2}$$

$$Z = \frac{\frac{\pi d^4}{64}}{d/2} = \frac{\pi d^3}{32}$$

Let σ_b be the max. stress offered by the beam section

Moment of resistance $M = \sigma_b Z = \frac{\pi}{32} d^3 \cdot \sigma_b$

4) Hollow circular section :-

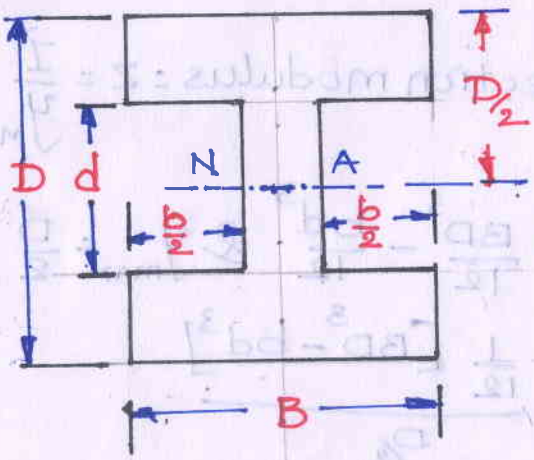


Section modulus $Z = \frac{I}{y_{max}}$

$$I = \frac{\pi}{64}\left(D^4 - d^4\right) \quad \& \quad y_{max} = D/2$$

$$Z = \frac{\frac{\pi}{64}\left(D^4 - d^4\right)}{D/2} = \frac{\pi}{32}\,\frac{\left(D^4 - d^4\right)}{D}$$

5) I Section :-
(Symmetrical about both axis)



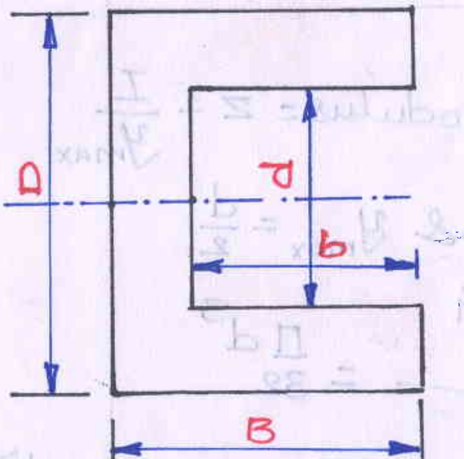
Sectional Modulus := $Z = \frac{I}{y_{max}}$

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} \quad \& \quad y_{max} = \frac{D}{2}$$

$$Z = \frac{\frac{1}{12}\left[BD^3 - bd^3\right]}{D/2}$$

$$= \frac{1}{6}\frac{\left[BD^3 - bd^3\right]}{D}$$

6) [Section :-



$$Z = \frac{I}{y_{max}}$$

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} \quad \& \quad y_{max} = \frac{D}{2}$$

$$Z = \frac{\frac{1}{12}\left[BD^3 - bd^3\right]}{D/2}$$

$$= \frac{1}{6}\frac{\left[BD^3 - bd^3\right]}{D}$$