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Moment of Inertia of planar Cross - Section

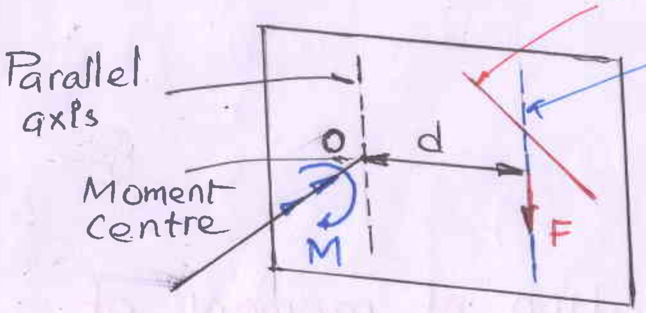
Moment of inertia (MI) :- $\int dMy^2$. where dM is an elementary mass & y is the distance of this elementary mass from an axis

1) Concepts: (Information)

→ **Moment of a force:-**

Moment is a vector quantity like force.

Moment = Force x distance



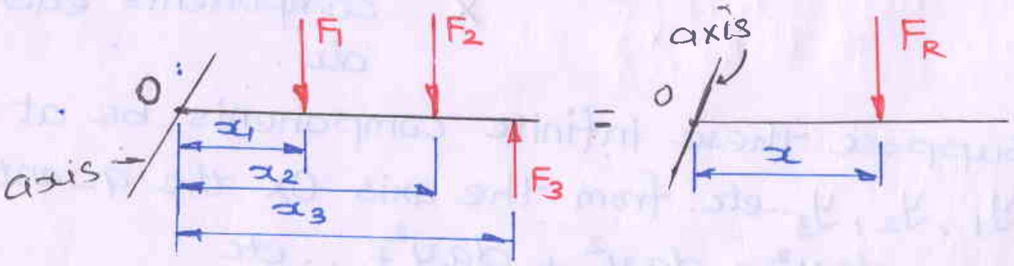
$M = F \times d$ N.m

Right-hand screw notation

→ Moment is measure of the rotating effect of a force

→ A force has no moment about an axis parallel to its line of action or intersecting it

Principle of moments or Varignon's theorem states that the algebraic sum of the moments of forces is equal to the moment of the resultant of the forces about the same axis.



$$F_1 \cdot x_1 + F_2 x_2 - F_3 \cdot x_3 = F_R \cdot x$$

$$\Sigma F_i x_i = F_R \cdot x$$

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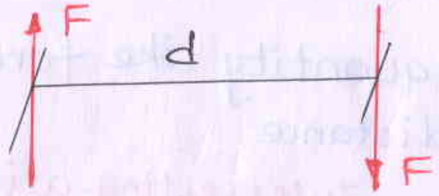
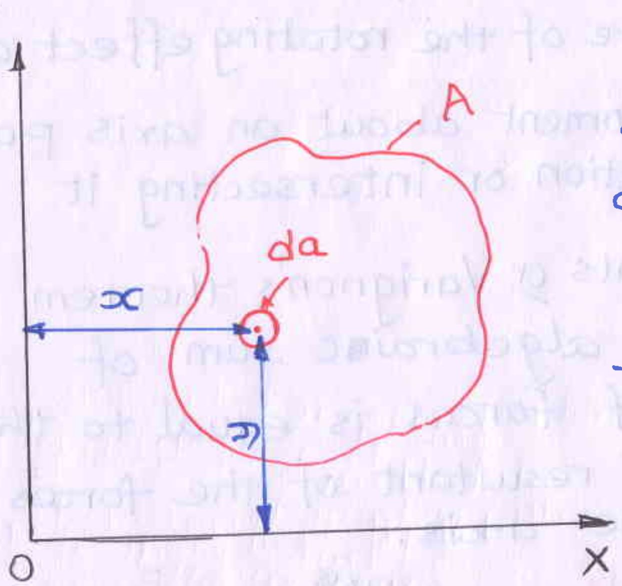
A Couple is a special case of a moment due to two equal and opposite forces acting at a distance.

∴ The couple has the same moment Fd about any point in its plane



2) Derivation of equation of moment of inertia of standard lamina



A = Area of Lamina

da = Area of an infinite number of elemental components

The lamina may be split up into an infinite number elemental components each of area da.

Suppose these infinite components be at distances y_1, y_2, y_3....etc. from the axis OX the quantity

$$da y_1^2 + da y_2^2 + da y_3^2 + \ldots \text{etc}$$

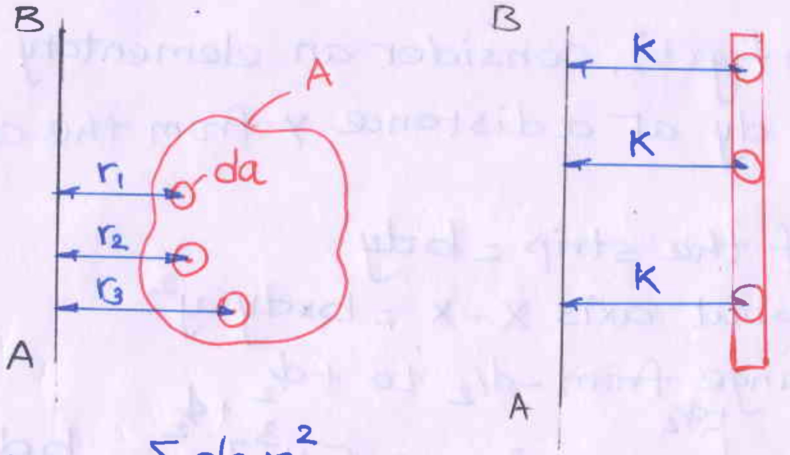
$I_{ox} = \sum da y^2$ is called the moment of inertia or the second moment of area of the lamina about the axis OX

Similarly if $x_1, x_2, x_3 \ldots$ etc are the distances of the various elemental components of area da each from the axis OY, then

$$da\,x_1^2 + da\,x_2^2 + da\,x_3^2 + \ldots$$

$I_{oy} = \Sigma\, da\,x^2$ is called the moment of inertia of the lamina about the axis OY

Radius of gyration of a given lamina about a given axis



$$I_{ab} = \Sigma\, da\,r^2$$

Let the infinite components of the lamina be arranged at the same distance k from the axis AB, the distance k being such that the moment of inertia about the axis AB remains unchanged. In this second position, each elemental area is at the same distance k from AB

Moment of inertia of the lamina about the axis AB

$$I_{ab} = dak^2 + dak^2 + dak^2 + \ldots$$

$$= k^2 \Sigma da = Ak^2$$

k is called the radius of gyration of the lamina about AB

$$K = \sqrt{\frac{I}{A}}$$

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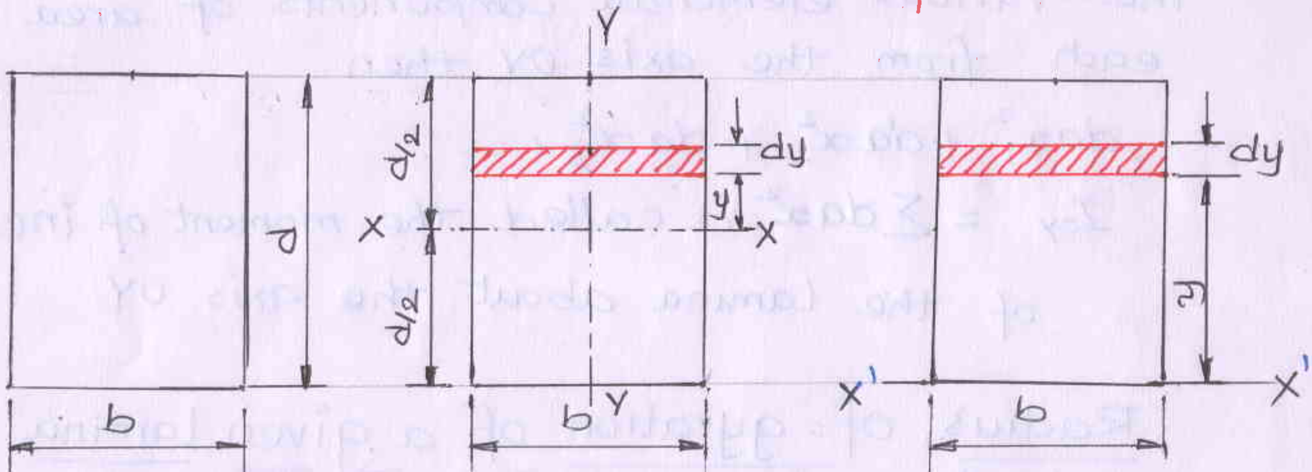
Computation of second moment of area



(a) (b) (c)

As shown in fig (b), consider an elementary strip of thickness dy at a distance y from the centroid axis X–X

Area of the strip = $b \cdot dy$

MI about axis X – X = $b \cdot dy \cdot y^2$

y has range from $-d/2$ to $+d/2$

$$I_{xx} = \int_{-d/2}^{+d/2} b \cdot dy \cdot y^2 = b\left[\frac{y^3}{3}\right]_{-d/2}^{+d/2} = \frac{bd^3}{12}$$

Similarly it can be show that

$$I_{yy} = \int_{-b/2}^{+b/2} d\,dx\,x^2 = \frac{db^3}{12}$$

fig (C)

$$I_{x'x'} = \int_0^d b\,dy\,y^2 = b\left[\frac{y^3}{3}\right]_0^d = \frac{bd^3}{3}$$

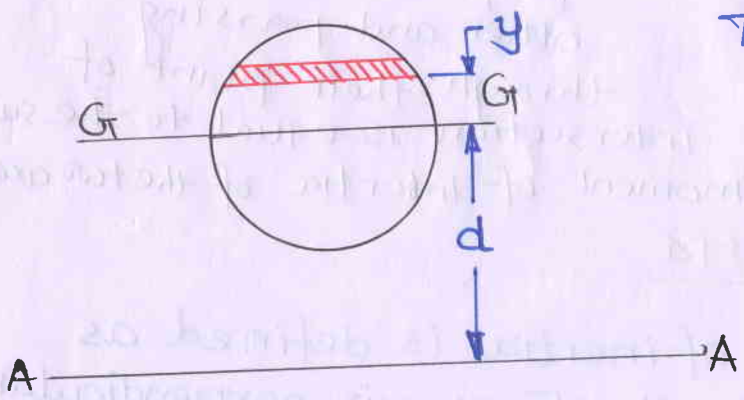
$$= \frac{bd^3}{3}$$

Parallel axis theorems :-

"The moment of inertia of a body about an axis parallel to the body passing through its center is equal to the sum of moment of inertia of body about the axis passing through the centre and product of mass of the body times the squre of distance between the two axes"

In previous example. We found the MI about a parallel axis from the fundamental principles. A general theorem to find MI about parallel axes is stated as follows.

If I_{GG} is the MI of an area about an axis through its its centroid, then the MI any axis A-A at a distance d from GG and parallel to it is $I_{AA} = I_{GG} + Ad^2$ where A is the area of the Lamina



The MI of the whole area about the axis A-A is

$$I_{AA} = \int dA\,(y+d)^2$$

$$= \int dA\,(y^2 + 2yd + d^2)$$

$$= \int dA y^2 + \int dA d^2 + \int dA \times 2yd$$

$$= I_{GG} + d^2 \int dA + 2d \int dAy$$

$$\int dAy = 0$$

$$\boxed{I_{AA} = I_{GG} + Ad^2}$$

"The moment of inertia of a plane area with respect to any axis plane is equal to the sum of moment of inertia with respect to parallel centrodal axis and the product of the total area & the square of the distance bet^n the two axes."

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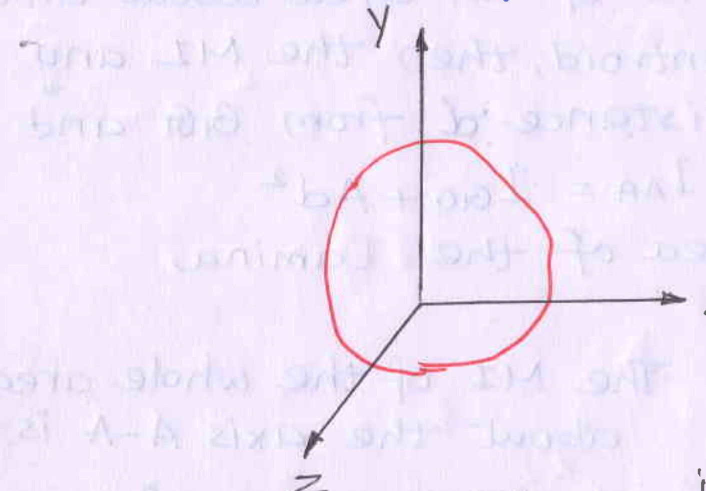
Perpendicular Axes Theorem

The perpendicular axes theorem states that "The second moment of an area about an axis perpendicular to the plane of the area through a point is equal to the sum of the second moment of areas about two mutually perpendicular axes through that point"

In some structural calculations, we need to find the second moment of an area about an axis passing through the centroid and perpendicular to the plane containing the area. This is required in the case of torsion of members.

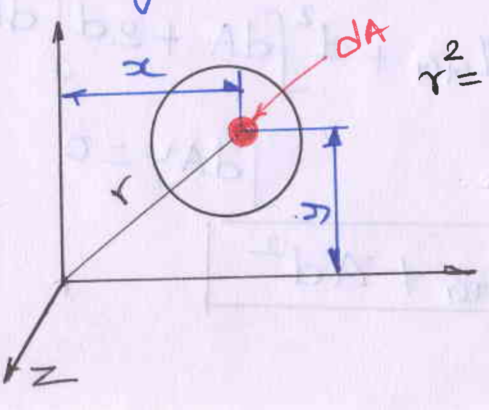


"The moment of inertia about an axis perpendicular to the plane containing two axes perpendicular to each other and passing through their point of intersection is equal to the sum of moment of intertia of the two axes"

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Polar Moment of Inertia

The polar moment of inertia is defined as the moment of inertia about an axis perpendicular to the plane of the area. Polar MI is denoted by the symbol J



$$r^2 = x^2 + y^2$$

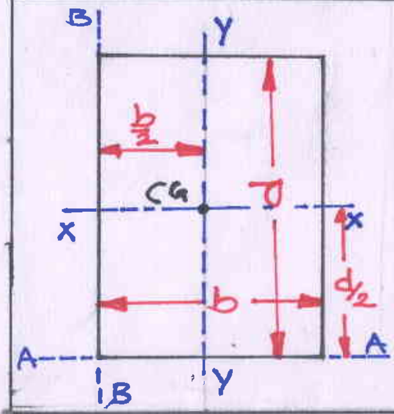
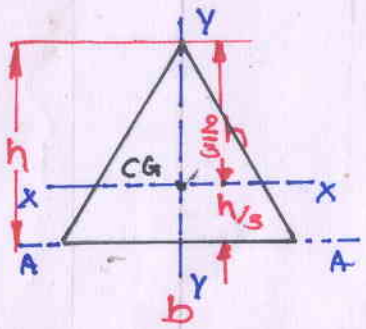
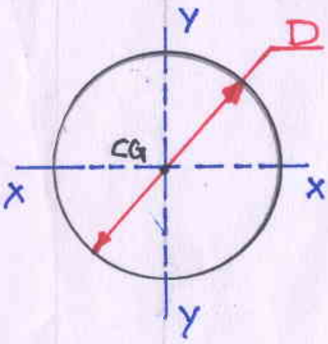
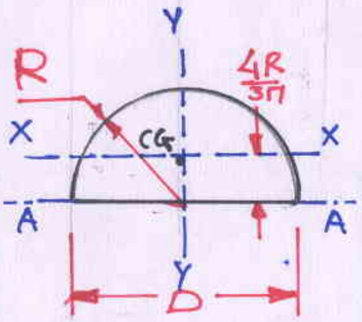
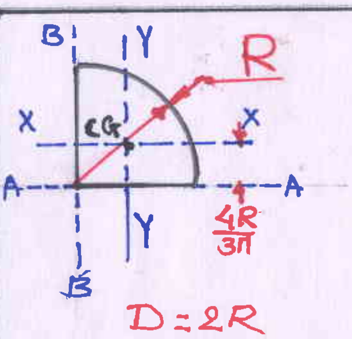
$$J_{zz} = \int_A dA\, r^2$$

$$J_{zz} = \int_A dA\,(x^2 + y^2)$$

$$= \int_A dA\, x^2 + \int_A dA\, y^2$$

$$= I_{xx} + I_{yy}$$

TABLE: M.I of Different Cross-Section. where I_{xx} & I_{yy} are centroidal Axes

Cross Section	Area	I_{xx}	I_{yy}	I_{AA}	I_{BB}
	bd	$\frac{bd^3}{12}$	$\frac{db^3}{12}$	$\frac{bd^3}{3}$	$\frac{db^3}{3}$
	$\frac{1}{2}bh$	$\frac{bh^3}{36}$	$\frac{hb^3}{12}$	$\frac{bh^3}{12}$	—
	$\frac{\pi D^2}{4}$	$\frac{\pi D^4}{64}$	$\frac{\pi D^4}{64}$	—	—
	$\frac{\pi D^2}{8}$	$0.11R^4$	$\frac{\pi D^4}{128}$	$\frac{\pi D^4}{128}$	—
 $D=2R$	$\frac{\pi D^2}{16}$	$0.055R^4$	$0.055R^4$	$\frac{\pi D^4}{256}$	$\frac{\pi D^4}{256}$