4 - Motion of Fluid particles and streams

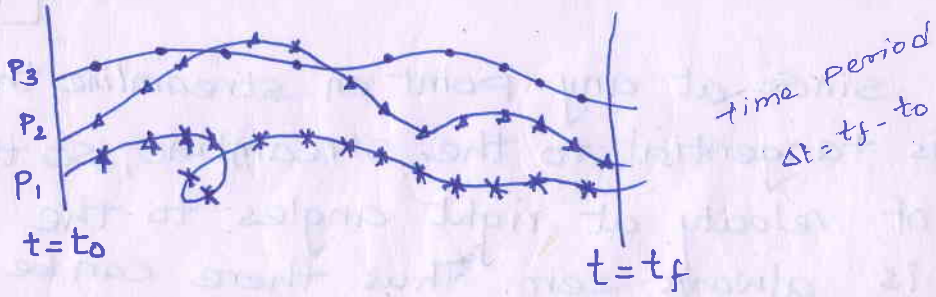
Prepared by: Prof. D.B. Patel

divyesheldragon@gmail.com

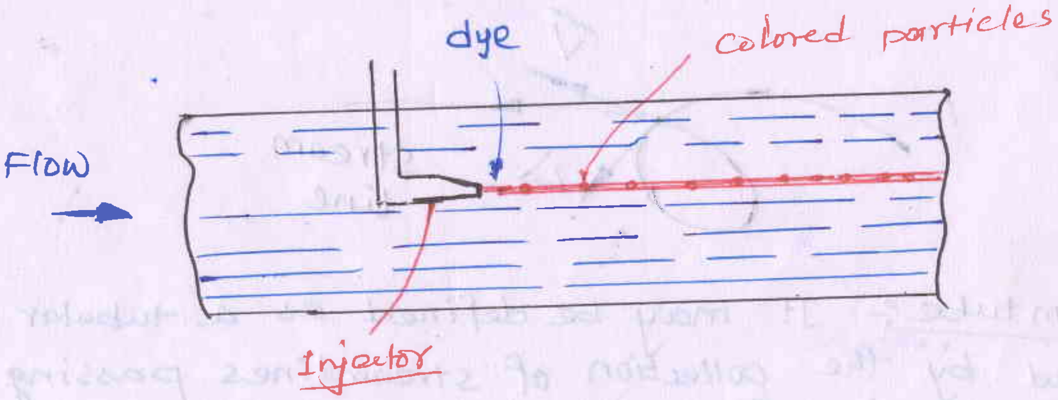
4.1 Introduction:-

The study of fluid in motion is called fluid kinematics. kinematics is the geometry of motion. In this analysis, the forces or energies responsible for accelerating or decelerating the flow are not considered

→ **Path line:** The path traced by a single fluid particle in motion over a period of time is called its path line. If an individual particle of fluid is coloured, it will describe a pathline



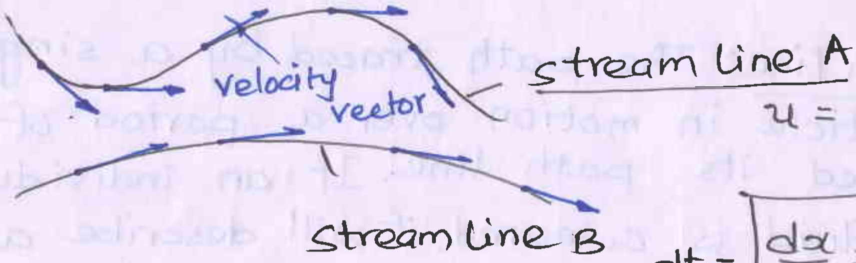
→ **Streakline or filament line:-**
It is an instantaneous picture of the position of all the fluid particles in the flow which have passed through a given fixed point. Injecting a stream of dye into a liquid, or smoke into a gas, the result will be a streakline



Streamline:-

A streamline is an imaginary line drawn through the flow field in such a way that the velocity vector of the fluid at each and every point on the streamline is tangent to the streamline at that instant.

$\vec{V} = f(u, v, w, t)$

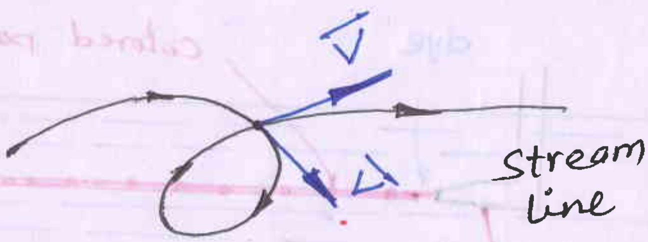


$\bar{S} = f(x, y, z)$

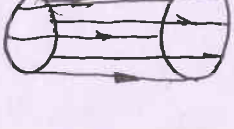
$u = \frac{dx}{dt}, \; v = \frac{dy}{dt}, \; w = \frac{dz}{dt}$

$$dt = \boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

Since at any point on streamline the velocity is tangential to the streamline, so the components of velocity at right angles to the streamline is always zero. Thus there can be no flow occurring across any streamline. Streamlines can not ever cross each other. If they cross the fluid particle will have two velocities at the point of intersection and that is physically impossible



→ **Stream tube:-** It may be defined as a tubular space formed by the collection of streamlines passing through the perimeter of a closed curve



4.2 Different type of flow or Classification of fluid flow.

→ **Steady and Unsteady Flow:-**

A steady flow is defined as a flow in which the various hydrodynamic parameters and fluid properties at any point do not change with time.

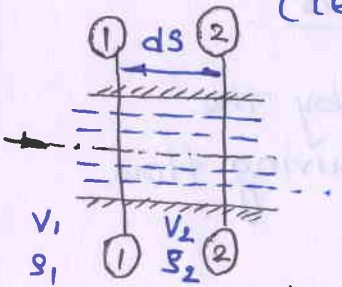
$$\frac{\partial V}{\partial t} = 0, \quad \frac{\partial P}{\partial t} = 0, \quad \frac{\partial \rho}{\partial t} = 0$$

Flow in which any of these parameters changes with time is termed as unsteady

$$\frac{\partial V}{\partial t} \neq 0, \quad \frac{\partial P}{\partial t} \neq 0, \quad \frac{\partial \rho}{\partial t} \neq 0$$

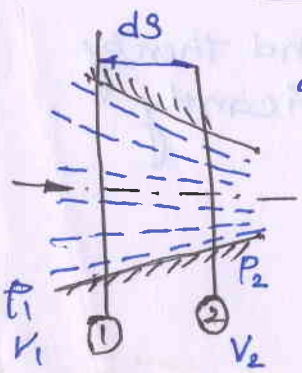
→ **Uniform flow and Non-uniform flow**

Uniform flow:- is defined as that type of flow in which the flow parameters like pressure, velocity, density etc. at given time do not change with respect to space (length of direction of flow)



$$\left.\frac{\partial V}{\partial s}\right|_{t=const} = 0, \quad \left.\frac{\partial P}{\partial s}\right|_{t=Const} = 0, \quad \left.\frac{\partial \rho}{\partial s}\right|_{t=const} = 0$$

Non-uniform flow flow in which the flow parameters like pressure, velocity, density etc. at a given time change with respect to space.

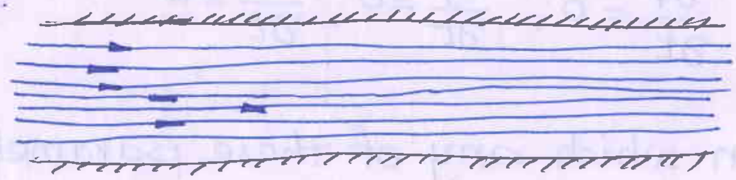


$$\left.\frac{\partial V}{\partial s}\right|_{t=Const} \neq 0, \quad \frac{\partial P}{\partial s} \neq 0, \quad \frac{\partial \rho}{\partial s} \neq 0$$

→ Laminar and Turbulent flow

A Laminar flow: is one in which the fluid particles move in Layers or Lamina with one Layers sliding over the other. Fluid elements move in well defined paths they retain the same relative position at successive cross-sections of the flow passage.

The Laminar flow is also called the streamline or viscous.



Turbulent flow :- is that type of flow in which the fluid particles move in zig-zag way. All the fluid particles are disturbed and they mix with each other. Thus there is a continuous transfer of momentum to adjacent Layers.

→ Compressible and Incompressible flows

Compressible: When the volume and thereby the density of fluid changes appreciably during flow, the flow is said to be compressible.

$$\rho \neq \text{Const.}$$

Incompressible flow:- When the volume and thereby the density of fluid changes insignificantly in the flow field.

$$\rho = \text{Const.}$$

— One, Two and Th~~ee~~ree-dimensional flow

One-dimensional (1-D) flow:- 1-D flow is that type of flow in which the flow parameters are the function of time and one space co-ordinate only

$V = \sqrt{u^2+v^2+w^2}$
u = Velocity - x - dir.
v = Velocity - y - dir.
w = Velocity - z - dir

Like flow parameter Velocity (V) one of the three co-ordinate direction say x

i.e $V = f(x,t)$ for unsteady flow

$V = f(x)$ for steady flow $u = f(x)$, $v = 0$, $w = 0$

Two-dimensional (2-D) flow is that type of flow in which the flow parameters are the function of time and two space co-ordinate say x and y

$V = f(x,y,t)$ for unsteady flow

$V = f(x,y)$ for steady flow, $u = f_1(x,y)$ $v = f_2(x,y)$ $w = 0$

Three dimensional (3-D) flow is that type of flow in which the flow parameters are the function of time and three space co-ordinate say x, y and z

$V = f(x,y,z,t)$ for unsteady flow

$V = f(x,y,z)$ to steady flow

$u = f_1(x,y,z)$, $v = f_2(x,y,z)$, $w = f_3(x,y,z)$

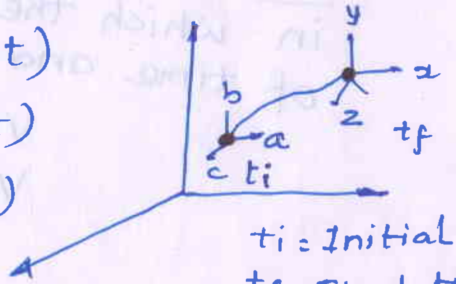
4.3 Acceleration of a fluid particle

There are two main frames of reference to study the motion of fluid particles
i) Lagrangian method ii) Eulerian method

i) Lagrangian method: -

In this method, the fluid motion is described by tracing the kinematic behaviour (velocity, acceleration) of each and every individual particle constituting the flow. The observer travels with the partical being studied. Identities of the particles are made by specifying their initial position (spatial location) at a given time. The position of a partical at any other instant of time the become a function of its identity and time. Let initially the co-cordinates of a fluid particle are a, b, c and final co-ordinates after time interval t are x, y, z. The kinetic flow pattern is described by following equations of motion.

$$x = x(a, b, c, t)$$
$$y = y(a, b, c\, t)$$
$$z = z(a, b, c\, t)$$



t_i : Initial time
t_f : Final time
$t_f - t_i = t$ time interval

The velocity components are

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}$$

acceleration components

$$a_x = \frac{du}{dt} = \frac{d^2x}{dt^2}, \quad a_y = \frac{dv}{dt} = \frac{d^2y}{dt^2}, \quad a_z = \frac{dw}{dt} = \frac{d^2z}{dt^2}$$

ii) Eulerian method

In this method, our coordinates are fixed in space and we observe the fluid as it passes by as if we had described a set of coordinate line on a glass window. The observer remain stationary and observes what happens at some particular point

The components of velocity vector

$$u = u(x, y, z, t)$$
$$v = v(x, y, z, t)$$
$$w = w(x, y, z, t)$$

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}$$

The velocity of a fluid particles is a function of both position and time

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$$

$$= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

Similarly

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

i) the term, $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ & $\frac{\partial w}{\partial t}$ are called the local acceleration

ii) The term $u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$, $u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$ & $u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$ are called convective acceleration

4.4 Discharge and mean velocity

The discharge is defined as the total quantity of fluid flowing per unit time past any particular cross-section of a stream. It is called a flow rate

In term of mass, mass flow rate (kg/s)

In term of volume, volume flow rate (m^3/s)

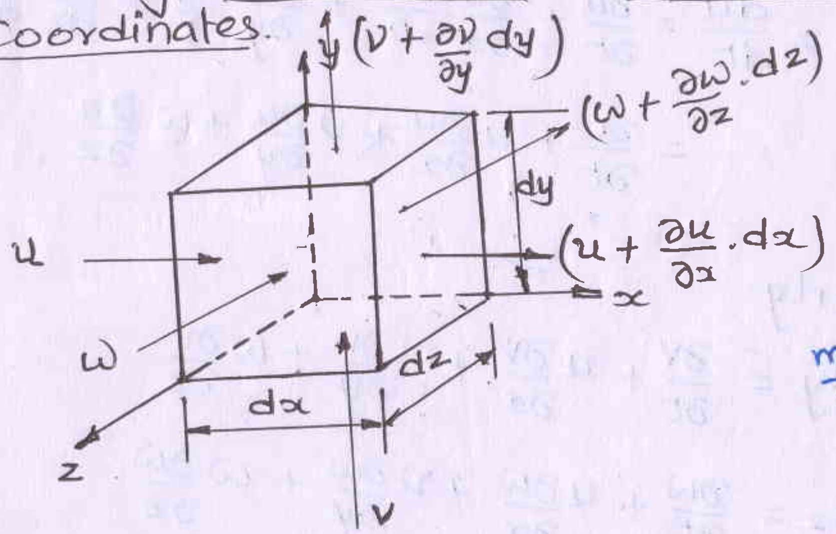
$$\text{Discharge } Q = \frac{\text{mass}}{\text{sec}}$$

$$Q = \frac{\text{Volume}}{\text{sec}} = \frac{\text{Length} \times \text{Area}}{\text{sec}} = V \cdot A$$

V = mean Velocity

divyesheldragon@gmail.com

4.5 Continuity equation for 3-D flow in Cartesion Coordinates.



$\frac{mass}{time} = \rho \bar{V} \times dx . dy . dz$

V = velocity

ρ = density

$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{cv}}{dt}$

Mass of fluid entering per unit time = Mass of fluid leaving per unit time + Increase of mass of fluid in the control volume per unit time

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

For Incompressible flow ρ = const, & steady flow $\frac{\partial \rho}{\partial t} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$