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*Maximum shear stress*

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right]$$
$$= 27.5 \text{ MPa } \textbf{Ans.}$$

## 5.9 Theories of Failure Under Static Load

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uni-axial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

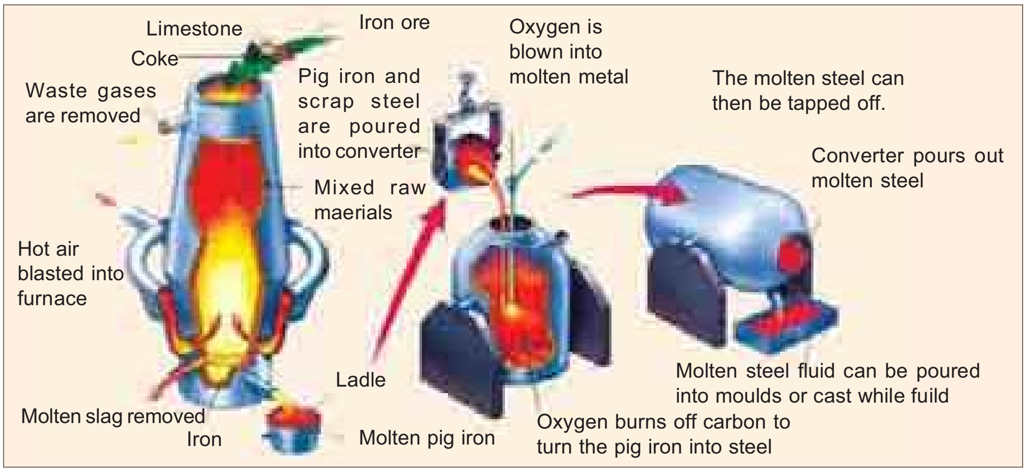
1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

## 5.10 Maximum Principal or Normal Stress Theory (Rankine's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according




*Pig iron is made from iron ore in a blast furnace. It is a brittle form of iron that contains 4-5 per cent carbon.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

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to the above theory, taking factor of safety (*F.S.*) into consideration, the maximum principal or normal stress ($\sigma_{t1}$) in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_{u}}{F.S.}, \text{ for brittle materials}$$

where $\sigma_{yt}$ = Yield point stress in tension as determined from simple tension test, and

$\sigma_u$ = Ultimate stress.

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

**Note :** The value of maximum principal stress ($\sigma_{t1}$) for a member subjected to bi-axial stress system may be determined as discussed in Art. 5.7.

## 5.11 Maximum Shear Stress Theory (Guest's or Tresca's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt}/F.S. \quad \text{...}(i)$$

where $\tau_{max}$ = Maximum shear stress in a bi-axial stress system,

$\tau_{yt}$ = Shear stress at yield point as determined from simple tension test, and

*F.S.* = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation (*i*) may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is mostly used for designing members of ductile materials.

**Note:** The value of maximum shear stress in a bi-axial stress system ($\tau_{max}$) may be determined as discussed in Art. 5.7.

## 5.12 Maximum Principal Strain Theory (Saint Venant's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (*i.e.* strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \, . \, E}$$

$\therefore$ According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \, . \, E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.S.} \quad \text{...}(i)$$

where $\sigma_{t1}$ and $\sigma_{t2}$ = Maximum and minimum principal stresses in a bi-axial stress system,

$\varepsilon$ = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

$E$ = Young's modulus, and

*F.S.* = Factor of safety.

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From equation ***(i)***, we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

## 5.13 Maximum Strain Energy Theory (Haigh's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (*i.e.* strain energy at the yield point ) per unit volume as determined from simple tension test.


*This double-decker A 380 has a passenger capacity of 555. Its engines and parts should be robust which can bear high torsional and variable stresses.*

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \quad \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory may be used for ductile materials.

## 5.14 Maximum Distortion Energy Theory (Hencky and Von Mises Theory)

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (*i.e.* distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

**Note:** The maximum distortion energy is the difference between the total strain energy and the strain energy due to uniform stress.

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**Example 5.16.** *The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to*

*1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory.*

*Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.*

**Solution.** Given : $P_{t1}$ = 10 kN ; $P_s$ = 5 kN ; $\sigma_{t(el)}$ = 100 MPa = 100 N/mm$^2$ ; $1/m$ = 0.3

Let $d$ = Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\, d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\, d^2} = \frac{6.365}{d^2} \text{ kN/mm}^2$$

**1.** ***According to maximum principal stress theory***

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4+4}\right] = \frac{15.365}{d^2} \text{ kN/mm}^2 = \frac{15\,365}{d^2} \text{ N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

∴ $d^2$ = 15 365/100 = 153.65 or $d$ = 12.4 mm **Ans.**

**2.** ***According to maximum shear stress theory***

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{9}{d^2} \text{ kN/mm}^2 = \frac{9000}{d^2} \text{ N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

∴ $d^2$ = 9000 / 50 = 180 or $d$ = 13.42 mm **Ans.**

**3. *According to maximum principal strain theory***

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2} \quad \text{...(As calculated before)}$$

and minimum principal stress,

*Front view of a jet engine. The rotors undergo high torsional and bending stresses.*

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-2.635}{d^2}\ \text{kN/mm}^2$$

$$= \frac{-2635}{d^2}\ \text{N/mm}^2$$

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$$d^2 = 16\,156 / 100 = 161.56 \quad \text{or} \quad d = 12.7 \text{ mm } \textbf{Ans.}$$

**4. *According to maximum strain energy theory***

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$$\therefore \quad d^4 = 26\,724 \quad \text{or} \quad d = 12.78 \text{ mm } \textbf{Ans.}$$

**5. *According to maximum distortion energy theory***

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\,391}{d^4} = 1$$

$$\therefore \quad d^4 = 32\,391 \quad \text{or} \quad d = 13.4 \text{ mm } \textbf{Ans.}$$

**Example 5.17.** *A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of bending moment 10 kN-m and a torsional moment 30 kN-m. Determine the diameter of the shaft using two different theories of failure, and assuming a factor of safety of 2. Take E = 210 GPa and poisson's ratio = 0.25.*

**Solution.** Given : $\sigma_{yt}$ = 700 MPa = 700 N/mm$^2$ ; $M$ = 10 kN-m = 10 × 10$^6$ N-mm ; $T$ = 30 kN-m = 30 × 10$^6$ N-mm ; $F.S.$ = 2 ; $E$ = 210 GPa = 210 × 10$^3$ N/mm$^2$ ; 1/$m$ = 0.25

Let $d$ = Diameter of the shaft in mm.

First of all, let us find the maximum and minimum principal stresses.

We know that section modulus of the shaft

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Bending (tensile) stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982\, d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

and shear stress due to torsional moment,

$$\tau = \frac{16\, T}{\pi\, d^3} = \frac{16 \times 30 \times 10^6}{\pi\, d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \text{...}(\because \sigma_2 = 0)$$

$$= \frac{101.8 \times 10^6}{2d^3} + \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{161 \times 10^6}{d^3} = \frac{211.9 \times 10^6}{d^3} \text{ N/mm}^2$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \text{...}(\because \sigma_2 = 0)$$

$$= \frac{50.9 \times 10^6}{d^3} - \frac{161 \times 10^6}{d^3} = \frac{-110.1 \times 10^6}{d^3} \text{ N/mm}^2$$

Let us now find out the diameter of shaft ($d$) by considering the maximum shear stress theory and maximum strain energy theory.

### 1. *According to maximum shear stress theory*

We know that maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\left[\frac{211.9 \times 10^6}{d^3} + \frac{110.1 \times 10^6}{d^3}\right] = \frac{161 \times 10^6}{d^3}$$

We also know that according to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{yt}}{2\, F.S.} \quad \text{or} \quad \frac{161 \times 10^6}{d^3} = \frac{700}{2 \times 2} = 175$$

∴ $d^3$ = 161 × 10$^6$ / 175 = 920 × 10$^3$ or $d$ = 97.2 mm **Ans.**

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**Note:** The value of maximum shear stress ($\tau_{max}$) may also be obtained by using the relation,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{101.8\times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8\times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\times 322 = \frac{161\times 10^6}{d^3}\ \text{N/mm}^2 \qquad \text{...(Same as before)}$$

## 2. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$\frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or
$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$\left[\frac{211.9\times 10^6}{d^3}\right]^2 + \left[\frac{-110.1\times 10^6}{d^3}\right]^2 - 2\times\frac{211.9\times 10^6}{d^3}\times\frac{-110.1\times 10^6}{d^3}\times 0.25 = \left(\frac{700}{2}\right)^2$$

or
$$\frac{44\ 902\times 10^{12}}{d^6} + \frac{12\ 122\times 10^{12}}{d^6} + \frac{11\ 665\times 10^{12}}{d^6} = 122\ 500$$

$$\frac{68\ 689\times 10^{12}}{d^6} = 122\ 500$$

$\therefore$ $d^6 = 68\ 689\times 10^{12}/122\ 500 = 0.5607\times 10^{12}$ or $d = 90.8$ mm **Ans.**

**Example 5.18.** *A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a torque T. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. the maximum shear stress; and 3. the maximum distortion strain energy theory of yielding.*

**Solution.** Given: $d$ = 50 mm ; $M$ = 2000 N-m = $2\times 10^6$ N-mm ; $\sigma_{yt}$ = 200 MPa = 200 N/mm$^2$

Let $T$ = Maximum torque without causing yielding of the shaft, in N-mm.

## 1. *According to maximum principal stress theory*

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32}\times d^3 = \frac{\pi}{32}\,(50)^3 = 12\ 273\ \text{mm}^3$$

$\therefore$ Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2\times 10^6}{12\ 273} = 163\ \text{N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16\,T}{\pi\,(50)^3} = 0.0407\times 10^{-3}\,T\ \text{N/mm}^2$$

$$\text{...}\left[\because T = \frac{\pi}{16}\times\tau\times d^3\right]$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} + \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407\times 10^{-3}\,T)^2}\right]$$

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$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

Minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} - \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \text{...(Taking } F.S. = 1)$$

$$\therefore \quad 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = 200$$

$$6642.5 + 1.65 + 10^{-9}\, T^2 = (200 - 81.5)^2 = 14\,042$$

$$T^2 = \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9$$

or $T = 2118 \times 10^3$ N-mm = 2118 N-m **Ans.**

**2. *According to maximum shear stress theory***

We know that according to maximum shear stress theory,

$$\tau_{max} = \tau_{yt} = \frac{\sigma_{yt}}{2}$$

$$\therefore \quad \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = \frac{200}{2} = 100$$

$$6642.5 + 1.65 \times 10^{-9}\, T^2 = (100)^2 = 10\,000$$

$$T^2 = \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9$$

$\therefore$ $T = 1426 \times 10^3$ N-mm = 1426 N-m **Ans.**

**3. *According to maximum distortion strain energy theory***

We know that according to maximum distortion strain energy theory

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = (\sigma_{yt})^2$$

$$\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]^2$$

$$-\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]\left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right] = (200)^2$$

$$2\left[(81.5)^2 + 6642.5 + 1.65 \times 10^{-9}\, T^2\right] - \left[(81.5)^2 - 6642.5 + 1.65 \times 10^{-9}\, T^2\right] = (200)^2$$

$$(81.5)^2 + 3 \times 6642.5 + 3 \times 1.65 \times 10^{-9}\, T^2 = (200)^2$$

$$26\,570 + 4.95 \times 10^{-9}\, T^2 = 40\,000$$

$$T^2 = \frac{40\,000 - 26\,570}{4.95 \times 10^{-9}} = 2713 \times 10^9$$

$\therefore$ $T = 1647 \times 10^3$ N-mm = 1647 N-m **Ans.**

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## 5.15 Eccentric Loading - Direct and Bending Stresses Combined

An external load, whose line of action is parallel but does not coincide with the centroidal axis of the machine component, is known as an ***eccentric load.*** The distance between the centroidal axis of the machine component and the eccentric load is called ***eccentricity*** and is generally denoted by *e*. The examples of eccentric loading, from the subject point of view, are *C*-clamps, punching machines, brackets, offset connecting links etc.

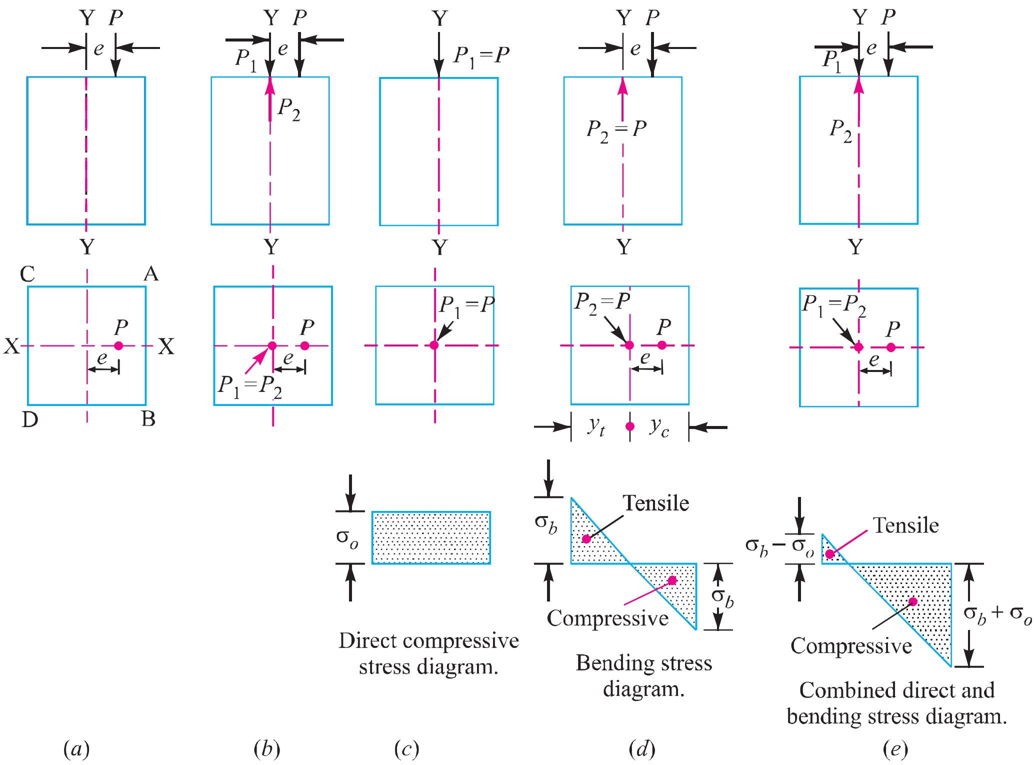


**Fig. 5.19.** Eccentric loading.

Consider a short prismatic bar subjected to a compressive load *P* acting at an eccentricity of *e* as shown in Fig. 5.19 (*a*).

Let us introduce two forces $P_1$ and $P_2$ along the centre line or neutral axis equal in magnitude to *P*, without altering the equilibrium of the bar as shown in Fig. 5.19 (*b*). A little consideration will show that the force $P_1$ will induce a direct compressive stress over the entire cross-section of the bar, as shown in Fig. 5.19 (*c*).

The magnitude of this direct compressive stress is given by

$$\sigma_o = \frac{P_1}{A} \text{ or } \frac{P}{A}, \text{ where } A \text{ is the cross-sectional area of the bar.}$$

The forces $P_1$ and $P_2$ will form a couple equal to $P \times e$ which will cause bending stress. This bending stress is compressive at the edge *AB* and tensile at the edge *CD*, as shown in Fig. 5.19 (*d*). The magnitude of bending stress at the edge *AB* is given by

$$\sigma_b = \frac{P \,.\, e \,.\, y_c}{I} \quad \text{(compressive)}$$

and bending stress at the edge *CD*,

$$\sigma_b = \frac{P \,.\, e \,.\, y_t}{I} \quad \text{(tensile)}$$

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where $y_c$ and $y_t$ = Distances of the extreme fibres on the compressive and tensile sides, from the neutral axis respectively, and

$I$ = Second moment of area of the section about the neutral axis *i.e.* $Y$-axis.

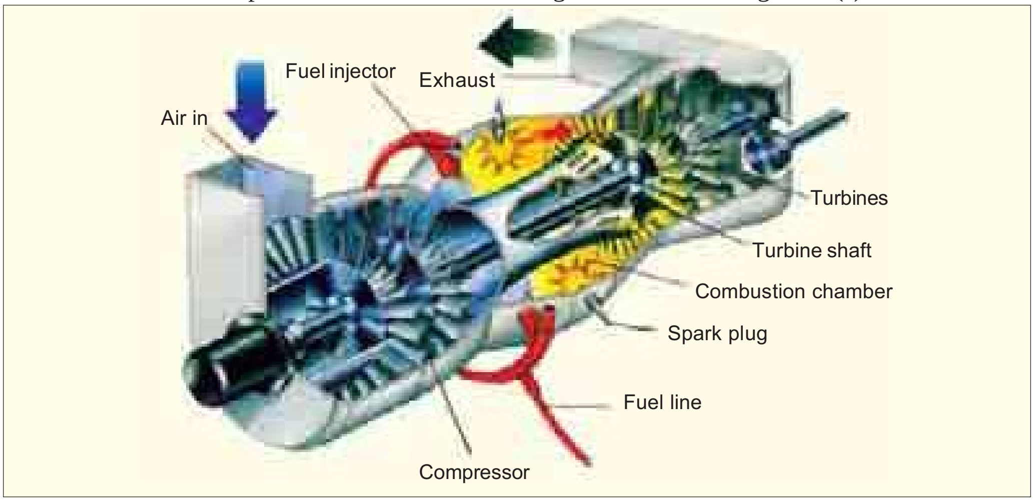
According to the principle of superposition, the maximum or the resultant compressive stress at the edge $AB$,

$$\sigma_c = \frac{P \cdot e \cdot y_c}{I} + \frac{P}{A} = {}^{*}\frac{M}{Z} + \frac{P}{A} = \sigma_b + \sigma_o$$

and the maximum or resultant tensile stress at the edge $CD$,

$$\sigma_t = \frac{P \cdot e \cdot y_t}{I} - \frac{P}{A} = \frac{M}{Z} - \frac{P}{A} = \sigma_b - \sigma_o$$

The resultant compressive and tensile stress diagram is shown in Fig. 5.19 (*e*).




*In a gas-turbine system, a compressor forces air into a combustion chamber. There, it mixes with fuel. The mixture is ignited by a spark. Hot gases are produced when the fuel burns. They expand and drive a series of fan blades called a turbine.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

**Notes: 1.** When the member is subjected to a tensile load, then the above equations may be used by interchanging the subscripts $c$ and $t$.

**2.** When the direct stress $\sigma_o$ is greater than or equal to bending stress $\sigma_b$, then the compressive stress shall be present all over the cross-section.

**3.** When the direct stress $\sigma_o$ is less than the bending stress $\sigma_b$, then the tensile stress will occur in the left hand portion of the cross-section and compressive stress on the right hand portion of the cross-section. In Fig. 5.19, the stress diagrams are drawn by taking $\sigma_o$ less than $\sigma_b$.

In case the eccentric load acts with eccentricity about two axes, as shown in Fig. 5.20, then the total stress at the extreme fibre

$$= \frac{P}{A} \pm \frac{P \cdot e_x \cdot x}{I_{XX}} \pm \frac{P \cdot e_y \cdot y}{I_{YY}}$$

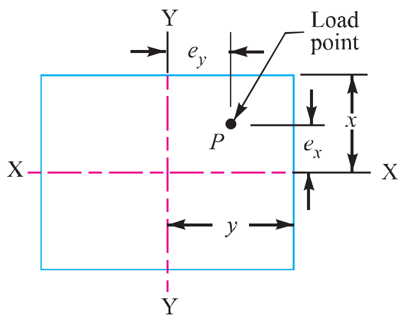



**Fig. 5.20.** Eccentric load with eccentricity about two axes.

* We know that bending moment, $M = P.e$ and section modulus, $Z = \dfrac{I}{y} = \dfrac{I}{y_c \text{ or } y_t}$

$\therefore$ Bending stress, $\sigma_b = M / Z$

**Example 5.19.** *A rectangular strut is 150 mm wide and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness as shown in Fig. 5.21. Find the maximum and minimum intensities of stress in the section.*

**Solution.** Given : $b$ = 150 mm ; $d$ = 120 mm ; $P$ = 180 kN = $180 \times 10^3$ N ; $e$ = 10 mm

We know that cross-sectional area of the strut,

$$A = b.d = 150 \times 120 = 18 \times 10^3 \text{ mm}^2$$

∴ Direct compressive stress,

$$\sigma_o = \frac{P}{A} = \frac{180 \times 10^3}{18 \times 10^3} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$

Section modulus for the strut,

$$Z = \frac{I_{YY}}{y} = \frac{d\,.\,b^3/12}{b/2} = \frac{d\,.\,b^2}{6} = \frac{120\,(150)^2}{6} = 450 \times 10^3 \text{ mm}^3$$

Bending moment, $M = P.e = 180 \times 10^3 \times 10 = 1.8 \times 10^6$ N-mm

∴ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{1.8 \times 10^6}{450 \times 10^3} = 4 \text{ N/mm}^2 = 4$ MPa

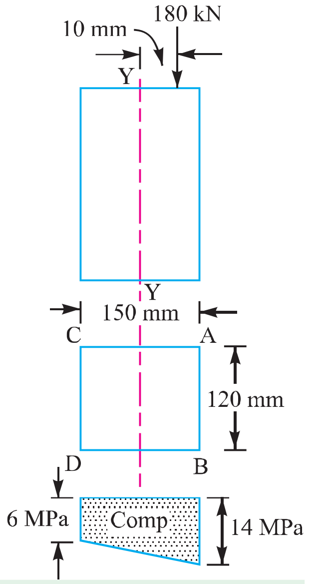


Fig. 5.21

Since $\sigma_o$ is greater than $\sigma_b$, therefore the entire cross-section of the strut will be subjected to compressive stress. The maximum intensity of compressive stress will be at the edge *AB* and minimum at the edge *CD*.

∴ Maximum intensity of compressive stress at the edge *AB*

$$= \sigma_o + \sigma_b = 10 + 4 = 14 \text{ MPa } \textbf{Ans.}$$

and minimum intensity of compressive stress at the edge *CD*

$$= \sigma_o - \sigma_b = 10 - 4 = 6 \text{ MPa } \textbf{Ans.}$$

**Example 5.20.** *A hollow circular column of external diameter 250 mm and internal diameter 200 mm, carries a projecting bracket on which a load of 20 kN rests, as shown in Fig. 5.22. The centre of the load from the centre of the column is 500 mm. Find the stresses at the sides of the column.*

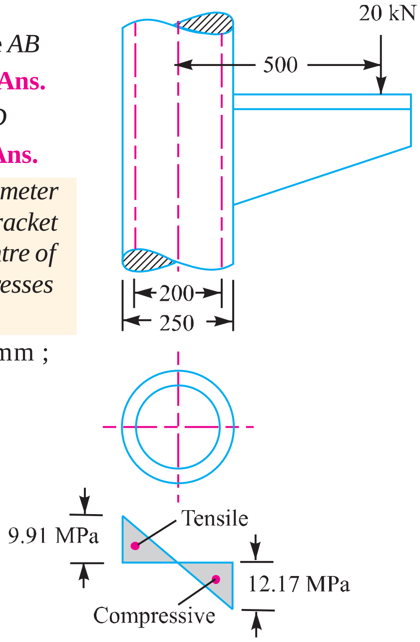
**Solution.** Given : $D$ = 250 mm ; $d$ = 200 mm ; $P$ = 20 kN = $20 \times 10^3$ N ; $e$ = 500 mm

We know that cross-sectional area of column,

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(250)^2 - (200)^2] = 17\,674 \text{ mm}^2$$

∴ Direct compressive stress,

$$\sigma_o = \frac{P}{A} = \frac{20 \times 10^3}{17\,674} = 1.13 \text{ N/mm}^2 = 1.13 \text{ MPa}$$



All dimensions in mm.
Fig. 5.22

Contents



Section modulus for the column,

$$Z = \frac{I}{y} = \frac{\frac{\pi}{64}\left[D^4 - d^4\right]}{D/2} = \frac{\frac{\pi}{64}\left[(250)^4 - (200)^4\right]}{250/2}$$

$$= 905.8 \times 10^3 \text{ mm}^3$$

Bending moment,

$$M = P.e$$
$$= 20 \times 10^3 \times 500$$
$$= 10 \times 10^6 \text{ N-mm}$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{10 \times 10^6}{905.8 \times 10^3}$$
$$= 11.04 \text{ N/mm}^2$$
$$= 11.04 \text{ MPa}$$

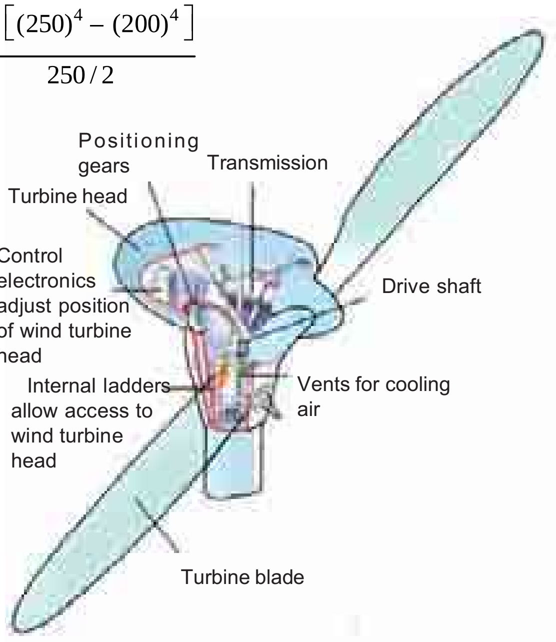
Since $\sigma_o$ is less than $\sigma_b$, therefore right hand side of the column will be subjected to compressive stress and the left hand side of the column will be subjected to tensile stress.

∴ Maximum compressive stress,

$$\sigma_c = \sigma_b + \sigma_o = 11.04 + 1.13$$
$$= 12.17 \text{ MPa } \textbf{Ans.}$$

and maximum tensile stress,

$$\sigma_t = \sigma_b - \sigma_o = 11.04 - 1.13 = 9.91 \text{ MPa } \textbf{Ans.}$$




*Wind turbine.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

**Example 5.21.** *A masonry pier of width 4 m and thickness 3 m, supports a load of 30 kN as shown in Fig. 5.23. Find the stresses developed at each corner of the pier.*

**Solution.** Given: $b = 4$ m ; $d = 3$ m ; $P = 30$ kN ; $e_x = 0.5$ m ; $e_y = 1$ m

We know that cross-sectional area of the pier,

$$A = b \times d = 4 \times 3 = 12 \text{ m}^2$$

Moment of inertia of the pier about *X*-axis,

$$I_{XX} = \frac{b\,.\,d^3}{12} = \frac{4 \times 3^3}{12} = 9 \text{ m}^4$$

and moment of inertia of the pier about *Y*-axis,

$$I_{YY} = \frac{d\,.\,b^3}{12} = \frac{3 \times 4^3}{12} = 16 \text{ m}^4$$

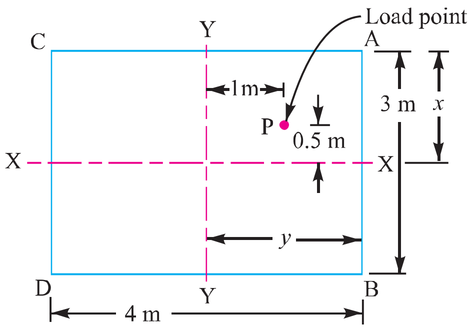



**Fig. 5.23**

Distance between *X*-axis and the corners *A* and *B*,

$$x = 3 / 2 = 1.5 \text{ m}$$

Distance between *Y*-axis and the corners *A* and *C*,

$$y = 4 / 2 = 2 \text{ m}$$

We know that stress at corner *A*,

$$\sigma_A = \frac{P}{A} + \frac{P\,.\,e_x\,.\,x}{I_{XX}} + \frac{P\,.\,e_y\,.\,y}{I_{YY}} \quad \text{... [∵ At } A\text{, both } x \text{ and } y \text{ are +ve]}$$

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$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 + 3.75 = 8.75 \text{ kN/m}^2$$ **Ans.**

Similarly stress at corner *B*,

$$\sigma_B = \frac{P}{A} + \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}} \qquad \text{... [}\because \text{ At } B, x \text{ is +ve and } y \text{ is –ve]}$$

$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 - 3.75 = 1.25 \text{ kN/m}^2$$ **Ans.**

Stress at corner *C*,

$$\sigma_C = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} + \frac{P \cdot e_y \cdot y}{I_{YY}} \qquad \text{... [At } C, x \text{ is –ve and } y \text{ is +ve]}$$

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 + 3.75 = 3.75 \text{ kN/m}^2$$ **Ans.**

and stress at corner *D*,

$$\sigma_D = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}} \qquad \text{... [At } D, \text{ both } x \text{ and } y \text{ are – ve]}$$

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 - 3.75 = -3.75 \text{ kN/m}^2 = 3.75 \text{ kN/m}^2 \text{ (tensile)}$$ **Ans.**

**Example 5.22.** *A mild steel link, as shown in Fig. 5.24 by full lines, transmits a pull of 80 kN. Find the dimensions b and t if b = 3t. Assume the permissible tensile stress as 70 MPa. If the original link is replaced by an unsymmetrical one, as shown by dotted lines in Fig. 5.24, having the same thickness t, find the depth $b_1$, using the same permissible stress as before.*

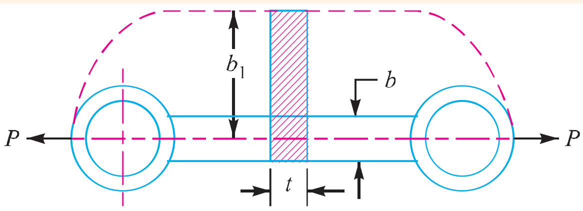


**Fig. 5.24**

**Solution.** Given : $P = 80$ kN $= 80 \times 10^3$ N ; $\sigma_t = 70$ MPa $= 70$ N/mm$^2$

When the link is in the position shown by full lines in Fig. 5.24, the area of cross-section,

$$A = b \times t = 3\,t \times t = 3\,t^2 \qquad \text{...(}\because\ b = 3t\text{)}$$

We know that tensile load (*P*),

$$80 \times 10^3 = \sigma_t \times A = 70 \times 3\,t^2 = 210\,t^2$$

$\therefore$ $$t^2 = 80 \times 10^3 / 210 = 381 \text{ or } t = 19.5 \text{ say } 20 \text{ mm}$$ **Ans.**

and $$b = 3\,t = 3 \times 20 = 60 \text{ mm}$$ **Ans.**

When the link is in the position shown by dotted lines, it will be subjected to direct stress as well as bending stress. We know that area of cross-section,

$$A_1 = b_1 \times t$$

$\therefore$ Direct tensile stress,

$$\sigma_o = \frac{P}{A} = \frac{P}{b_1 \times t}$$

and bending stress, $$\sigma_b = \frac{M}{Z} = \frac{P \cdot e}{Z} = \frac{6\,P \cdot e}{t\,(b_1)^2} \qquad \ldots \left(\because Z = \frac{t\,(b_1)^2}{6}\right)$$

$\therefore$ Total stress due to eccentric loading

$$= \sigma_b + \sigma_o = \frac{6\,P \cdot e}{t\,(b_1)^2} + \frac{P}{b_1 \times t} = \frac{P}{t \cdot b_1}\left(\frac{6\,e}{b_1} + 1\right)$$

Since the permissible tensile stress is the same as 70 N/mm$^2$, therefore

$$70 = \frac{80 \times 10^3}{20\,b_1}\left(\frac{6 \times b_1}{b_1 \times 2} + 1\right) = \frac{16 \times 10^3}{b_1} \qquad \ldots \left(\because \text{Eccentricity, } e = \frac{b_1}{2}\right)$$

$\therefore$ $b_1 = 16 \times 10^3 / 70 = 228.6$ say 230 mm **Ans.**

**Example 5.23.** *A cast-iron link, as shown in Fig. 5.25, is to carry a load of 20 kN. If the tensile and compressive stresses in the link are not to exceed 25 MPa and 80 MPa respectively, obtain the dimensions of the cross-section of the link at the middle of its length.*

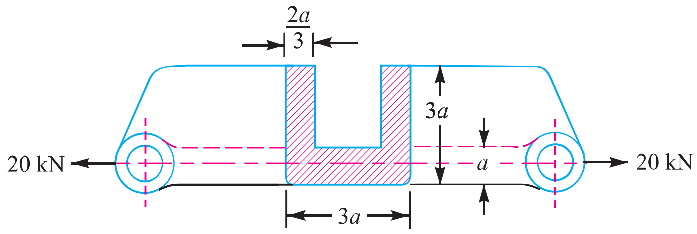


Fig. 5.25

**Solution.** Given : $P$ = 20 kN = 20 × 10$^3$ N ; $\sigma_{t(max)}$ = 25 MPa = 25 N/mm$^2$ ; $\sigma_{c(max)}$ = 80 MPa = 80 N/mm$^2$

Since the link is subjected to eccentric loading, therefore there will be direct tensile stress as well as bending stress. The bending stress at the bottom of the link is tensile and in the upper portion is compressive.

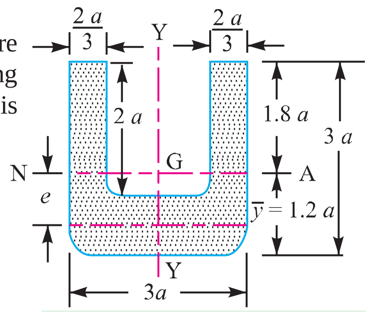


Fig. 5.26

We know that cross-sectional area of the link,

$$A = 3a \times a + 2 \times \frac{2a}{3} \times 2a$$
$$= 5.67\,a^2 \text{ mm}^2$$

$\therefore$ Direct tensile stress,

$$\sigma_o = \frac{P}{A} = \frac{20 \times 10^3}{5.67\,a^2} = \frac{3530}{a^2} \text{ N/mm}^2$$

Now let us find the position of centre of gravity (or neutral axis) in order to find the bending stresses.

Let $\bar{y}$ = Distance of neutral axis (N.A.) from the bottom of the link as shown in Fig. 5.26.

$$\therefore \quad \bar{y} = \frac{3a^2 \times \frac{a}{2} + 2 \times \frac{4a^2}{3} \times 2a}{5.67\,a^2} = 1.2\,a \text{ mm}$$

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Moment of inertia about N.A.,

$$I = \left[\frac{3a \times a^3}{12} + 3a^2\,(1.2a - 0.5a)^2\right] + 2\left[\frac{\frac{2}{3}\,a \times (2a)^3}{12} + \frac{4a^2}{3}\,(2a - 1.2a)^2\right]$$

$$= (0.25\,a^4 + 1.47\,a^4) + 2\,(0.44a^4 + 0.85\,a^4) = 4.3\,a^4 \text{ mm}^4$$

Distance of N.A. from the bottom of the link,

$$y_t = \bar{y} = 1.2\,a \text{ mm}$$

Distance of N.A. from the top of the link,

$$y_c = 3\,a - 1.2\,a = 1.8\,a \text{ mm}$$

Eccentricity of the load (*i.e.* distance of N.A. from the point of application of the load),

$$e = 1.2\,a - 0.5\,a = 0.7\,a \text{ mm}$$

We know that bending moment exerted on the section,

$$M = P.e = 20 \times 10^3 \times 0.7\,a = 14 \times 10^3\,a \text{ N-mm}$$

∴ Tensile stress in the bottom of the link,

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{I / y_t} = \frac{M \,.\, y_t}{I} = \frac{14 \times 10^3\,a \times 1.2\,a}{4.3\,a^4} = \frac{3907}{a^2}$$

and compressive stress in the top of the link,

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{I / y_c} = \frac{M \,.\, y_c}{I} = \frac{14 \times 10^3\,a \times 1.8\,a}{4.3\,a^4} = \frac{5860}{a^2}$$

We know that maximum tensile stress [$\sigma_{t\,(max)}$],

$$25 = \sigma_t + \sigma_c = \frac{3907}{a^2} + \frac{5860}{a^2} = \frac{9767}{a^2}$$

∴ $a^2 = 9767 / 25 = 390.7$ or $a = 19.76$ mm ...(*i*)

and maximum compressive stress [$\sigma_{c(max)}$],

$$80 = \sigma_c - \sigma_0 = \frac{5860}{a^2} - \frac{3530}{a^2} = \frac{2330}{a^2}$$

∴ $a^2 = 2330 / 80 = 29.12$ or $a = 5.4$ mm ...(*ii*)

We shall take the larger of the two values, *i.e.*

$a = 19.76$ mm **Ans.**

**Example 5.24.** *A horizontal pull P = 5 kN is exerted by the belting on one of the cast iron wall brackets which carry a factory shafting. At a point 75 mm from the wall, the bracket has a T-section as shown in Fig. 5.27. Calculate the maximum stresses in the flange and web of the bracket due to the pull.*

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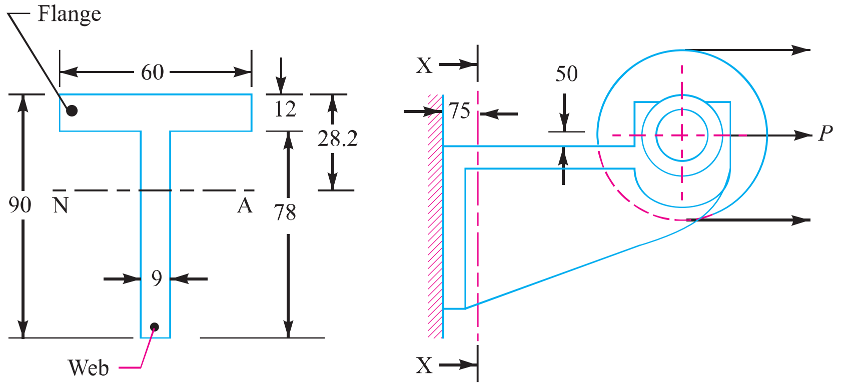


All dimensions in mm.

**Fig. 5.27**

**Solution.** Given : Horizontal pull, $P$ = 5 kN = 5000 N

Since the section is subjected to eccentric loading, therefore there will be direct tensile stress as well as bending stress. The bending stress at the flange is tensile and in the web is compressive.

We know that cross-sectional area of the section,

$$A = 60 \times 12 + (90 - 12)9 = 720 + 702 = 1422 \text{ mm}^2$$

$\therefore$ Direct tensile stress, $\sigma_0 = \dfrac{P}{A} = \dfrac{5000}{1422} = 3.51 \text{ N/mm}^2 = 3.51 \text{ MPa}$

Now let us find the position of neutral axis in order to determine the bending stresses. The neutral axis passes through the centre of gravity of the section.

Let $\bar{y}$ = Distance of centre of gravity (*i.e.* neutral axis) from top of the flange.

$$\therefore \quad \bar{y} = \frac{60 \times 12 \times \dfrac{12}{2} + 78 \times 9 \left(12 + \dfrac{78}{2}\right)}{720 + 702} = 28.2 \text{ mm}$$

Moment of inertia of the section about N.A.,

$$I = \left[\frac{60\,(12)^3}{12} + 720\,(28.2 - 6)^2\right] + \left[\frac{9\,(78)^3}{12} + 702\,(51 - 28.2)^2\right]$$

$$= (8640 + 354\,845) + (355\,914 + 364\,928) = 1\,084\,327 \text{ mm}^4$$

*This picture shows a reconnoissance helicopter of air force. Its dark complexion absorbs light that falls on its surface. The flat and sharp edges deflect radar waves and they do not return back to the radar. These factors make it difficult to detect the helicopter.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

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Distance of N.A. from the top of the flange,

$$y_t = \bar{y} = 28.2 \text{ mm}$$

Distance of N.A. from the bottom of the web,

$$y_c = 90 - 28.2 = 61.8 \text{ mm}$$

Distance of N.A. from the point of application of the load (*i.e.* eccentricity of the load),

$$e = 50 + 28.2 = 78.2 \text{ mm}$$

We know that bending moment exerted on the section,

$$M = P \times e = 5000 \times 78.2 = 391 \times 10^3 \text{ N-mm}$$

∴ Tensile stress in the flange,

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{I / y_t} = \frac{M \,.\, y_t}{I} = \frac{391 \times 10^3 \times 28.2}{1\,084\,327} = 10.17 \text{ N/mm}^2$$
$$= 10.17 \text{ MPa}$$

and compressive stress in the web,

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{I / y_c} = \frac{M \,.\, y_c}{I} = \frac{391 \times 10^3 \times 61.8}{1\,084\,327} = 22.28 \text{ N/mm}^2$$
$$= 22.28 \text{ MPa}$$

We know that maximum tensile stress in the flange,

$$\sigma_{t(max)} = \sigma_b + \sigma_o = \sigma_t + \sigma_o = 10.17 + 3.51 = 13.68 \text{ MPa} \quad \textbf{Ans.}$$

and maximum compressive stress in the flange,

$$\sigma_{c(max)} = \sigma_b - \sigma_o = \sigma_c - \sigma_o = 22.28 - 3.51 = 18.77 \text{ MPa} \quad \textbf{Ans.}$$

**Example 5.25.** *A mild steel bracket as shown in Fig. 5.28, is subjected to a pull of 6000 N acting at 45° to its horizontal axis. The bracket has a rectangular section whose depth is twice the thickness. Find the cross-sectional dimensions of the bracket, if the permissible stress in the material of the bracket is limited to 60 MPa.*

**Solution.** Given : $P = 6000$ N ; $\theta = 45°$ ; $\sigma = 60$ MPa $= 60$ N/mm$^2$

Let $t$ = Thickness of the section in mm, and

$b$ = Depth or width of the section $= 2\,t$ ...(Given)

We know that area of cross-section,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

and section modulus,

$$Z = \frac{t \times b^2}{6} = \frac{t\,(2t)^2}{6} = \frac{4\,t^3}{6} \text{ mm}^3$$

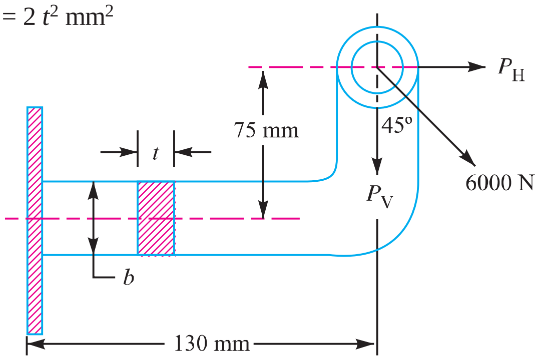

**Fig. 5.28**

Horizontal component of the load,

$$P_H = 6000 \cos 45° = 6000 \times 0.707 = 4242 \text{ N}$$

∴ Bending moment due to horizontal component of the load,

$$M_H = P_H \times 75 = 4242 \times 75 = 318\,150 \text{ N-mm}$$

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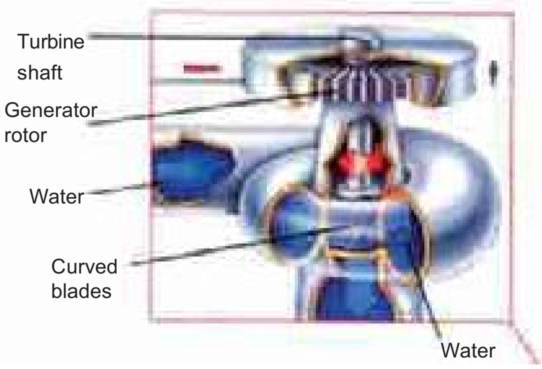
A little consideration will show that the bending moment due to the horizontal component of the load induces tensile stress on the upper surface of the bracket and compressive stress on the lower surface of the bracket.

∴ Maximum bending stress on the upper surface due to horizontal component,

$$\sigma_{bH} = \frac{M_H}{Z}$$

$$= \frac{318\ 150 \times 6}{4\ t^3}$$




*Schematic of a hydel turbine.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$= \frac{477\ 225}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

Vertical component of the load,

$$P_V = 6000 \sin 45° = 6000 \times 0.707 = 4242 \text{ N}$$

∴ Direct stress due to vertical component,

$$\sigma_{oV} = \frac{P_V}{A} = \frac{4242}{2t^2} = \frac{2121}{t^2} \text{ N/mm}^2 \text{ (tensile)}$$

Bending moment due to vertical component of the load,

$$M_V = P_V \times 130 = 4242 \times 130 = 551\ 460 \text{ N-mm}$$

This bending moment induces tensile stress on the upper surface and compressive stress on the lower surface of the bracket.

∴ Maximum bending stress on the upper surface due to vertical component,

$$\sigma_{bV} = \frac{M_V}{Z} = \frac{551\ 460 \times 6}{4\ t^3} = \frac{827\ 190}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

and total tensile stress on the upper surface of the bracket,

$$\sigma = \frac{477\ 225}{t^3} + \frac{2121}{t^2} + \frac{827\ 190}{t^3} = \frac{1\ 304\ 415}{t^3} + \frac{2121}{t^2}$$

Since the permissible stress ($\sigma$) is 60 N/mm$^2$, therefore

$$\frac{1\ 304\ 415}{t^3} + \frac{2121}{t^2} = 60 \quad \text{or} \quad \frac{21\ 740}{t^3} + \frac{35.4}{t^2} = 1$$

∴ $t$ = 28.4 mm **Ans.** ... (By hit and trial)

and $b = 2t = 2 \times 28.4 = 56.8$ mm **Ans.**

**Example 5.26.** *A C-clamp as shown in Fig. 5.29, carries a load P = 25 kN. The cross-section of the clamp at X-X is rectangular having width equal to twice thickness. Assuming that the clamp is made of steel casting with an allowable stress of 100 MPa, find its dimensions. Also determine the stresses at sections Y-Y and Z-Z.*

**Solution.** Given : $P = 25$ kN $= 25 \times 10^3$ N ; $\sigma_{t(max)} = 100$ MPa $= 100$ N/mm$^2$

***Dimensions at X-X***

Let $t$ = Thickness of the section at *X-X* in mm, and

$b$ = Width of the section at *X-X* in mm = $2t$ ...(Given)

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We know that cross-sectional area at X-X,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

∴ Direct tensile stress at X-X,

$$\sigma_o = \frac{P}{A} = \frac{25 \times 10^3}{2\,t^2}$$

$$= \frac{12.5 \times 10^3}{t^3} \text{ N/mm}^2$$

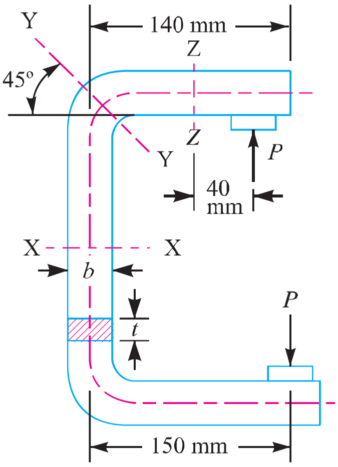


Fig. 5.29

Bending moment at X-X due to the load P,

$$M = P \times e = 25 \times 10^3 \times 140$$

$$= 3.5 \times 10^6 \text{ N-mm}$$

Section modulus, $$Z = \frac{t\,.\,b^2}{6} = \frac{t\,(2t)^2}{6} = \frac{4\,t^3}{6} \text{ mm}^3$$

...(∵ $b = 2t$)

∴ Bending stress at X-X,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6 \times 6}{4\,t^3} = \frac{5.25 \times 10^6}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

We know that the maximum tensile stress [$\sigma_{t\,(max)}$],

$$100 = \sigma_o + \sigma_b = \frac{12.5 \times 10^3}{t^2} + \frac{5.25 \times 10^6}{t^3}$$

or $$\frac{125}{t^2} + \frac{52.5 \times 10^3}{t^3} - 1 = 0$$

∴ $t = 38.5$ mm **Ans.** ...(By hit and trial)

and $b = 2\,t = 2 \times 38.5 = 77$ mm **Ans.**

***Stresses at section Y-Y***

Since the cross-section of frame is uniform throughout, therefore cross-sectional area of the frame at section Y-Y,

$$A = b \sec 45° \times t = 77 \times 1.414 \times 38.5 = 4192 \text{ mm}^2$$

Component of the load perpendicular to the section

$$= P \cos 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform tensile stress over the section.

∴ Uniform tensile stress over the section,

$$\sigma = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

Component of the load parallel to the section

$$= P \sin 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform shear stress over the section.

∴ Uniform shear stress over the section,

$$\tau = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

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We know that section modulus,

$$Z = \frac{t\,(b \sec 45°)^2}{6} = \frac{38.5\,(77 \times 1.414)^2}{6} = 76 \times 10^3 \text{ mm}^3$$

Bending moment due to load (*P*) over the section *Y-Y*,

$$M = 25 \times 10^3 \times 140 = 3.5 \times 10^6 \text{ N-mm}$$

∴ Bending stress over the section,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6}{76 \times 10^3} = 46 \text{ N/mm}^2 = 46 \text{ MPa}$$

Due to bending, maximum tensile stress at the inner corner and the maximum compressive stress at the outer corner is produced.

∴ Maximum tensile stress at the inner corner,

$$\sigma_t = \sigma_b + \sigma_o = 46 + 4.2 = 50.2 \text{ MPa}$$

and maximum compressive stress at the outer corner,

$$\sigma_c = \sigma_b - \sigma_o = 46 - 4.2 = 41.8 \text{ MPa}$$

Since the shear stress acts at right angles to the tensile and compressive stresses, therefore maximum principal stress (tensile) on the section *Y-Y* at the inner corner

$$= \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{50.2}{2} + \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] \text{ MPa}$$

$$= 25.1 + 25.4 = 50.5 \text{ MPa} \quad \textbf{Ans.}$$

and maximum principal stress (compressive) on section *Y-Y* at outer corner

$$= \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right] = \frac{41.8}{2} + \frac{1}{2}\left[\sqrt{(41.8)^2 + 4 \times (4.2)^2}\right] \text{ MPa}$$

$$= 20.9 + 21.3 = 42.2 \text{ MPa} \quad \textbf{Ans.}$$

Maximum shear stress $= \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] = 25.4 \text{ MPa}$ **Ans.**

***Stresses at section Z-Z***

We know that bending moment at section *Z-Z*,

$$= 25 \times 10^3 \times 40 = 1 \times 10^6 \text{ N-mm}$$

and section modulus,

$$Z = \frac{t\,.\,b^2}{6} = \frac{38.5\,(77)^2}{6} = 38 \times 10^3 \text{ mm}^3$$

∴ Bending stress at section *Z-Z*,

$$\sigma_b = \frac{M}{Z} = \frac{1 \times 10^6}{38 \times 10^3} = 26.3 \text{ N/mm}^2 = 26.3 \text{ MPa} \quad \textbf{Ans.}$$

The bending stress is tensile at the inner edge and compressive at the outer edge. The magnitude of both these stresses is 26.3 MPa. At the neutral axis, there is only transverse shear stress. The shear stress at the inner and outer edges will be zero.

We know that *maximum transverse shear stress,

$$\tau_{max} = 1.5 \times \text{Average shear stress} = 1.5 \times \frac{P}{b.t} = 1.5 \times \frac{25 \times 10^3}{77 \times 38.5}$$

$$= 12.65 \text{ N/mm}^2 = 12.65 \text{ MPa} \quad \textbf{Ans.}$$

* Refer Art. 5.16

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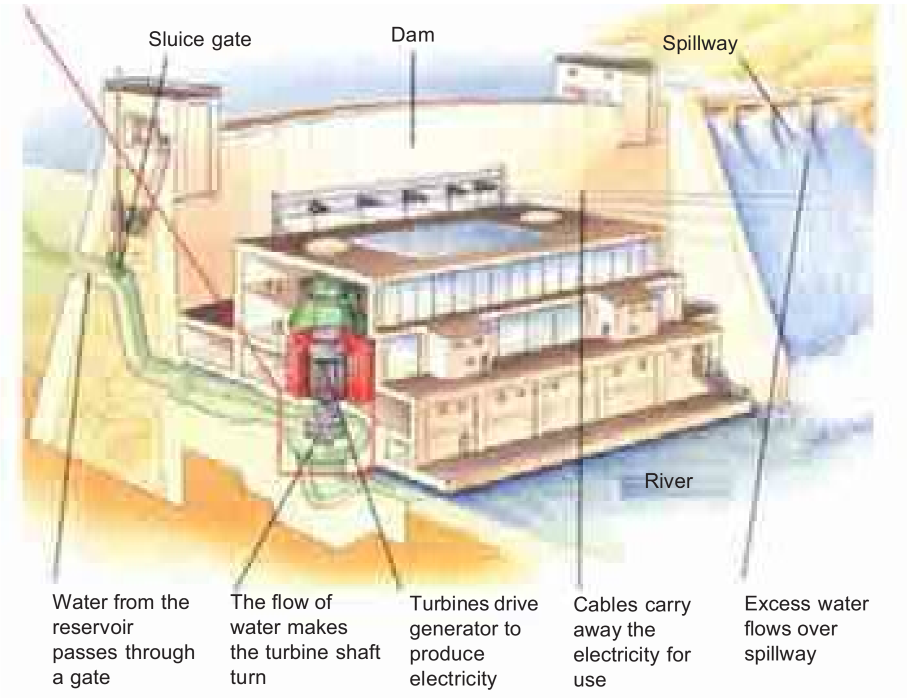


*General layout of a hydroelectric plant.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

## 5.16 Shear Stresses in Beams

In the previous article, we have assumed that no shear force is acting on the section. But, in actual practice, when a beam is loaded, the shear force at a section always comes into play along with the bending moment. It has been observed that the effect of the shear stress, as compared to the bending stress, is quite negligible and is of not much importance. But, sometimes, the shear stress at a section is of much importance in the design. It may be noted that the shear stress in a beam is not uniformly distributed over the cross-section but varies from zero at the outer fibres to a maximum at the neutral surface as shown in Fig. 5.30 and Fig. 5.31.

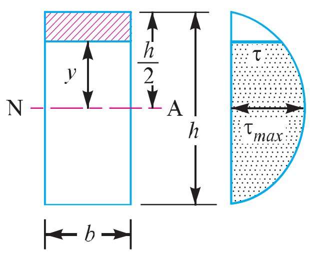


**Fig. 5.30.** Shear stress in a rectangular beam.

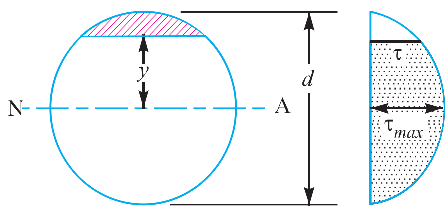


**Fig. 5.31.** Shear stress in a circular beam.

The shear stress at any section acts in a plane at right angle to the plane of the bending stress and its value is given by

$$\tau = \frac{F}{I \cdot b} \times A.\bar{y}$$

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where $F$ = Vertical shear force acting on the section,

$I$ = Moment of inertia of the section about the neutral axis,

$b$ = Width of the section under consideration,

$A$ = Area of the beam above neutral axis, and

$\bar{y}$ = Distance between the C.G. of the area and the neutral axis.

The following values of maximum shear stress for different cross-section of beams may be noted :

**1.** For a beam of rectangular section, as shown in Fig. 5.30, the shear stress at a distance $y$ from neutral axis is given by

$$\tau = \frac{F}{2I}\left(\frac{h^2}{4} - y^2\right) = \frac{3F}{2b \cdot h^3}(h^2 - 4y^2) \qquad \ldots \left[\because I = \frac{b \cdot h^3}{12}\right]$$

and maximum shear stress,

$$\tau_{max} = \frac{3F}{2b \cdot h} \qquad \ldots \left(\text{Substituting } y = \frac{h}{2}\right)$$

$$= 1.5\,\tau_{(average)} \qquad \ldots \left[\because \tau_{(average)} = \frac{F}{\text{Area}} = \frac{F}{b.h}\right]$$

The distribution of stress is shown in Fig. 5.30.

**2.** For a beam of circular section as shown in Fig. 5.31, the shear stress at a distance $y$ from neutral axis is given by

$$\tau = \frac{F}{3I}\left(\frac{d^2}{4} - y^2\right) = \frac{16\,F}{3\,\pi\,d^4}(d^2 - 4y^2)$$

and the maximum shear stress,

$$\tau_{max} = \frac{4F}{3 \times \frac{\pi}{4} d^2} \qquad \ldots \left(\text{Substituting } y = \frac{d}{2}\right)$$

$$= \frac{4}{3}\tau_{(average)} \qquad \ldots \left[\because \tau_{(average)} = \frac{F}{\text{Area}} = \frac{F}{\frac{\pi}{4}d^2}\right]$$

The distribution of stress is shown in Fig. 5.31.

**3.** For a beam of *I*-section as shown in Fig. 5.32, the maximum shear stress occurs at the neutral axis and is given by

$$\tau_{max} = \frac{F}{I \cdot b}\left[\frac{B}{8}(H^2 - h^2) + \frac{b.h^2}{8}\right]$$

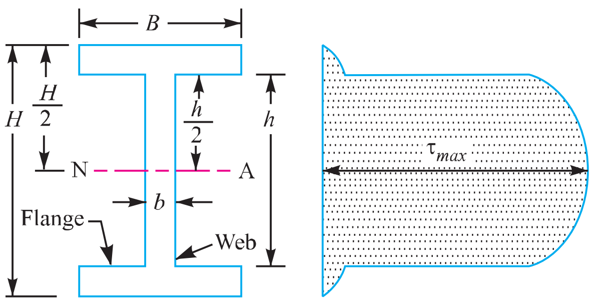


**Fig. 5.32**

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Shear stress at the joint of the web and the flange

$$= \frac{F}{8I}(H^2 - h^2)$$

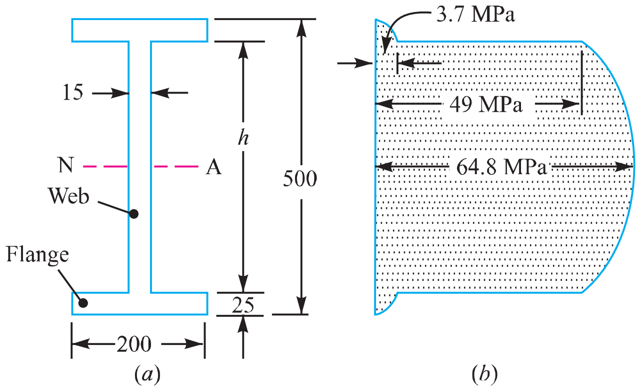
and shear stress at the junction of the top of the web and bottom of the flange

$$= \frac{F}{8I} \times \frac{B}{b}(H^2 - h^2)$$

The distribution of stress is shown in Fig. 5.32.

**Example 5.27.** *A beam of I-section 500 mm deep and 200 mm wide has flanges 25 mm thick and web 15 mm thick, as shown in Fig. 5.33 (a). It carries a shearing force of 400 kN. Find the maximum intensity of shear stress in the section, assuming the moment of inertia to be 645 × 10⁶ mm⁴. Also find the shear stress at the joint and at the junction of the top of the web and bottom of the flange.*

**Solution.** Given : $H$ = 500 mm ; $B$ = 200 mm ; $h$ = 500 – 2 × 25 = 450 mm ; $b$ = 15 mm ; $F$ = 400 kN = 400 × $10^3$ N ; $I$ = 645 × $10^6$ mm$^4$




All dimensions in mm.

**Fig. 5.33**

### *Maximum intensity of shear stress*

We know that maximum intensity of shear stress,

$$\tau_{max} = \frac{F}{I \cdot b}\left[\frac{B}{8}(H^2 - h^2) + \frac{b \cdot h^2}{8}\right]$$

$$= \frac{400 \times 10^3}{645 \times 10^6 \times 15}\left[\frac{200}{8}(500^2 - 450^2) + \frac{15 \times 450^2}{8}\right] \text{N/mm}^2$$

$= 64.8$ N/mm$^2$ = 64.8 MPa **Ans.**

The maximum intensity of shear stress occurs at neutral axis.

**Note :** The maximum shear stress may also be obtained by using the following relation :

$$\tau_{max} = \frac{F \cdot A \cdot \bar{y}}{I \cdot b}$$

We know that area of the section above neutral axis,

$$A = 200 \times 25 + \frac{450}{2} \times 15 = 8375 \text{ mm}^2$$

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Distance between the centre of gravity of the area and neutral axis,

$$\bar{y} = \frac{200 \times 25\,(225 + 12.5) + 225 \times 15 \times 112.5}{8375} = 187 \text{ mm}$$

$$\therefore \quad \tau_{max} = \frac{400 \times 10^3 \times 8375 \times 187}{645 \times 10^6 \times 15} = 64.8 \text{ N/mm}^2 = 64.8 \text{ MPa } \textbf{Ans.}$$

***Shear stress at the joint of the web and the flange***

We know that shear stress at the joint of the web and the flange

$$= \frac{F}{8I}(H^2 - h^2) = \frac{400 \times 10^3}{8 \times 645 \times 10^6}\left[(500)^2 - (450)^2\right] \text{ N/mm}^2$$

$$= 3.7 \text{ N/mm}^2 = 3.7 \text{ MPa } \textbf{Ans.}$$

***Shear stress at the junction of the top of the web and bottom of the flange***

We know that shear stress at junction of the top of the web and bottom of the flange

$$= \frac{F}{8I} \times \frac{B}{b}(H^2 - h^2) = \frac{400 \times 10^3}{8 \times 645 \times 10^6} \times \frac{200}{15}\left[(500)^2 - (450)^2\right] \text{ N/mm}^2$$

$$= 49 \text{ N/mm}^2 = 49 \text{ MPa } \textbf{Ans.}$$

The stress distribution is shown in Fig. 5.33 (*b*)

## EXERCISES

1. A steel shaft 50 mm diameter and 500 mm long is subjected to a twisting moment of 1100 N-m, the total angle of twist being 0.6°. Find the maximum shearing stress developed in the shzaft and modulus of rigidity. **[Ans. 44.8 MPa; 85.6 kN/m$^2$]**
2. A shaft is transmitting 100 kW at 180 r.p.m. If the allowable stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more than 1° in a length of 3 metres. Take $C$ = 80 GPa. **[Ans. 105 mm]**
3. Design a suitable diameter for a circular shaft required to transmit 90 kW at 180 r.p.m. The shear stress in the shaft is not to exceed 70 MPa and the maximum torque exceeds the mean by 40%. Also find the angle of twist in a length of 2 metres. Take $C$ = 90 GPa. **[Ans. 80 mm; 2.116°]**
4. Design a hollow shaft required to transmit 11.2 MW at a speed of 300 r.p.m. The maximum shear stress allowed in the shaft is 80 MPa and the ratio of the inner diameter to outer diameter is 3/4. **[Ans. 240 mm; 320 mm]**
5. Compare the weights of equal lengths of hollow shaft and solid shaft to transmit a given torque for the same maximum shear stress. The material for both the shafts is same and inside diameter is 2/3 of outside diameter in case of hollow shaft. **[Ans. 0.56]**
6. A spindle as shown in Fig. 5.34, is a part of an industrial brake and is loaded as shown. Each load $P$ is equal to 4 kN and is applied at the mid point of its bearing. Find the diameter of the spindle, if the maximum bending stress is 120 MPa. **[Ans. 22 mm]**

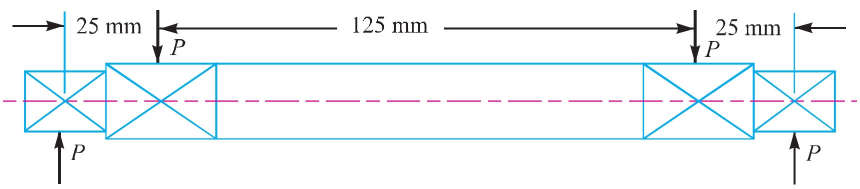


**Fig. 5.34**

7. A cast iron pulley transmits 20 kW at 300 r.p.m. The diameter of the pulley is 550 mm and has four straight arms of elliptical cross-section in which the major axis is twice the minor axis. Find the dimensions of the arm, if the allowable bending stress is 15 MPa. **[Ans. 60 mm; 30 mm]**

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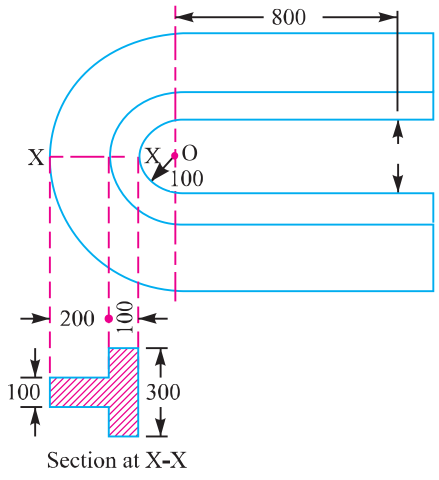
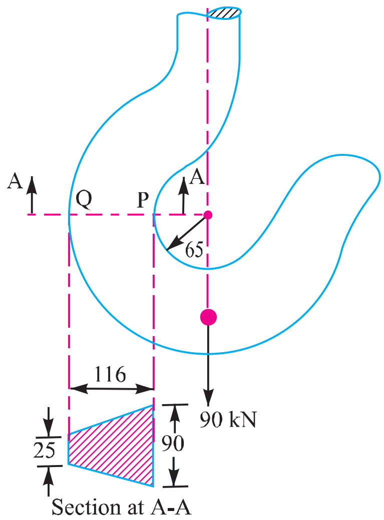
**8.** A shaft is supported in bearings, the distance between their centres being 1 metre. It carries a pulley in the centre and it weighs 1 kN. Find the diameter of the shaft, if the permissible bending stress for the shaft material is 40 MPa. **[Ans. 40 mm]**

**9.** A punch press, used for stamping sheet metal, has a punching capacity of 50 kN. The section of the frame is as shown in Fig. 5.35. Find the resultant stress at the inner and outer fibre of the section.
**[Ans. 28.3 MPa (tensile); 17.7 MPa (compressive)]**




All dimensions in mm.

**Fig. 5.35**




All dimensions in mm.

**Fig. 5.36**

**10.** A crane hook has a trapezoidal section at *A-A* as shown in Fig. 5.36. Find the maximum stress at points *P* and *Q*. **[Ans. 118 MPa (tensile); 62 MPa (compressive)]**

**11.** A rotating shaft of 16 mm diameter is made of plain carbon steel. It is subjected to axial load of 5000 N, a steady torque of 50 N-m and maximum bending moment of 75 N-m. Calculate the factor of safety available based on 1. Maximum normal stress theory; and 2. Maximum shear stress theory.

Assume yield strength as 400 MPa for plain carbon steel. If all other data remaining same, what maximum yield strength of shaft material would be necessary using factor of safety of 1.686 and maximum distortion energy theory of failure. Comment on the result you get.
**[Ans. 1.752; 400 MPa]**

**12.** A hand cranking lever, as shown in Fig. 5.37, is used to start a truck engine by applying a force *F* = 400 N. The material of the cranking lever is 30C8 for which yield strength = 320 MPa; Ultimate tensile strength = 500 MPa ; Young's modulus = 205 GPa ; Modulus of rigidity = 84 GPa and poisson's ratio = 0.3.

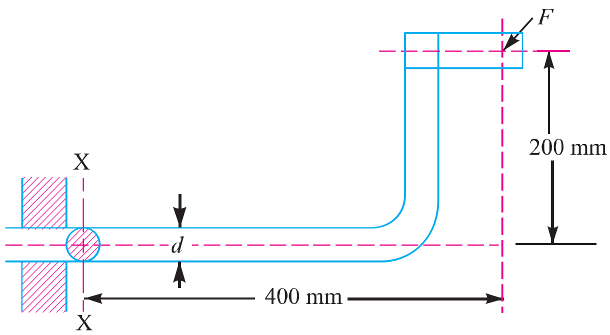



**Fig. 5.37**

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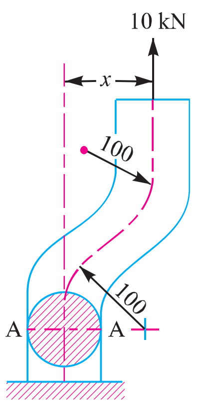
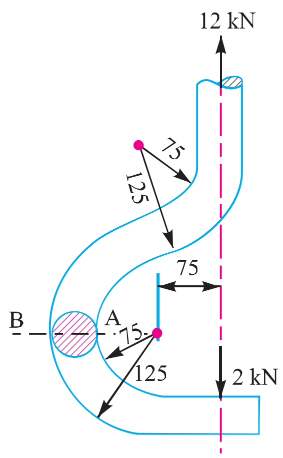
Assuming factor of safety to be 4 based on yield strength, design the diameter '*d*' of the lever at section *X-X* near the guide bush using : 1. Maximum distortion energy theory; and 2. Maximum shear stress theory. **[Ans. 28.2 mm; 28.34 mm]**

**13.** An offset bar is loaded as shown in Fig. 5.38. The weight of the bar may be neglected. Find the maximum offset (*i.e.*, the dimension *x*) if allowable stress in tension is limited to 70 MPa. **[Ans. 418 mm]**




All dimensions in mm.

**Fig. 5.38**

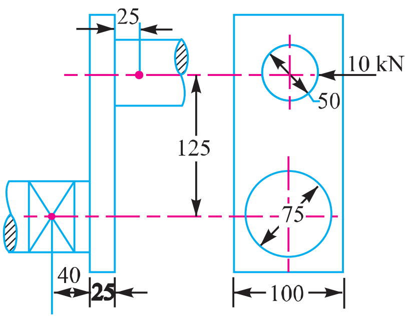



All dimensions in mm.

**Fig. 5.39**

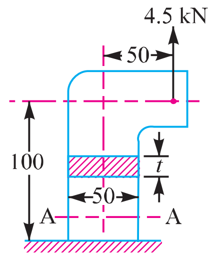
**14.** A crane hook made from a 50 mm diameter bar is shown in Fig. 5.39. Find the maximum tensile stress and specify its location. **[Ans. 35.72 MPa at A]**

**15.** An overhang crank, as shown in Fig. 5.40 carries a tangential load of 10 kN at the centre of the crankpin. Find the maximum principal stress and the maximum shear stress at the centre of the crank-shaft bearing. **[Ans. 29.45 MPa; 18.6 MPa]**




All dimensions in mm.

**Fig. 5.40**




All dimensions in mm.

**Fig. 5.41**

**16.** A steel bracket is subjected to a load of 4.5 kN, as shown in Fig. 5.41. Determine the required thickness of the section at A-A in order to limit the tensile stress to 70 MPa. **[Ans. 9 mm]**

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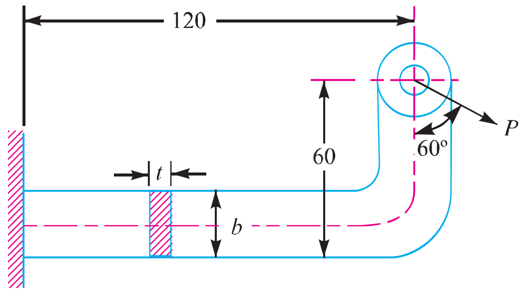
17. A wall bracket, as shown in Fig. 5.42, is subjected to a pull of $P$ = 5 kN, at 60° to the vertical. The cross-section of bracket is rectangular having $b = 3t$ . Determine the dimensions $b$ and $t$ if the stress in the material of the bracket is limited to 28 MPa. **[Ans. 75 mm; 25 mm]**



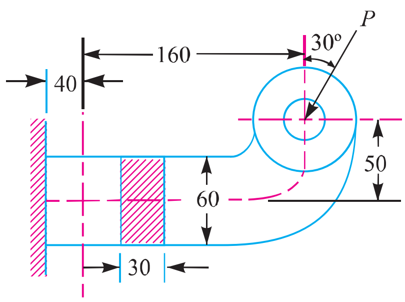



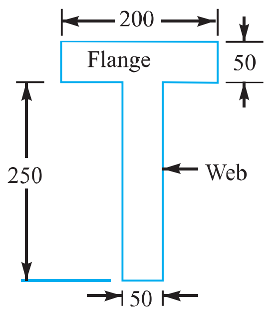

**Fig. 5.42** **Fig. 5.43**

18. A bracket, as shown in Fig. 5.43, is bolted to the framework of a machine which carries a load $P$. The cross-section at 40 mm from the fixed end is rectangular with dimensions, 60 mm × 30 mm. If the maximum stress is limited to 70 MPa, find the value of $P$.

**[Ans. 3000 N]**

19. A $T$-section of a beam, as shown in Fig. 5.44, is subjected to a vertical shear force of 100 kN. Calculate the shear stress at the neutral axis and at the junction of the web and the flange. The moment of inertia at the neutral axis is 113.4 × $10^6$ $mm^4$.

**[Ans. 11.64 MPa; 11 MPa; 2.76 MPa]**



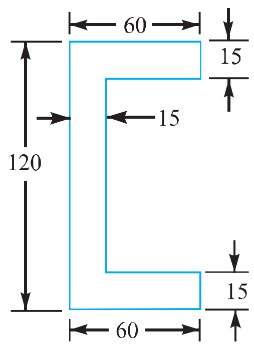




**Fig. 5.44** **Fig. 5.45**

20. A beam of channel section, as shown in Fig. 5.45, is subjected to a vertical shear force of 50 kN. Find the ratio of maximum and mean shear stresses. Also draw the distribution of shear stresses.

**[Ans. 2.22]**

## QUESTIONS

1. Derive a relation for the shear stress developed in a shaft, when it is subjected to torsion.
2. State the assumptions made in deriving a bending formula.

3. Prove the relation: $M/I = \sigma/y = E/R$

   where $M$ = Bending moment; $I$ = Moment of inertia; $\sigma$ = Bending stress in a fibre at a distance $y$ from the neutral axis; $E$ = Young's modulus; and $R$ = Radius of curvature.
4. Write the relations used for maximum stress when a machine member is subjected to tensile or compressive stresses along with shearing stresses.
5. Write short note on maximum shear stress theory *verses* maximum strain energy theory.
6. Distinguish clearly between direct stress and bending stress.
7. What is meant by eccentric loading and eccentricity?
8. Obtain a relation for the maximum and minimum stresses at the base of a symmetrical column, when it is subjected to

   (*a*) an eccentric load about one axis, and (*b*) an eccentric load about two axes.

## OBJECTIVE TYPE QUESTIONS

1. When a machine member is subjected to torsion, the torsional shear stress set up in the member is
   (*a*) zero at both the centroidal axis and outer surface of the member
   (*b*) Maximum at both the centroidal axis and outer surface of the member
   (*c*) zero at the centroidal axis and maximum at the outer surface of the member
   (*d*) none of the above
2. The torsional shear stress on any cross-section normal to the axis is ......... the distance from the centre of the axis.
   (*a*) directly proportional to (*b*) inversely proportional to
3. The neutral axis of a beam is subjected to
   (*a*) zero stress (*b*) maximum tensile stress
   (*c*) maximum compressive stress (*d*) maximum shear stress
4. At the neutral axis of a beam,
   (*a*) the layers are subjected to maximum bending stress
   (*b*) the layers are subjected to tension (*c*) the layers are subjected to compression
   (*d*) the layers do not undergo any strain
5. The bending stress in a curved beam is
   (*a*) zero at the centroidal axis (*b*) zero at the point other than centroidal axis
   (*c*) maximum at the neutral axis (*d*) none of the above
6. The maximum bending stress, in a curved beam having symmetrical section, always occur, at the
   (*a*) centroidal axis (*b*) neutral axis
   (*c*) inside fibre (*d*) outside fibre
7. If $d$ = diameter of solid shaft and $\tau$ = permissible stress in shear for the shaft material, then torsional strength of shaft is written as
   (*a*) $\frac{\pi}{32} d^4 \tau$ (*b*) $d \log_e \tau$
   (*c*) $\frac{\pi}{16} d^3 \tau$ (*d*) $\frac{\pi}{32} d^3 \tau$
8. If $d_i$ and $d_o$ are the inner and outer diameters of a hollow shaft, then its polar moment of inertia is
   (*a*) $\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$ (*b*) $\frac{\pi}{32}\left[(d_o)^3 - (d_i)^3\right]$
   (*c*) $\frac{\pi}{32}\left[(d_o)^2 - (d_i)^2\right]$ (*d*) $\frac{\pi}{32}(d_o - d_i)$

**9.** Two shafts under pure torsion are of identical length and identical weight and are made of same material. The shaft *A* is solid and the shaft *B* is hollow. We can say that

(*a*) shaft *B* is better than shaft *A*

(*b*) shaft *A* is better than shaft *B*

(*c*) both the shafts are equally good

**10.** A solid shaft transmits a torque *T*. The allowable shear stress is τ. The diameter of the shaft is

(*a*) $\sqrt[3]{\dfrac{16\,T}{\pi\,\tau}}$  (*b*) $\sqrt[3]{\dfrac{32\,T}{\pi\,\tau}}$

(*c*) $\sqrt[3]{\dfrac{64\,T}{\pi\,\tau}}$  (*d*) $\sqrt[3]{\dfrac{16\,T}{\tau}}$

**11.** When a machine member is subjected to a tensile stress ($\sigma_t$) due to direct load or bending and a shear stress (τ) due to torsion, then the maximum shear stress induced in the member will be

(*a*) $\frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$  (*b*) $\frac{1}{2}\left[\sqrt{(\sigma_t)^2 - 4\,\tau^2}\right]$

(*c*) $\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$  (*d*) $(\sigma_t)^2 + 4\,\tau^2$

**12.** Rankine's theory is used for

(*a*) brittle materials  (*b*) ductile materials

(*c*) elastic materials  (*d*) plastic materials

**13.** Guest's theory is used for

(*a*) brittle materials  (*b*) ductile materials

(*c*) elastic materials  (*d*) plastic materials

**14.** At the neutral axis of a beam, the shear stress is

(*a*) zero  (*b*) maximum

(*c*) minimum

**15.** The maximum shear stress developed in a beam of rectangular section is ........ the average shear stress.

(*a*) equal to  (*b*) $\frac{4}{3}$ times

(*c*) 1.5 times

## ANSWERS

| | | | | |
|---|---|---|---|---|
| **1.** (*b*) | **2.** (*a*) | **3.** (*a*) | **4.** (*d*) | **5.** (*b*) |
| **6.** (*c*) | **7.** (*c*) | **8.** (*a*) | **9.** (*a*) | **10.** (*a*) |
| **11.** (*a*) | **12.** (*a*) | **13.** (*b*) | **14.** (*b*) | **15.** (*c*) |

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C H A P T E R

# 6

# Variable Stresses in Machine Parts



## 6.1 Introduction

We have discussed, in the previous chapter, the stresses due to static loading only. But only a few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses.

## 6.2 Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load *W*, as shown in Fig. 6.1. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (*i.e.* at point *A*) are under compressive stress and the lower fibres (*i.e.* at point *B*) are under tensile stress. After



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