LUKHDHIRJI ENGINEERING COLLEGE, MORBI

GENERAL DEPARTMENT

SUBJECT: CV& PDE(3140610) Civil SEM-4

Tutorial: 1 (Complex Number & Function)

Q-1 Attempt the following :

1. Show that if c is any nth root of unity other than unity itself, then $1+c+c^2+\text{----}+c^{n-1}=0$
2. Find and plot all roots of $\sqrt[3]{8i}$
3. Solve the equation $z^2-(5+i)z+8+i=0$
4. Find and plot all the roots of $(1+i)^{1/3}$
5. Find all solutions of sinz=2
6. Find all roots of the equation $\log z = \frac{i\pi}{2}$
7. Show that $\cos\left(i\bar{z}\right)=\overline{\cos(iz)}$ for all z.
8. Find the Principal argument of $z=\frac{-2}{1+i\sqrt{3}}$
9. Find the Principal Value of $\left[\frac{e}{2}\left(-1-i\sqrt{3}\right)\right]^{3\pi i}$
10. Define Log(x+iy) .Determine Log (1-i).
11. Find real and imaginary part of $(-1-i)^7+(-1+i)^7$

Q-2 Attempt the following:

i) State the necessary and sufficient condition for a function to be analytic and prove the necessary condition.

ii) Show that if f(z) is analytic in a domain D and |f(z)| = k= const. in D, then f(z)= const. in D.

iii) Check whether the following functions are analytic or not.

(a) $f(z) = z^{5/2}$ (b) $f(z)= \bar{z}$

iv) Show that f(z)= z Im(z) is differentiable only at z=0 and f'(0)=0.

v) Check whether the function f(z)=sin z is analytic or not. If analytic, find its derivative.

vi) Check whether the following functions are analytic or not at any point: (a)$f(z) = e^{\bar{z}}$ (b) $f(z) = 2x+ ixy^2$

vii) Simplify the following using DeMoivre's theorem.

(1) $\frac{(\cos 2\theta+i\sin 2\theta)^{\frac{3}{2}}(\cos\theta-i\sin\theta)^3}{(\cos 3\theta-i\sin 3\theta)^2(\cos 5\theta-i\sin 5\theta)^{\frac{2}{5}}}$ (2) $(1+i\sqrt{3})^{90}+(1-i\sqrt{3})^{90}$

viii) Determine a such that function $u=e^{ax}\cos 5y$ is harmonic and find a conjugate harmonic.

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Tutorial: 2 (Complex Integration)

1. Evaluate by using the definition of an integral as the limit of a sum
   a. (i) $\int_C dz$ (ii) $\int_C |dz|$ (iii) $\int_C z dz$
   b. Where C is the arc joining the points z = a and z = B.
2. Evaluate $\int_C \left(x^2 + ixy\right) dz$ from (1, 1) to (2, 4) along the curve x = t, y = t^2.
3. (Important Little Integral)
   a. Prove that (i) $\oint_C \frac{dz}{z-a} = 2\pi i$ (ii) $\oint_C (z-a)^n dz = 0$ [n is an integer $\neq -1$]
   b. Over the circle | z – a | = r
4. Evaluate $\int_C \frac{z+2}{z} dz$ where C is the semi circle | z | = 2.
5. Evaluate $\int_C (z - z^2) dz$, where c is the upper half of the circle |z|=1.
6. Evaluate: $\oint_c \frac{z^2+5}{z-3} dz$, where C is the circle |z|=4.
7. Evaluate: $\oint_c \frac{e^z}{z^2+1} dz$,where C is the circle |z|=2.
8. Evaluate: $\oint_c \frac{z dz}{z^2-1}$ where C is the circle |z|=2.
9. Evaluate: $\oint_c \frac{z+4}{z^2+2z+5} dz$,where C is the circle |z+1|=1.
10. Evaluate $\oint_c \frac{e^{-z}}{z+1} dz$, Where C is the circle (1) |z|=1/2 (2) |z|=2
11. Evaluate: $\oint_c \frac{z-1}{(z+1)^2 (z-2)} dz$, where C is the circle |z-i|=2
12. Evaluate: $\oint_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle |z|=3
13. Evaluate: $\oint_c \frac{z}{z^2+1} dz$, where C is the circle (1) |z+1/z|=2 (2) |z+i|=1
14. Evaluate: $\oint_c \frac{3z^2+z}{z^2-1} dz$, where C is the circle |z-1|=1

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15. Evaluate: $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle |z|=3

16. Evaluate: $\oint_c \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle |z|=3/2

17. Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ & $y = \pm 2$ where C is described in positive sense. . Evaluate

    (1) $\oint_C \frac{z}{2z+1} dz$ (2) $\oint_C \frac{\cos z}{z(z^2+8)} dz$

18. Evaluate: $\oint_c \frac{zdz}{(9-z^2)(z+i)}$, where C is the circle |z|=2

19. Evaluate: $\oint_c \frac{e^{2z}}{(z+1)^4} dz$, Where C is the circle |z|=2]

20. Evaluate: $\oint_c \frac{\sin^2 z}{(z-\frac{\pi}{6})^3} dz$, where C is the circle |z|=1

21. Evaluate the integral $\oint_c \frac{zdz}{z^4-1}$, where C is the circle |z-2|=2

22. Evaluate: $\oint_c \frac{1}{(z^3-1)^2} dz$, where C is the circle |z-1|=1

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Tutorial: 3 (Power Series and Residue)

1. Expand $f(z)=\frac{1}{z}$ in Taylor series about $z_o=1$
2. What is the ROC of the Taylor series of $\frac{1}{z^2-3z+2}$ about z = 3i.
3. Expand $\frac{1}{z(z^2-3z+2)}$ about z=0 for the region a) $1<|z|<2$ b) $|z|=2$
4. Obtain the Taylor and Laurent series which represent the function

   $\frac{z^2-1}{(z+2)(z+3)}$ in the region $a)|z|<2$ $b)|z|>3$ $c)2<|z|<3$
5. Find the residue of the function f(z)= $\frac{1}{z^4+1}$
6. Find the residue at the singular point $a)\frac{1}{z^4+z^3-2z^2}$ $b)\frac{z}{z^2+16}$
7. Using Cauchy's residue thm. (Counter clockwise) Evaluate the following integrals
   a. $\oint_C \frac{2z+6}{z^2+4}\, dz \;, C: |z-i|=2$
   b. $\oint_C \frac{e^z}{z^4+5z^3}\, dz \;, C: |z|-2$
8. Determine the residues of $f(z)=\frac{3z-4}{z(z-1)(2-2)}$ at each of its poles in the finite z-plane
9. Determine the residues at each poles

   (a) $(\frac{z+1}{z-1})^3$ (b) $\frac{z+1}{(z^2-16)(z+2)}$ (c) $\frac{1-e^{2z}}{z^4}$
10. Determine the residue of $f(z)=\frac{z^2-2z}{(z+1)^2(z^2+4)}$, at each of its poles in the finite z-plane
11. Find the sum of the residues of the function $f(z)=\frac{\sin z}{z\cos z}$ at its poles inside the circle |z|=2
12. Determine the residue of $f(z)=\left(\frac{\sin z}{z^2}\right)^3$ at z=0
13. Cauchy's Residue Theorem Evaluate, $\oint_c \frac{z}{(2z-1)^2}dz$ where C is the circle.

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Tutorial: 4 (Contour and conformal mapping)

Q-1 Defn. of Bilinear Transformation, Cross ratio, Normal form of Bilinear Transformation, Fixed point.

Q-2 Find Bilinear Transformation which transforms z=2, 1, 0 into w=1,0,i

Q-3 Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$.

Q-4 Find the bilinear transformation which maps the points z=-1, 1, ∞ on w= $-i$,-1, i.

Q-5 Find the bilinear transformation that maps the points z=∞, i., 0 into the points w=0, i., ∞.

Q-6 Determine the bilinear transformation which maps z=0, 1, ∞ into w= i.,-1,- i. respectively. Under this transformation show that the interior of the unit circle of the z-plane maps onto the half plane above the real axis of w-plane.

Q-7 Find the bilinear transformation which maps the points z=0, - i.,-1 into w= i., 1, 0 respectively

Q-8 Find the fixed points and the normal forms of the following transformations

(1) $w = \frac{z-1}{z+1}$ (2) $w = -\frac{2z+4i}{iz+1}$

(3) $w = -\frac{z+4}{2z+5}$ (4) $w = -\frac{z-3}{z+1}$

Q-10 Under the transformation $w = \frac{1}{z}$, find the image of $|z-2i| = 2$ in w-plane.

Q-11 Find the image of the triangle with vertices i, 1+i, 1-i in the z plane under the transformation $w = e^{\frac{5\pi i}{3}} z - 2 + 4i$.

Q-12 Find the image of the region $\frac{1}{2} \le x \le 1$ and $\frac{1}{2} \le y \le 1$ mapped by the transformation $w = z^2$ in the w-plane.

Q-13 Discuss the application of the transformation $w = z^2$ to the area in the first quadrant of the z-plane bounded by the axes and the circles |z|=a, |z|=b
(a>b>0)

Q-14 Show that the transformation $w = \frac{1}{z}$ maps the circle $|z-3| = 5$ into the circle $\left|w + \frac{3}{16}\right| = \frac{5}{16}$

Q-15 Find the image of the strip 2<x<3 under the transformation $w = \frac{1}{z}$.

Q-16 Find the image of the real axis of the z-plane on the w-plane under the transformation $w = \frac{1}{z+i}$.

Q-17 Show that the transformation $w = i\frac{1-z}{1+z}$ transforms the circle |z|=1 into the real axis of w-plane and the interior of the circle |z|<1 into the upper half of the w-plane.

Q. 18 Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}$ using the residue theorem.