

# LUKHDHIRJI ENGINEERING COLLEGE, MORBI

Semester: 1

Subject - Mathematics - 1

Branch: ALL

## Tutorial-2 Sequence & Series

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1. Discuss the convergence of the following series:

$$(1) 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots + \frac{2}{3^{n-1}} + \dots$$

$$(4) 1 - 2 + 4 - 8 + \dots + (-1)^{n-1} 2^{n-1} + \dots$$

$$(2) \frac{9}{100} + \frac{9}{100^2} + \frac{9}{100^3} + \dots + \frac{9}{100^n} + \dots$$

$$(5) \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{(n+1)(n+2)} + \dots$$

$$(3) 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + (-1)^{n-1} \frac{1}{2^{n-1}} + \dots$$

2. Discuss the convergence of the following series (Geometric Series Test):

$$(1) \sum_{n=0}^{\infty} \left( \frac{1}{2^n} + \frac{(-1)^n}{5^n} \right)$$

$$(2) \sum_{n=0}^{\infty} \left( \frac{2^{n+1}}{5^n} \right)$$

3. Use partial fraction to find the sum of each of the following series:

$$(1) \sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

$$(3) \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$$

$$(2) \sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$$

$$(4) \sum_{n=1}^{\infty} \left( \tan^{-1} n - \tan^{-1}(n+1) \right)$$

4. In following, which series converge, and which diverge? Find sum if series converge.

$$(1) \sum_{n=0}^{\infty} \cos n\pi$$

$$(6) \sum_{n=1}^{\infty} \left( 1 + \frac{1}{n} \right)^n$$

$$(2) \sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$$

$$(7) \sum_{n=1}^{\infty} \ln \left( \frac{n}{n+1} \right)$$

$$(3) \sum_{n=0}^{\infty} e^{-2n}$$

$$(8) \sum_{n=1}^{\infty} \ln \left( \frac{n}{2n+1} \right)$$

$$(4) \sum_{n=0}^{\infty} \frac{2}{10^n}$$

$$(9) \sum_{n=1}^{\infty} \left( \frac{e}{\pi} \right)^n$$

$$(5) \sum_{n=0}^{\infty} \frac{2^n - 1}{3^n}$$

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5. Find condition on  $x$  such

s.

$$(1) \sum_{n=0}^{\infty} 2^n x^n$$

$$(3) \sum_{n=0}^{\infty} (-1)^n (x+1)^n$$

$$(2) \sum_{n=0}^{\infty} (-1)^n x^{-2n}$$

$$(4) \sum_{n=0}^{\infty} 3 \left( \frac{x-1}{2} \right)^n$$

6. Discuss the convergence of the following series:

$$(1) \sum_{n=1}^{\infty} e^{-n}$$

$$(6) \sum_{n=1}^{\infty} \frac{-8}{n}$$

$$(2) \sum_{n=1}^{\infty} \frac{n}{n+1}$$

$$(7) \sum_{n=1}^{\infty} \frac{5^n}{4^n + 3}$$

$$(3) \sum_{n=1}^{\infty} \frac{5}{n+1}$$

$$(8) \sum_{n=1}^{\infty} \frac{5}{n+1}$$

$$(4) \sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$$

$$(9) \sum_{n=1}^{\infty} \frac{1}{(\ln 2)^2}$$

$$(5) \sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}}$$

7. Check the convergence of the following series:

$$(1) \sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$$

$$(5) \sum_{n=1}^{\infty} \frac{3^n}{n^3 2^n}$$

$$(2) \sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$$

$$(6) \sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$$

$$(3) \sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$$

$$(7) \sum_{n=1}^{\infty} \frac{(3n)!}{n!(n+1)!(n+2)!}$$

$$(4) \sum_{n=1}^{\infty} \frac{n!}{n^n}$$

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8. Which of the following series converges, and which diverges?

$$(1) \sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$$

$$(2) \sum_{n=1}^{\infty} \frac{3}{n + \sqrt{n}}$$

$$(3) \sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$$

$$(4) \sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2}$$

$$(5) \sum_{n=1}^{\infty} \frac{2n}{3n-1}$$

$$(6) \sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$$

$$(7) \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$$

$$(8) \sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$$

$$(9) \sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$$

$$(10) \sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^3}$$

$$(11) \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}}$$

$$(12) \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

$$(13) \sum_{n=1}^{\infty} \frac{1-n}{n2^n}$$

$$(14) \sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$$

9. Check the convergence of the following series:

$$(1) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$$

$$(2) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+3}$$

$$(3) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^4+1}$$

$$(4) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3+n}{5+n}$$

$$(5) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{\frac{3}{2}}}$$

$$(7) \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln n}$$