* Today's Topics:

1. Types of Elasticity:

(i) Young's Modulus (Y)
(ii) Bulk Modulus (K)
(iii) Modulus of Rigidity (σ) or Shear Modulus (S)

2. Elastic Behaviours of Solids

(i) Elastic After Effect
(ii) Elastic Hysteresis
(iii) Elastic Fatigue

3. Working Stress & Factor of Safety.

4. Factors Affecting Elasticity.

(i) Effect of Hammering, Rolling and Annealing
(ii) Effect of Impurities
(iii) Effect of change of Temperature.

5. Poisson's Ratio. (σ)

(i) Young's Modulus (Y):

$$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$= \frac{F_\perp / A}{\Delta L / L}$$

$$= \frac{F_\perp \cdot L}{A (\Delta L)}$$

- If the elongation produced is not proportional to the applied force, Young's modulus can be determined as follows.

$$Y = \frac{dF_\perp / A}{dL / L}$$

$$= \frac{dF_\perp \cdot L}{A \cdot (dL)}$$

Here, $\frac{dF_\perp}{A}$ = Very small (infinitesimal) increase in the longitudinal stress

$\frac{dL}{L}$ = Corresponding increase in longitudinal strain

(ii) Bulk Modulus (K)

$$K = \frac{\text{Normal Stress}}{\text{Volume Strain}}$$

$$= \frac{F_\perp / A}{\Delta V / V}$$

$$= \frac{P \cdot V}{\Delta V}$$

- If the change in volume is not proportional to the normal stress then bulk modulus can be determined as follows,

$$K = \frac{dp}{dV} \cdot V$$

Here, dV = very small (infinitesimal) change in volume.
dp = corresponding small change in pressure.

- Inverse of bulk modulus is known as compressibility and denoted by "C"

$\therefore$ Compressibility ... $C = \frac{1}{K}$

(iii) Modulus of Rigidity (G) or Shear Modulus (S):

$$G = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$= \frac{F_{\parallel}/A}{\theta} = \frac{F_{\parallel}/A}{\Delta x/L} \quad \left\{ \theta = \frac{\Delta x}{L} \right.$$

$$= \frac{F_{\parallel} \cdot L}{A \cdot (\Delta x)}$$

- If the shear stress is not proportional to the shear strain then modulus of rigidity can be determined as follows,

$$G = \frac{dF_{\parallel}/A}{d\theta}$$

3. Working Stress & Factor of Safety.

- **Working stress:**

"The maximum stress applied on a given material within the elastic limit is called working stress or working strength (or tenacity) of the material."

- **Working Load:**

The load corresponding to the working stress is called working load.

- **Factor of safety:**

The ratio of ultimate stress (or breaking stress) to working stress is defined as Factor of safety.

$$\therefore \text{Factor of safety} = \frac{\text{Breaking stress}}{\text{Working stress}}$$

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5. Poisson's Ratio (σ)

When a load is applied on a wire of length L and radius r, its length increases by ΔL and the radius decreases by Δr.

Therefore two types of strains are produced by this deforming force.

(i) Longitudinal strain ... $\frac{\Delta L}{L}$, and

(ii) Lateral strain ... $\frac{-\Delta r}{r}$.

The ratio of lateral strain to longitudinal strain is called Poisson's ratio.

$$\therefore \text{Poisson's ratio} \ldots \sigma = \frac{\text{Lateral Strain}}{\text{Longitudinal strain}}$$

$$\therefore \sigma = \frac{-\Delta r/r}{\Delta L/L}$$

$$\therefore \sigma = \frac{-\Delta r}{r} \times \frac{L}{\Delta L}$$

* Note: (i) Here, negative sign indicates the inverse relationship of change in length and change in radius.
(ii) The practical value of Poisson's ratio lies between 0 and 0.5.

2. Elastic Behaviours of solids :

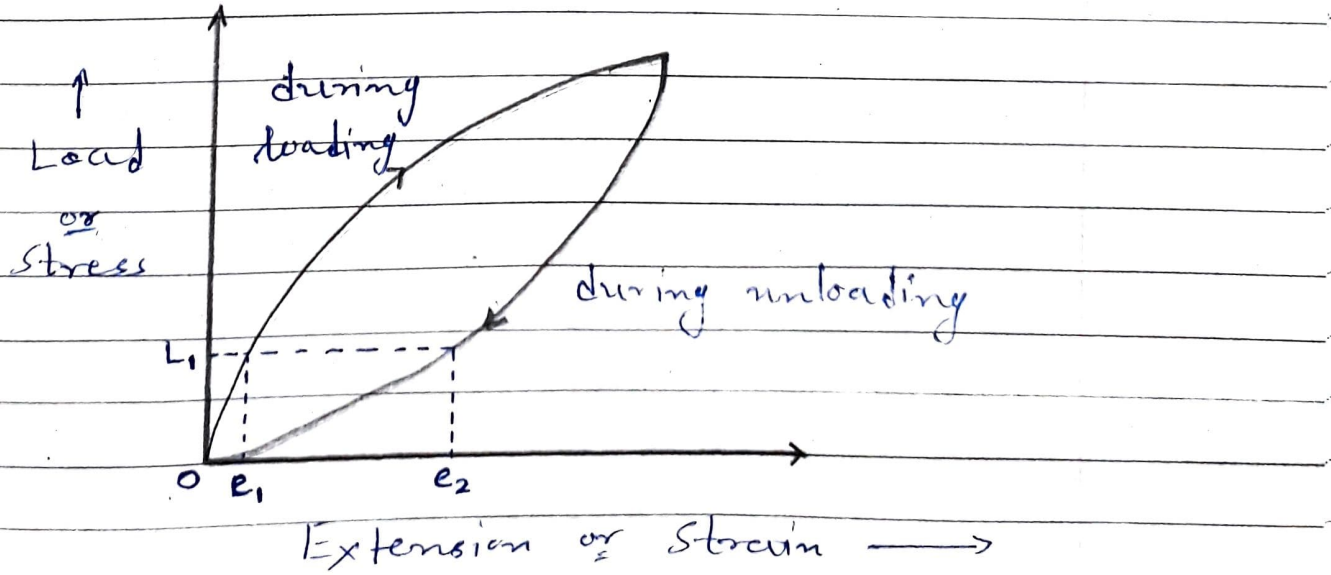
(i) Elastic After-effect.

Some solids regain their original condition on removal of load within the elastic limit but they take some time to do so.

This delay in coming back to the original condition on removing the deforming force is called elastic after-effect.

- Note: Quartz, phosphor-bronze, gold and silver do not show elastic after effect. They regain their initial condition as soon as the load is removed.

(ii) Elastic Hysteresis :



- Due to the elastic after effect, the strain in a material tends to remain present or lag behind the stress applied, with the result that during a rapidly changing stress, the strain is smaller for the same value of stress when it is increasing than when it is decreasing.

- This is shown in the figure. For the same load L_1, while increasing and then decreasing the stress, two different values of extensions e_1 and e_2 ($e_1 < e_2$) are observed respectively.

- This lag between load and extension, or more precisely, a lag between stress and strain is called elastic hysteresis.

(iii) Elastic Fatigue:

It is observed that the torsional vibrations die off much faster in the case of wire kept vibrating continuously for a long time than that if a fresh wire is selected for torsional vibration.

- This is observed for any elastic body under an alternating strain.
- The continuously vibrating wire gets "fatigued" and stops.
- Lord Kelvin called it as elastic fatigue.

- Thus, a body under repeated strains above its elastic limit loses its elastic properties and may break under a stress less than its normal breaking stress even within its elastic limit.