1 Find the force required to increase the length of a wire of 10^{-6} m^2 area of cross-section by 50 % whose Young's modulus is 2×10^{11} N/m^2.

Here, $A = 10^{-6}\ m^2$

$\Delta L = 50\ \%$ of L

$\therefore \Delta L = \frac{L}{2}$

$Y = 2\times10^{11}\ N/m^2$

$F = ?$

Now, Young's modulus is given by

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{F/A}{\Delta L/L}$$

$$\therefore F = \frac{Y(\Delta L)A}{L}$$

$$\therefore F = \frac{2\times10^{11}\times L\times10^{-6}}{2L}$$

$$\underline{\underline{F = 10^{5}\ N}}$$

2. A bronze bar having a 50 mm diameter and 170 cm length is subjected to a tensile stress of 70 MN/m^2. Calculate the extension produced in the bar and the work done during the process. (Young's modulus of bronze = 85×10^9 N/m^2.)

Here, $d = 50$ mm $\Rightarrow r = 25$ mm $= 25 \times 10^{-3}$ m

$L = 170$ cm $= 1.7$ m

Stress $= 70$ MN/m^2 $= 70 \times 10^6$ N/m^2

$Y = 85 \times 10^9$ N/m^2

$\Delta L = ?$

$W = ?$

Now, Young's modulus is,

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$\therefore Y = \frac{\text{Stress}}{\Delta L / L}$$

$$\therefore \Delta L = \frac{\text{Stress} \times L}{Y}$$

$$= \frac{70 \times 10^6 \times 1.7}{85 \times 10^9}$$

$$= 1.4 \times 10^{-3} \text{ m}$$

$$= 1.4 \text{ mm}$$

Now, work done is given by

$$W = \frac{1}{2} \times \text{Stretching Force} \times \text{Stretch}$$

$$= \frac{1}{2} \times (\text{Stress} \times A) \times \Delta L \quad \{\because \text{Stress} =$$

$$= \frac{1}{2} \times \text{Stress} \times (\pi r^2) \times \Delta L$$

$$= \frac{1}{2} \times 70 \times 10^6 \times 3.14 \times (25 \times 10^{-3})^2 \times 1.4 \times$$

$$\therefore W = 96.16 \text{ J}$$

3. Calculate the work done in stretching a wire through 0.1×10^{-3} m of a certain material whose cross-section is 1×10^{-6} m^2 and length is 2 m. Young's modulus is 2×10^{11} N/m^2.

Here, $\Delta L = 0.1 \times 10^{-3}$ m

$A = 1 \times 10^{-6}$ m^2

$L = 2$ m

$Y = 2 \times 10^{11}$ N/m^2

$W = ?$

Now, Work done is given by

$$W = \frac{1}{2} \times \text{Stretching Force} \times \text{Stretch}$$

$$= \frac{1}{2} \times F \times (\Delta L)$$

$$\left\{ \because Y = \frac{F/A}{\Delta L/L} \right.$$

$$= \frac{1}{2} \times \left[\frac{Y \cdot A \cdot (\Delta L)}{L} \right] \times \Delta L$$

$$\left\{ \therefore F = \frac{YA(\Delta L)}{L} \right.$$

$$= \frac{1}{2} \times \frac{2 \times 10^{11} \times 10^{-6} \times (0.1 \times 10^{-3})^2}{2}$$

$$= 0.5 \times 10^{-3} \text{ J}$$

$$\therefore W = 5 \times 10^{-4} \text{ J}$$

4. Calculate the factor of safety in the case of a piston rod of 0.2 m diameter, the maximum tensile pull on which, under working conditions, is 1 MN, and the tensile strength for the material of which is 600 MN/m².

Here,

$$\begin{bmatrix}\text{the maximum tensile pull}\\ \text{under working conditions}\end{bmatrix} = \begin{bmatrix}\text{the maximum load or force}\\ \text{within the elastic limit}\end{bmatrix}$$

$\therefore$ Maximum load $= 1\ MN = 10^6\ N$

$d = 0.2\ m \Rightarrow r = 0.1\ m$

Tensile strength $= 600\ MN/m^2 = 600 \times 10^6\ N/m^2$

Factor of safety $= ?$

Now, Factor of safety is given by the following eqn.

$$\text{Factor of safety} = \frac{\text{ultimate stress or ultimate strength}}{\text{working stress}}$$

$$= \frac{\text{maximum tensile strength}}{\text{maximum load/Area}}$$

$$= \frac{600 \times 10^6}{10^6/\pi r^2}$$

$$= 600 \times 3.14 \times (0.1)^2$$

$\therefore$ Factor of safety $= 18.84$

5. A disc of 1 kg mass and 0.1 m radius is suspended in a horizontal Plane by a vertical wire attached to its centre. The time period of torsional vibration of the disc is 5 sec. Find the torsional rigidity of the material of the wire.

Here, $M = 1\ kg$

$r = 0.1\ m$

$T = 5\ sec.$

$C = ?$

Now, time period of torsional pendulum is given by,

$$T = 2\pi\sqrt{\frac{I}{C}}$$

where, $I = \frac{Mr^2}{2}$ (For a solid disc rotated about its axis)

$$\therefore T^2 = 4\pi^2\left(\frac{I}{C}\right)$$

$$\therefore I = \frac{1 \times (0.1)^2}{2}$$

$$= 5 \times 10^{-3}\ kg \cdot m^2$$

$$\therefore C = \frac{4\pi^2 I}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times 5 \times 10^{-3}}{(5)^2}$$

$$\therefore C = 7.89 \times 10^{-3}\ \frac{kg \cdot m^2}{s^2}$$ ← Torsional rigidity of the material of the wire.

6. In the above problem-5, if the length and diameter of the wire is 1.5 m and 1.0×10^{-3} m respectively then find the rigidity (rigidity modulus) of the material of the wire.

Here, $L = 1.5$ m

$D = 1.0 \times 10^{-3}$ m $\Rightarrow R = 5 \times 10^{-4}$ m

$G = ?$

Now, torsional rigidity of the material of the wire is given by the equation,

$$C = \frac{\pi G R^4}{2L}$$

$$G = \frac{2LC}{\pi R^4}$$

$$= \frac{2 \times 1.5 \times 7.89 \times 10^{-3}}{3.14 \times (5 \times 10^{-4})^4}$$

$\left\{ C = 7.89 \times 10^{-3} \ \frac{kg \cdot m^2}{s^2} \right.$; From problem-5.

$\therefore G = 1.206 \times 10^{11}$ N/m^2 ← Rigidity of the material of the wire.

7. Calculate the twisting couple on a solid shaft of 1.5 m length and 0.12 m diameter when it is twisted through an angle of 0.0104 rad. The coefficient of rigidity of the material is $9.3 \times 10^{10}\ N/m^2$

Here,

$L = 1.5\ m$

$D = 0.12\ m \Rightarrow R = 0.06\ m$

$\theta = 0.0104\ rad$

$G = 9.3 \times 10^{10}\ N/m^2$

$\tau = ?$

Now, the twisting couple (Torque) is given by the equation.

$$\tau = \frac{\pi G R^4 \theta}{2L}$$

$$= \frac{3.14 \times 9.3 \times 10^{10} \times (0.06)^4 \times 0.0104}{2 \times 1.5}$$

* Remember that, torsional rigidity C is given by $C = \frac{\tau}{\theta}$

$\tau = 1.31 \times 10^4\ N{\cdot}m$ ← Torque on a solid shaft.

8. A copper wire of 3 m length, 0.001 m diameter having a young's modulus of 12.5×10^{10} N/m² is taken. What is the extension produced if a weight of 10 kg is attached to one free end? What lateral compression is produced if the Poisson's ratio is 0.26 ?

Here, $L = 3$ m

$d = 0.001$ m $\Rightarrow \therefore r = 0.0005$ m $= 5 \times 10^{-4}$ m

$Y = 12.5 \times 10^{10}$ N/m²

$\Delta L = ?$

$m = 10$ kg

$\sigma = 0.26$

Lateral Compression $= 2 \times \Delta r = ?$

Now, From the equation of Young's modulus,

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{F/A}{\Delta L/L}$$

$$= \frac{mg}{A} \times \frac{L}{\Delta L} \quad \left\{ \because F = mg \right.$$

$$\therefore \Delta L = \frac{m \cdot g \cdot L}{\pi r^2 Y} \quad \left\{ \because A = \pi r^2 \right.$$

$$= \frac{10 \times 9.8 \times 3}{3.14 \times (5 \times 10^{-4})^2 \times 12.5 \times 10^{10}}$$

$\therefore \Delta L = 2.996 \times 10^{-3}$ m $\leftarrow$ Extension produced in the copper wire.

Now, Poisson's ratio is given by the equation,

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= -\frac{\Delta r/r}{\Delta L/L}$$

$$\therefore \frac{\Delta r}{r} = \sigma\left(\frac{\Delta L}{L}\right) \quad \{ \text{Omitting the minus sign}$$

$$\therefore \Delta r = \frac{\sigma \times \Delta L \times r}{L}$$

$$= \frac{0.26 \times 2.996 \times 10^{-3} \times 5 \times 10^{-4}}{3}$$

$$= 1.30 \times 10^{-7}\ m$$

$$\therefore \text{Lateral compression} = 2 \times \Delta r = 2 \times 1.30 \times 10^{-7}\ m$$

$$= 2.60 \times 10^{-7}\ m$$