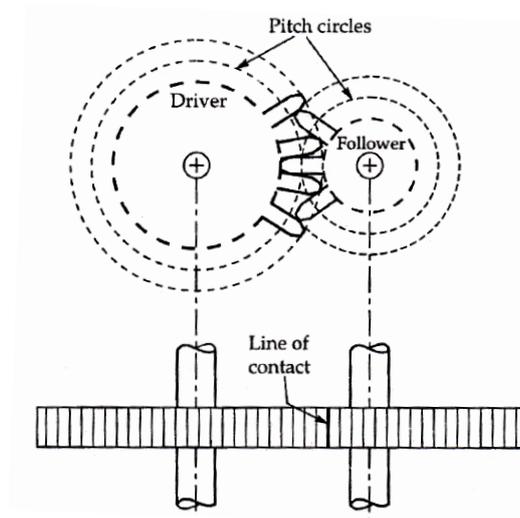


1.1 Introduction

- Any toothed member designed to transmit motion to another one, or receive motion from it, by means of successively engaging tooth is called a (toothed) gear.
- A gear is a toothed wheel with teeth cut on the periphery of a cylinder or a cone.
- Teeth of one gear meshes with teeth of the other gear, hence it is called a mesh drive or positive drive.
- It is preferred when medium or larger power is to be transmitted.
- Rotation of one gear will cause rotation of the other in the opposite direction.
- Fig. 1.1 shows two spur gears in mesh, the smaller gear is called the pinion and



the bigger one the gear wheel.

Fig. 1.1 Spur Gears in Mesh

1.1.1 ADVANTAGES OF GEARS

Gears offer the following advantages:

- I. Compact drive on account of small centre distance.
- II. High efficiency, reliable service and simple operation.
- III. Positive drive due to negligible slip between contacting surfaces.
- IV. Give higher speed ratio and can transmit higher power.
- V. It is possible to transmit power between parallel, non-parallel, intersecting, and nonintersecting shafts.
- VI. Power can be transmitted at higher speeds.

1.1.2 LIMITATIONS OF GEARS

The limitations of gear drive are:

- I. The error in tooth meshing may cause undesirable vibrations and noise during operation.
- II. Costlier than belts and chain drives.

Vision:

To deliver quality engineering education for Mechanical Engineers with Professional competency, Human values and Acceptability in the society.

Mission

- To nurture engineers with basic and advance mechanical engineering concepts
- To impart Techno-Managerial skill in students to meet global engineering challenges
- To create ethical engineers who can contribute for sustainable development of society

- III. Power cannot be transmitted over long distances.
- IV. Precise alignment of shafts is required.
- V. Require continuous lubrication.

1.1.3 APPLICATIONS OF GEARS

The gear drive has wide applications in the following fields:

- I. Metal cutting machine tools
- II. Automobiles
- III. Tractors
- IV. Hoisting and transporting machinery
- V. Rolling mills
- VI. Marine engines, etc.

Table 1.1 indicates the most useful fields of application of the main classes of gears.

Table 1.1 Choice of Type of Gear

<i>Relation between shaft axes</i>	<i>Gear ratio upto</i>	<i>Tooth speed m/s max.</i>	<i>Type of tooth</i>	<i>Wheel load MN/mm, max.</i>
Parallel	10	5	Helical or straight	1.4
		25	Helical	3.5
		200	Helical	0.35
Intersecting	7	2.5 60	Spiral bevel or straight bevel Spiral bevel	0.14 0.07
Non-intersecting Crossed at 90°	50	50	Worm and worm wheel	0.44
			Crossed helical	0.26
Non-intersecting Cross at 80° to 100° but not 90°	50	50	Worm and worm wheel	0.17
			Crossed helical	0.26

1.1.4 CLASSIFICATION OF GEARS

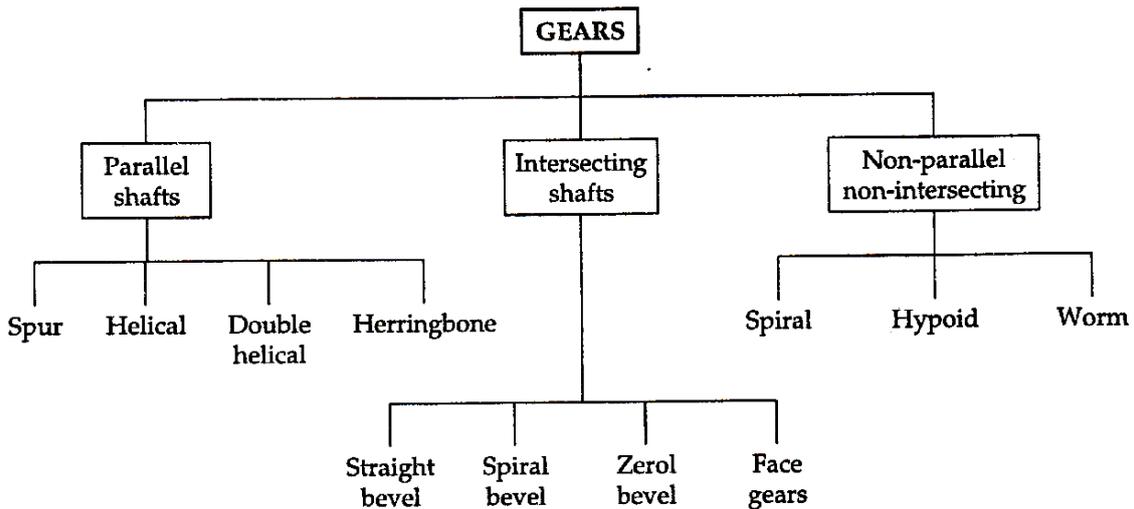


Fig.1.2 CLASSIFICATION OF GEARS

Gears may be classified as shown in Fig. 1.2.

- A spur gear is a cylindrical gear whose tooth traces are straight line generators of the reference cylinder.

- In helical gears the tooth traces are helices.
- A cylindrical gear in which a part of the face width is right hand and the other left hand, with or without a gap between them is called a double helical or herring-bone gear.
- The various types of gears are shown in Fig. 1.3.

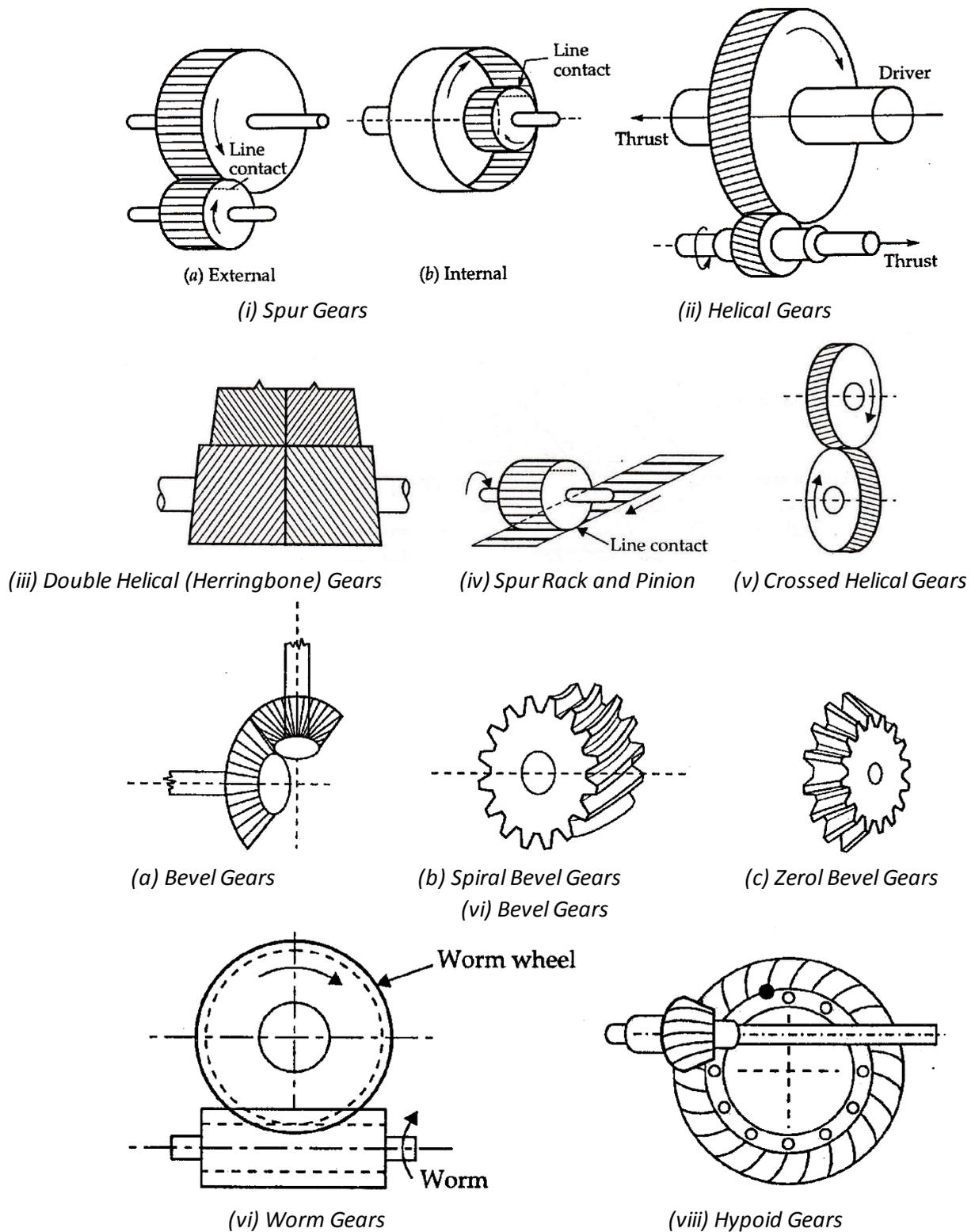


Fig. 1.3 Types of gears

- In spiral gears the tooth traces are curved lines other than helices.
- A gear pair (or train comprising such gears) one of whose axes, instead of being fixed in position in the mechanism of which the gear pair is a part, moves around the other is called planetary gear train.
- In bevel gears the reference surface is a cone. The bevel gears may be straight, spiral, zerol and face gears. In zerol bevel gears the teeth are curved in the lengthwise direction and are arranged in such a manner that the effective spiral angle is zero. In face gears, the teeth are cut on the flat face of the blank. They mesh at right angles with spur or helical pinions.
- A crown gear is a bevel gear with a reference cone angle of 90° .
- The hypoid gears are similar to the spiral bevel gears with the difference that the axes of the shafts do not intersect.
- In worm gears, one gear has screw threads. They are used on non-parallel non-intersecting shafts.

1.1.5 GEAR MATERIALS

The desirable properties of gear materials are:

1. The gear material should have sufficient static strength (ultimate or yield) and endurance strength against fluctuating loads to resist failure due to the breakage of tooth.
2. It should have sufficient surface endurance strength to avoid failure due to destructive pitting caused by excessive wear.
3. The material should have a low coefficient of friction to avoid failure due to scoring caused by high sliding velocities during high speed power transmission.
4. The coefficient of thermal expansion of material should be low to limit the thermal stresses causing distortion and warping.

The Bureau of Indian Standards (BIS) has recommended numerous materials for gears as per IS: 4460-1967. These materials are: Malleable cast iron, cast steel, forged steel, surface hardened steels, case hardened steels, and phosphor bronze.

1.2 SPUR GEAR TERMINOLOGY

- Spur gears are used to transmit power and rotary motion between parallel shafts.
- If teeth of the gear wheels are parallel to the axis of wheel, the gears are called spur gears.
- It is used when axes of the driving and driven shafts are parallel and co-planar.
- The smaller of the two gears in mesh is called the pinion, and the larger is designated as the gear.
- The terminology of a spur gear is shown in Fig. 1.4.

The important definitions are explained below:

Gear ratio (i): The ratio of the number of teeth of the wheel (gear) to that of the pinion is called gear ratio.

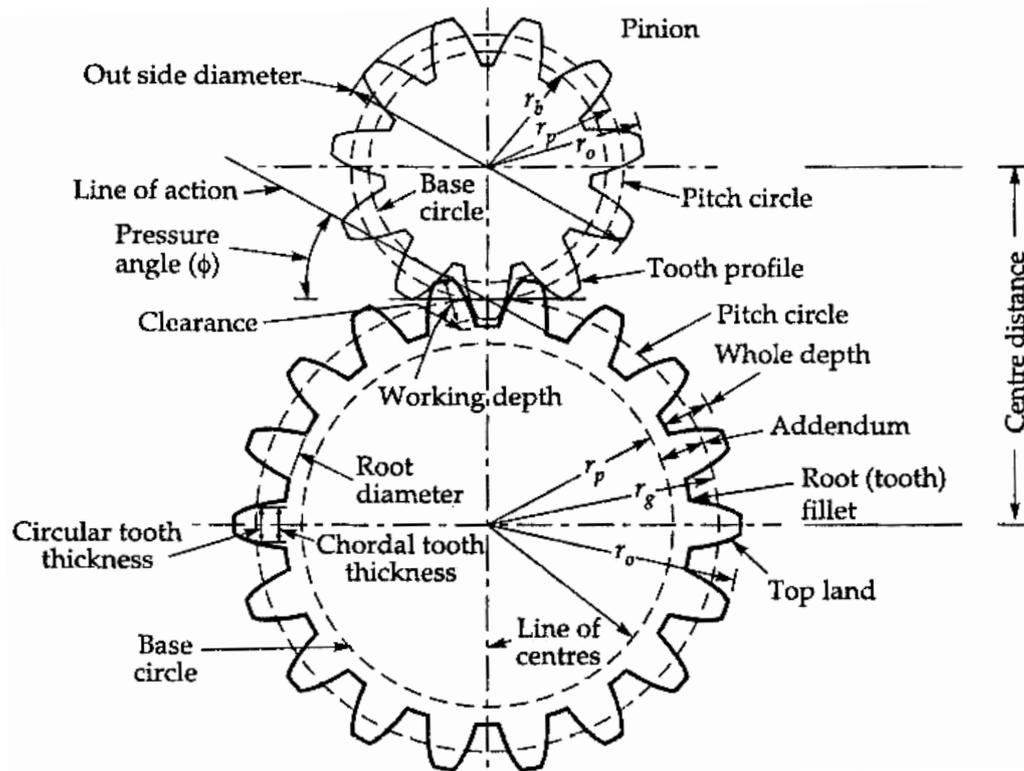


Fig. 1.4 Terminology of spur gears

Transmission ratio (i): The ratio of the angular speed of the first driving gear of a train of gears to that of the last driven gear is called transmission ratio.

Cycloid: A plane curve described by a point on a circle (generating circle), which rolls without slip on a fixed line (base line) is known as cycloid.

Involute: A plane curve described by a point on a straight line which rolls without slip on a fixed circle is known as involute.

Face width: The width over the toothed part of a gear, measured along a straight line generator of the reference cylinder is known as face width.

Base circle: In an involute cylindrical gear, the base circle of the involutes of the tooth profiles is known as base circle.

Circular pitch (p): The distance on the pitch circle from a point on a tooth to the corresponding point of the adjacent tooth.

$$\text{Thus, } p = \frac{\pi d}{z}$$

Diametral pitch (P): It is defined as the number of teeth of the gear divided by the pitch circle diameter.

$$P = \frac{z}{d}$$

$$\text{Thus, } pP = \pi$$

Module (m): It is defined as the pitch circle diameter per unit number of teeth, i.e.,

$$m = \frac{d}{z} = \frac{1}{P}$$

Pressure angle (ϕ): The angle between the line of action (a line through the pitch point and tangential to the base circles) and a line perpendicular to the line of centers at the pitch point is known as pressure angle.

Centre distance (C): The distance between the centers of the two gears in mesh is known as centre distance.

$$\text{Thus, } C = \frac{1}{2}(d_1 + d_2) = \frac{m}{2}(z_1 + z_2)$$

Backlash: The difference between tooth space and tooth width is known as backlash.

Clearance: The difference between the dedendum of one gear and the addendum of the mating gear is known as clearance.

Fundamental Law of Gearing: This law may be stated as "The shape of the teeth of a gear must be such that the common normal at the point of contact between two teeth must always pass through a fixed point on the line of centers".

1.3 BEAM STRENGTH OF SPUR GEAR TEETH (Design for Static Load)

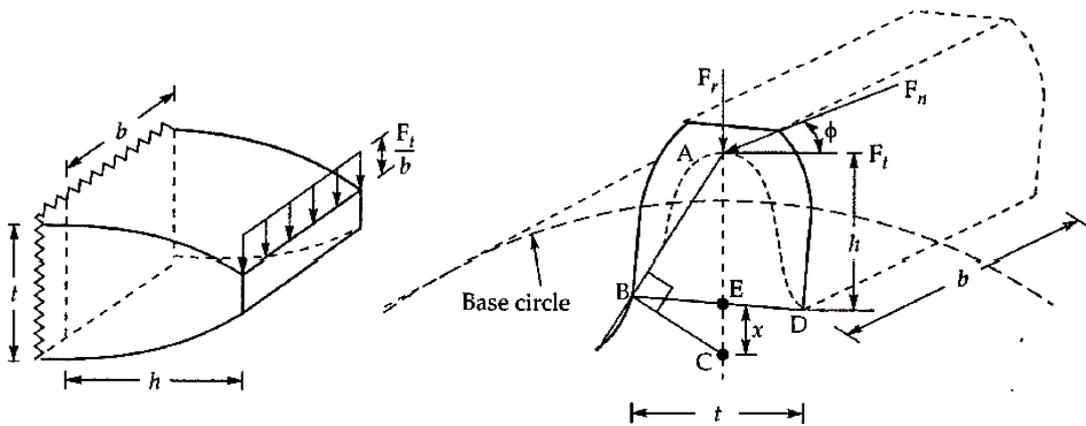


Fig. 1.5(a) Gear tooth as a cantilever beam

Fig. 1.5 (b) Loading on a gear tooth

- The determination of the proper gears to use in a particular application is a complex problem because of the many factors involved.
- First, the gears must operate together without tooth interference, with a proper length of contact and without undue noise.
- Second, the gear teeth must have the ability to transmit the applied loads without failure and with a certain margin of safety. This involves the ability of the teeth to resist not only the load resulting from the power transmitted but also the increases in load due to impact and shock caused by inaccuracy of tooth contour, tooth deflection tooth acceleration and stress-concentration at the root of the tooth or fatigue strength. The total resulting load is commonly referred to as the dynamic load.
- Third, the wearing qualities of teeth must be considered. This is known as the wear load.

Determination of Lewis equation:

- The static strength of the tooth is determined by assuming the tooth to be a cantilever beam (Fig. 1.5(a)) acted upon by the moment resulting from the

transmitted load obtained from the power transmitted. The design stress is based upon the ultimate strength of the material with a factor of safety of about 3. This analysis was given by Wilfred Lewis in 1892 and the design equation is known as the Lewis Equation.

- In order to take into account the effects of tooth fabrication and additional loads due to impact, the design is further modified by a velocity factor. These modifications in design were presented by Earle Buckingham in 1932, after which the gear design has been based upon the dynamic load and the endurance limit of the material and the wear load.

The Lewis equation is based on the following assumptions:

1. The gear tooth is treated as a cantilever beam.
 2. The effect of the radial component, which induces compressive stresses, is neglected.
 3. It is assumed that the tangential component is uniformly distributed over the face width of the gear. This is possible when the gears are rigid and accurately machined.
 4. The effect of stress concentration is neglected.
 5. It is assumed that at any time, only one pair of teeth is in contact and takes the total load.
- Fig. 1.5 (b) shows a gear tooth with the force acting at the tip of the tooth. The normal force F_n , is resolved into its components F_r and F_t acting at point A, the intersection of the line of action of the normal tooth load and the centre of the tooth.
 - The radial component F_r produces compressive stress in the tooth and the tangential component F_t causes bending stresses. The direct compressive stress is small enough as compared to the bending stress and is ignored in determining the strength of the tooth.
 - The maximum bending stress may be located and computed as follows: Through the point A in Fig. 1.5 (b), draw a parabola (shown in dash line) tangent to the tooth curves at B and D. This parabola represents the outline of a beam of uniform strength, and therefore the maximum stress in the actual tooth will be the point of tangency B or D. This stress is:

$$\sigma_b = \frac{M c}{I} = \frac{6 F_t h}{b t^2}$$

$$\therefore F_t = \frac{\sigma_b b t^2}{6 h}$$

Both t and h are based upon the size of the tooth and its profile; hence the equation may be written in the form,

$$h = \left(\frac{\sigma_b b}{6 F_t} \right) t^2 = \text{constant} \times t^2$$

Clearly this is the equation of a parabola. Triangles ABE and BCE are similar, thus

$$\frac{x}{(t/2)} = \frac{(t/2)}{h} \quad \text{or} \quad h = \frac{t^2}{4x}$$

$$\therefore F_t = \frac{\sigma_b b t^2}{6t^2} \cdot 4x = \sigma_b b \frac{4x}{6}$$

If we define a factor $y = 2x/3p$, called the Lewis form factor, based on circular pitch then, we get

$$F_t = \sigma_b b y p = \frac{\sigma_b b Y}{P} = \sigma_b b Y m = \sigma_b b \pi y m$$

Where, y = Lewis form factor based upon circular pitch

Y = Lewis form factor based upon diametral pitch

b = face width, mm = $3 \pi m$ to $4 \pi m$

p = circular pitch, mm

σ_b = Permissible bending stress, N/mm²

The permissible bending stress in the Lewis equation is taken as $\frac{1}{3}$ of the ultimate tensile strength

$$\therefore \sigma_b = \frac{1}{3} \sigma_{ut}$$

The values of y may be obtained from the following relations:

$$y = 0.124 - \frac{0.684}{z}, \text{ for } 14.5^\circ \text{ involute}$$

$$y = 0.154 - \frac{0.912}{z}, \text{ for } 20^\circ \text{ involute full depth (FD)}$$

$$y = 0.170 - \frac{0.95}{z}, \text{ for } 20^\circ \text{ involute stub}$$

1.3.1 Velocity Factor

Slight inaccuracies in profile and tooth spacing both, teeth being not absolutely rigid, variations in the applied load and repetitions of the loading cause impact and fatigue stresses that become more severe as the pitch line velocity increase. To allow for these additional stresses, a velocity factor C_v is introduced into the Lewis equation. This factor is given by:

$$C_v = \frac{3.05}{3.05 + v_m}, \text{ for ordinary industrial gears operating at velocity upto } 10 \text{ m/s}$$

$$C_v = \frac{6.1}{6.1 + v_m}, \text{ for accurately cut gears operating at velocity upto } 20 \text{ m/s}$$

$$C_v = \frac{5.56}{5.56 + v_m}, \text{ for precision gears cut with a high degree of accuracy and}$$

operating at velocity of 20 m/s and over

where v_m is the mean speed in m/s.

1.3.2 Tangential Load on Gear Tooth

The tangential load acting on the gear tooth is the load perpendicular to the pitch circle radius. The tangential load on the tooth at the pitch line is given by,

$$F_t = 10^3 \times \frac{P}{v}, \quad N$$

where P = power transmitting in kW

$$v = \text{mean pitch line velocity} = \frac{\pi d n}{60 \times 10^3} \text{ m/s}$$

d = pitch circle diameter, mm

n = speed in rpm.

1.3.3 Service Factor, C_s

The service factor accounts for increase in the tangential force due to fluctuation of the torque developed by the prime mover and the torque required to run the machine. It depends upon the prime mover and the driven machine.

$$C_s = \frac{\text{maximum torque}}{\text{rated torque}}$$

1.3.4 Load Distribution Factor, C_m For Spur Gears

The load on the gear tooth is not same along the face width. The variation of the load on the gear tooth is called the load distribution. The load distribution factor accounts for the non-uniform distribution of load across the face width of the gear. It depends upon the following factors.

1. Accuracy of alignment of gears and bearing mountings.
2. Bearing clearances.
3. Elastic rigidity of shafts, gears, bearings and housings, etc.
4. Manufacturing accuracy of gear tooth.

The values of load distribution factors are given in Table 1.2

Table 1.2 Load Distribution Factor for Spur Gears, C_m

Sl. No.	Characteristics of support	Face width, upto, mm			
		50	150	225	400
1.	Accurate mountings, small bearing clearances, minimum deflection, precision gears,	1.3	1.4	1.5	1.8
		1.6	1.7	1.8	2.0
2.	Less rigid mountings and less accurate gears, contact across the full face				
3.	Accuracy and mounting such that less full face contact exists.	More than 2.0			

1.3.5 Maximum Tangential Load, (F_t)

It is the tangential load which the gear tooth is required to sustain after accounting for service factor and load distribution factor.

$$(F_t) = C_s C_m F_t$$

1.4 DESIGN FOR DYNAMIC LOAD

The dynamic force is introduced in the gear teeth due to the following factors:

1. Inaccuracies of the tooth profile,
-

2. Errors in tooth spacing,
3. Misalignment between bearings
4. Elasticity of parts, and
5. Inertia of rotating masses

1.4.1 Buckingham's Dynamic Load Equation

$$F_d = (F_t) + \frac{21v(b \times C + F_t)}{21v + \sqrt{b \times C + F_t}}$$

where F_d = Total load on gear including load due to dynamic action,

$$C = \text{Load stress factor,} = \frac{k \times e}{\left(\frac{1}{E_p} + \frac{1}{E_g} \right)} \text{ in N/mm,}$$

$k = 0.107$ for 14.5° involute Full Depth

$= 0.111$ for 20° involute Full depth

$= 0.115$ for 20° involute Stub,

E_p & E_g = modulus of elasticity of pinion and gear materials respectively,

e = Sum of errors between two meshing teeth, mm

$= e_p + e_g$

e_p = error for pinion

e_g = error for gear

1.5 DESIGN FOR WEAR

- The failure of the gear tooth due to pitting occurs when the Hertz's contact stresses between two meshing teeth exceed the surface endurance strength of the material.
- Pitting is a surface fatigue failure which is characterized by small pits on the surface of the gear tooth. In order to avoid this type of failure, the proportions of gear tooth and surface hardness should be selected in such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth.
- The analysis of wear strength was done by Earle Buckingham, which gives the wear strength of gear tooth. This equation is based on Hertz's theory of contact stresses.
- The wear load is determined by the surface endurance limit of the material, curvature of the surface, and relative hardness of the surfaces.
- The pinion should always be harder to allow for work hardening of the gear to preserve the involute profile and to allow for greater abrasive wear on the pinion, and to decrease the possibility of seizing.

$$F_w = \frac{\sigma_{es}^2 \times b \sin \phi}{1.4} \left[\frac{2d_p d_g}{d_p + d_g} \right] \left[\frac{1}{E_p} + \frac{1}{E_g} \right]$$

where σ_{es} = Surface endurance limit,

b = Face width

ϕ = Pressure angle

E_p & E_g = Modulus of elasticity of pinion and gear materials respectively,

d_p & d_g = Pitch circle diameter of pinion and gear respectively,

For a safe design, $F_w \geq F_d$

If pinion and gear are of steel, $\sigma_{es} = (2.76 \times BHN - 70)$, N/mm^2

1.6 GEAR TOOTH FAILURES

The two basic modes for gear tooth failure are:

1. Breakage of the tooth due to static and dynamic loads,
2. Surface destruction.

1.6.1. Breakage of Tooth

- The complete breakage of the tooth can be avoided by adjusting module and face width so that the beam strength of the gear tooth is more than the sum of static and dynamic loads.
- The static beam strength of a gear tooth was suggested by Wilfred Lewis. The dynamic load is caused due to small machining errors resulting into inertia and impact loads on the gear tooth.

1.6.2. Surface Destruction

- The wear of gear tooth takes place due to the combined action of rolling and sliding. Rolling causes contact stresses and sliding causes rubbing action. Pinion is subjected to more rubbing action as it rotates faster than the gear.
- The principal types of gear tooth wear are: abrasive wear, corrosive wear, pitting, and scoring.
 - i. Abrasive wear:** The tooth surface is scratched by foreign particles in the lubricant, such as dirt, rust and weld spatter of metallic debris. This can be reduced by oil filter, using high viscosity lubricants, and surface hardness.
 - ii. Corrosive wear:** The corrosion of the tooth surface is caused by corrosive elements, such as extreme pressure (EP) additives present in the lubricating oils and foreign materials due to external contamination.
 - These elements attack the tooth surface, resulting in fine wear uniformly distributed over the entire surface.
 - The corrosive wear can be controlled by complete enclosure of the gears, selecting proper additives and replacing the lubricant at regular intervals of use.
 - iii. Pitting:** It is a type of fatigue failure caused by repeated applications of stress cycles. Pitting phenomenon is of two types: initial pitting and destructive pitting.
 - **Initial (or corrective) pitting** is a localized phenomenon, characterized by small pits at high spots.

- Such high spots are progressively worn out and the load is redistributed. Initial pitting is caused by the errors in tooth profile, surface irregularities and misalignment.
 - This controlled by precise- machining of gears and their correct alignment so that the load is uniformly distributed across the full face width and the dynamic load is reduced.
 - **Destructive pitting** is a surface fatigue failure which occurs when the load on the gear tooth exceeds the surface endurance strength of the gear material.
 - This type of failure is characterized by pits, which continue to grow resulting in complete destruction of the tooth surface. In some cases, this may even break the tooth permanently.
 - This type of failure can be avoided by ensuring that the wear strength of entire gear tooth is more than the sum of static and dynamic loads.
 - The surface endurance strength is a function of the hardness and can be increased by improving the surface hardness of the gear tooth surface by using an appropriate heat treatment process.
 - iv. **Scoring:** The oil film between the gear teeth may breakdown under excessive surface pressure, high sliding velocity and inadequate supply of lubricant. This results in generation of excessive frictional heat and overheating of the contacting surfaces of gear teeth. This may lead to metal-to-metal contact. Scoring is a stick-slip phenomenon, in which alternate welding and shearing takes place rapidly at the high spots. This increases the wear rate faster.
 - Scoring can be controlled by selecting proper surface speed, surface pressure and flow rate of lubricant to keep the temperature of contacting surfaces within permissible limits. The bulk temperature of lubricant can be reduced by providing fins on the gear box, air cooling by a fan or circulating cold water.
-

EXAMPLE 1.1

Design a pair of spur gear with 20° FD involute teeth to transmit 10 kW at 1440 rpm of the pinion. The speed ratio is 4: 1. The pinion is made of plain carbon steel Fe 410 ($S_{ut} = 410$ MPa) and the gear is made of grey cast iron FG 200($S_{ut} = 200$ MPa). The factor of safety desired is 3 and the load is steady with medium shocks running for 8 to 10 hours per day. The gears are commercial gears.

Solution:

Data given:

Pressure angle, $\phi = 20^\circ$ FD involute,

$P = 10$ kW,

$n_p = 1440$ rpm,

$i = 4:1$,

$n = 3$,

$(\sigma_{ut})_p = 410$ MPa,

$(\sigma_{ut})_g = 200$ MPa.

- Design for Static Beam Strength

Minimum number of teeth on pinion to avoid interference, $z_p = 18$ for $\phi = 20^\circ$ involute.

Assume, module, $m = 5$ mm.

$\sigma_p = 410/3$ MPa

$\sigma_g = 200/3$ MPa

Number of teeth on the gear $z_g = i z_p = 4 \times 18 = 72$

Pitch diameters of pinion and gear,

$d_p = m z_p = 5 \times 18 = 90$ mm

$d_g = m z_g = 5 \times 72 = 360$ mm

Lewis form factors:

$$y = 0.154 - \frac{0.912}{z}, \text{ for } 20^\circ \text{ involute full depth (FD)}$$

$$y_p = 0.154 - \frac{0.912}{z_p} = 0.154 - \frac{0.912}{18} = 0.1033$$

$$y_g = 0.154 - \frac{0.912}{z_g} = 0.154 - \frac{0.912}{72} = 0.1431$$

$$\sigma_p y_p = 410 \times 0.1033 = 42.35$$

$$\sigma_g y_g = 200 \times 0.1431 = 28.26$$

Hence gear is the weaker and shall be considered for design.

Pitch line velocity,

$$v = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi \times 90 \times 1440}{60 \times 10^3} = 6.79 \text{ m/s}$$

Tangential load,

$$F_t = 10^3 \times \frac{P}{v} = \frac{10^3 \times 10}{6.79} = 1472.75 \text{ N}$$

For given conditions, value of $C_s = 1.5$ and $C_m = 1.3$

Maximum load, $F_t = C_s C_m F_t = 1.5 \times 1.3 \times 1472.75 = 2871.87 \text{ N}$

Velocity factor for ($v < 8 \text{ m/s}$), $C_v = \frac{3.05}{3.05 + v} = \frac{3.05}{3.05 + 6.79} = 0.31$

Effective load, $F_{eff} = \frac{F_t}{C_v} = \frac{2871.87}{0.31} = 9264 \text{ N}$

Lewis equation for beam strength of gear tooth,

$$F_b = \sigma_g y_g b \pi m = \left(\frac{200}{3} \right) \times 0.1413 \times \pi \times b \times 5 = 147.96 b \text{ N}$$

Now, $F_b = F_{eff}$

$$\therefore 147.96 b = 9264$$

$$\therefore b = 62.6 \text{ mm}$$

The limits for face width are:

$$9.5 m = 9.5 \times 5 = 47.5 \text{ mm and } 12.5 \times 5 = 62 \text{ mm}$$

Adopt $b = 62 \text{ mm}$ and $m = 5 \text{ mm}$

- Design for Wear

Wear load, $F_w = d_p b Q K$

Here, $d_p = 90 \text{ mm}$, $b = 62 \text{ mm}$

$$Q = \frac{2 z_g}{z_g + z_p} = \frac{2 \times 72}{72 + 18} = 1.6$$

For given condition, load stress factor, $K = 0.2413 \left(\frac{BHN}{100} \right)^2$

Pinion material is given as Fe 410, hence hardness = 300 BHN

$$K = 0.2413 \left(\frac{300}{100} \right)^2 = 2.172$$

$$F_w = d_p b Q K = 90 \times 62 \times 1.6 \times 2.172 = 19391.6 \text{ N}$$

$$FOS = \frac{F_w}{F_d} = \frac{19391.6}{11461.2} = 1.69$$

Hence design is safe.

EXAMPLE 1.2

A compressor running at 250 rpm is driven by a 15 kW 750 rpm electric motor through a pair of 20° full depth spur gears. The pinion is made of 45C8 steel (heat treated) and gear of 40C8 steel (untreated). Estimate the module, face width, and number of teeth on each gear. Check for dynamic and wear loads.

Solution:

Data given:

$$n_p = 750 \text{ rpm,}$$

$$n_g = 250 \text{ rpm,}$$

Pressure angle, $\phi = 20^\circ$ FD involute,

$$P = 15 \text{ kW,}$$

For 45C8 steel pinion, $(\sigma_{ut})_p = 630 \text{ MPa,}$

For 40C8 steel gear, $(\sigma_{ut})_g = 580 \text{ MPa.}$

Minimum number of teeth on pinion to avoid interference, $z_p = 18$ for $\phi = 20^\circ$ involute.

Assume, module, $m = 5 \text{ mm.}$

$$\sigma_p = 410/3 \text{ MPa}$$

$$\sigma_g = 200/3 \text{ MPa}$$

$$\text{Speed ratio, } i = \frac{n_p}{n_g} = \frac{750}{250} = 3$$

$$\text{Number of teeth on the gear } z_g = i z_p = 3 \times 18 = 54$$

Pitch diameters of pinion and gear,

$$d_p = m z_p = 5 \times 18 = 90 \text{ mm}$$

$$d_g = m z_g = 5 \times 54 = 270 \text{ mm}$$

Lewis form factors:

$$y = 0.154 - \frac{0.912}{z}, \text{ for } 20^\circ \text{ involute full depth (FD)}$$

$$y_p = 0.154 - \frac{0.912}{z_p} = 0.154 - \frac{0.912}{18} = 0.1033$$

$$y_g = 0.154 - \frac{0.912}{z_g} = 0.154 - \frac{0.912}{54} = 0.1371$$

$$\text{For pinion, } e_p = 32.0 + 2.50(5 + 0.25\sqrt{90}) = 50.42 \mu\text{m}$$

$$\text{For gear, } e_g = 32.0 + 2.50(5 + 0.25\sqrt{270}) = 54.70 \mu\text{m}$$

$$e = e_p + e_g = 50.42 + 54.70 = 105.18 \mu m = 105.18 \times 10^{-3} m$$

Pitch line velocity,

$$v = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi \times 90 \times 750}{60 \times 10^3} = 3.53 \text{ m/s}$$

Tangential load,

$$F_t = 10^3 \times \frac{P}{v} = \frac{10^3 \times 10}{3.53} = 2829.42 \text{ N}$$

For given conditions, value of $C_s = 1.5$ and $C_m = 1.3$

Maximum load, $F_t = C_s C_m F_t = 1.5 \times 1.3 \times 2829.42 = 5517.37 \text{ N}$

$$C = \frac{k \times e}{\left(\frac{1}{E_p} + \frac{1}{E_g} \right)} = \frac{0.111 \times 105.18 \times 10^{-3}}{\left(\frac{1}{207 \times 10^3} + \frac{1}{100 \times 10^3} \right)} = 787.2 \text{ N/mm}$$

$$F_d = (F_t) + \frac{21v(b \times C + F_t)}{21v + \sqrt{b \times C + F_t}}$$

$$F_d = 5517.37 + \frac{21 \times 3.53(50 \times 787.2 + 5517.37)}{21 \times 3.53 + \sqrt{50 \times 787.2 + 5517.37}} = 17150.7 \text{ N}$$

Wear load, $F_w = d_p b Q K$

Here, $d_p = 90 \text{ mm}$, $b = 50 \text{ mm}$

$$Q = \frac{2z_g}{z_g + z_p} = \frac{2 \times 54}{54 + 18} = 1.5$$

For given condition, load stress factor, $K = 0.1536 \left(\frac{BHN}{100} \right)^2$

Pinion material is given as 45C8, hence hardness = 500 BHN

$$K = 0.1536 \left(\frac{500}{100} \right)^2 = 3.84$$

$$F_w = d_p b Q K = 90 \times 50 \times 1.5 \times 3.84 = 25920 \text{ N}$$

$F_w > F_d$

Hence design is safe.
