LUKHDHIRJI ENGINEERING COLLEGE, MORBI

GENERAL DEPARTMENT SEM-2 SUBJECT: MTHEMATICS-2 (3110015)

Tutorial-1 Vector Calculus

Ex-1 If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then show that $\mathrm{div}\left(r^n \bar{r}\right) = (n+3)r^n$.

Ex-2 Find the constant a if $\vec{A} = \left(x + 3y^2\right)\hat{i} + \left(2y + 2z^2\right)\hat{j} + \left(x^2 + az\right)\hat{k}$ is solenoidal.

Ex-3 If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then show that (a) $\mathrm{div}\,\vec{r} = 3$, (b) $\mathrm{curl}\,\vec{r} = 0$.

Ex-4 Show that $\vec{F} = \left(3x^2 y\right)\hat{i} + \left(x^3 - 2yz^2\right)\hat{j} + \left(z^2 - 2y^2 z\right)\hat{k}$ is irrotational but not solenoidal.

Ex -5 Evaluate the line integral of f (x,y,z) = x^2 - y^2 + 3xyz – yz over the straight line segment from A(0,0,0) to B(1,2,3).

Ex-6 Using Green's theorem, evaluate $\oint_C (\sin y\, dx + \cos x\, dy)$, where C is the boundary of the triangle with vertices (0,0),(π,0), (π,1).

Ex-7 Verify Green's theorem for the field $\overrightarrow{f(x,y)}$ = (x-y) $\hat{i}$ +x $\hat{j}$and the region R bounded by the unit circle C : $\overrightarrow{r(t)}$ =($\cos t$)$\hat{i}$+($\sin t$) $\hat{j}$, $0 \le t \le 2\pi$.

Ex-8 Verify Green's theorem for vector function $\vec{F}$ = (y^2 – 7y)$\hat{i}$ + (2xy +2x)$\hat{j}$ and curve

C : $x^2 + y^2 = 1$.

Tutorial-2 Laplace Transform

Ex-1 Find $L[3\sinh 4t + 5\sin 7t]$

Ex-2 Find $L[\cos^3 2t]$

Ex-3 Find $L\{[f(t)]\}$, if $f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$

Ex-4 Find $L[e^{2t}\cos t]$

Ex-5 Find $L[\sinh 2t \sin 3t]$

Ex-6 Find $L[t^2 \sin \pi t]$

Ex-7 Find $L[te^{2t}\sin 3t]$

Ex-8 Find $L\left[\dfrac{e^{-bt} - e^{-at}}{t}\right](a \neq b)$

Ex-9 Find $L\left[\dfrac{1-e^{t}}{t}\right]$

Ex-10 Find $L\left[\sqrt{e^{3(t+1)}}\right]$

Ex-11 Find $L^{-1}\left[\dfrac{6s-7}{s^2+5}\right]$

Ex-12 Find inverse Laplace of $\dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$

Ex-13 Find inverse Laplace of $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$

Ex-14 Find inverse Laplace of $\dfrac{2s^2-1}{(s^2+1)(s^2+4)}$

Ex-15 Find $t * \sin t$

Ex-16 Find $L^{-1}\left[\dfrac{1}{(s^2+a^2)^2}\right]$ by using convolution theorem.

Ex-17 Find $L^{-1}\left\{\log\left(\dfrac{s+a}{s-b}\right)\right\}$

Ex-18 Solve the equation by using Laplace transform $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 5x = e^{-t}\sin t$; x(0)=0, $x'(0) = 1$

Ex-19 Solve the equation by using Laplace transform $y'' + 3y' + 2y = e^{t}$, $y(0) = 1$, $y'(0) = 0$

Ex-20 Find the inverse Laplace transform of (a) $\dfrac{e^{-2s}}{s+1}$, (b) $\dfrac{1+e^{-s}}{s^2+4}$

Tutorial-3 Fourier Integral

1. Express the fuction $f(x) = \begin{cases} 1 \\ 0 \end{cases}$ for $|x| \leq 1$ *and for* $|x| > 1$, as aFourier integral.

   Hence evaluate $\int_0^{\infty} \frac{\sin\rho \quad \cos\rho x}{\rho} \, d\rho$

2. Using Fourier Sine integral,show that $\int_0^{\infty} \frac{1-\cos\pi\rho}{\rho} \sin x\rho \, d\rho = \begin{cases} \frac{1}{2}\pi, & 0 < x < \pi \\ o, & x > \pi \end{cases}$.

3. Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 2 & ; |x| < 2 \\ 0 & ; |x| > 2 \end{cases}$$

4. Find Fourier cosine integral of $f(x) = e^{-ax}$; x>0, a>0. Hence prove that

$$\int_0^{\infty} \frac{\cos\lambda x}{a^2 + \lambda^2} d\lambda = \frac{\pi e^{-ax}}{2a}$$

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Tutorial-4 FIRST ORDER ORDINARY DIFFERENTIAL EQUATION

1. Solve $\left(e^{x}+1\right)ydy=\left(y+1\right)e^{x}dx$
2. Solve $3e^{x}\cos^{2}ydx+\left(1-e^{x}\right)\cot ydy=0$
3. Solve $\left(x+\sin y\right)dx+\left(x\cos y-2y\right)dy=0$
4. Solve $\left(xy-2y^{2}\right)dx=\left(x^{2}-3xy\right)dy$
5. Solve $\left(x^{2}y^{2}+2\right)ydx+\left(2-x^{2}y^{2}\right)xdy=0$
6. Solve $\dfrac{dy}{dx}+y=-\dfrac{x}{y}$
7. Solve $\left(y^{3}-2x^{2}y\right)dx+\left(2xy^{2}-x^{3}\right)dy=0$
8. Solve $y'-\left(1+3x^{-1}\right)y=x+2,\ \ y(1)=e-1$
9. Solve $\left(1+y^{2}\right)dx=\left(\tan^{-1}y-x\right)dy$
10. Solve $\dfrac{dy}{dx}+\dfrac{1}{x}=\dfrac{e^{y}}{x^{2}}$
11. Solve $\left(y\sin 2x\right)dx-\left(1+y^{2}+\cos^{2}x\right)dy=0$
12. Solve $\left(y+\dfrac{y^{3}}{3}+\dfrac{1}{2}x^{2}\right)dx+\dfrac{1}{4}\left(x+xy^{2}\right)dy=0$
13. Solve $y'\tan y=\sin(x+y)+\sin(x-y)$
14. Solve $\left(x^{2}-y^{2}\right)dx+2xydy=0$

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Tutorial-5 Higher Order Differential Equations

Ex-1 Solve $y''-4y'+4y=0$.

Ex-2 Solve $y''+y=0$.

Ex-3 Solve $y''-3y'+2y=0$.

Ex-4 Solve $y''-2y'+2y=0$.

Ex-5 Solve $(D^4+2D^2+1)y=0$.

Ex-6 Solve $\frac{d^3y}{dx^3}+y=0$.

Ex-7 Solve $y''+y'-2y=0$, $y(0)=4$ & $y'(0)=-5$.

Ex-8 Solve $(D^2-7D+10)y=5x+7$.

Ex-9 Solve $(D^2-4D+4)y=x^3e^{2x}$.

Ex-10 Solve $(D^2-2D+1)y=e^x$

Ex-11 Solve $(D^2-3D+2)y=\cosh x$.

Ex-12 Solve $(D^2-5D+6)y=\sin 3x$

Ex-13 Using the method of variation of parameters solve the following differential equations:

1. $y''+4y=\sec 2x$
2. $y''+y=x\sin x$

Ex-14 Find the second linearly independent solution of $y''+6y'+9y=0$
Given that $y_1(x)=e^{-3x}$ is one solution.

Ex-15 Using the method of undetermined coefficients solve $y''+y'-12y=e^{3x}$

Ex-16 Solve $x^2D^2y-3xDy+5y=x^2\sin\log x$

Ex-17 Solve $(x^2D^2+xD-1)y=0$

Ex-18 Solve $(x^3D^3+6x^2D^2-12)y=12/x^2$

Tutorial-6 Series solution of ODE

1. Find the power series solution of the equation $\frac{d^2y}{dx^2}+y=0$ about $x_0=0$
2. Find the series solution of $y"=2y'$ in power of x.
3. Find the power series solution of $\left(x^2+1\right)y"++xy'-xy=0$ about $x=0$
4. Using Frobenius method, solve differential equation $4x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$.
5. Find the series solution of the equation $xy"+y'-y=0$ about $x_0=0$
6. Find a series solution of the differential equation $x^2y"+x^3y'+\left(x^2-2\right)y=0$ about $x=0$
7. Prove that (a) $\int_{-1}^{1}\mathrm{Pn}\ (\mathrm{x})\mathrm{dx}=2 \text{ if } \mathrm{n}=0$

   (b) $\int_{-1}^{1}\mathrm{Pn}(\mathrm{x})\mathrm{dx}=0 \text{ if } \mathrm{n}\geq 1$
8. Show that $\int_{-1}^{1}\mathrm{Pn}(\mathrm{x})\mathrm{Pm}(\mathrm{x})\mathrm{dx}=0\ ,\mathrm{m}\neq \mathrm{n}$

   $=\frac{2}{2n+1}, m=n$.(Orthogonality of Legendre`s polynomials) Also using it obyaine the value of $\int_{-1}^{1}{P_n}^2(x)dx$.
9. Prove that $\mathrm{J}_{-\mathrm{n}}(\mathrm{x})=(-1)^{\mathrm{n}}\mathrm{J}_{\mathrm{n}}(\mathrm{x})$.
10. Determine the values of (1)$\mathrm{J}_{\frac{3}{2}}(\mathrm{x})$ (2)$\mathrm{J}_0(\mathrm{x})\&\mathrm{J}_1(\mathrm{x})$
11. Show that $\mathrm{J}_1(\mathrm{x})=\mathrm{J}_0(\mathrm{x})-\mathrm{x}^{-1}\mathrm{J}_1(\mathrm{x})$.
12. Prove that $\frac{\mathrm{d}}{\mathrm{dx}}[\mathrm{x}^{\mathrm{n}+1}\mathrm{J}_{\mathrm{n}+1}(\mathrm{x})]=\mathrm{x}^{\mathrm{n}+1}\mathrm{J}_{\mathrm{n}}(\mathrm{x})$.