

Tutorials for
Operation Research
(3151910)

B.E. Semester 5 (Mechanical)



Directorate of Technical Education,
Gandhinagar, Gujarat

Vision of the DTE

- To provide globally competitive technical education;
- Remove geographical imbalances and inconsistencies;
- Develop student friendly resources with a special focus on girls' education and support to weaker sections;
- Develop programs relevant to industry and create a vibrant pool of technical professionals.

Vision of the Institute

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MECHANICAL ENGINEERING DEPARTMENT

Vision of the Department

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Lukhdhirji Engineering College, Morbi

Certificate

This is to certify that Mr./Ms. _____

_____ Enrollment No. _____ of B.E. Semester

_____ Mechanical Engineering of this Institute/College (GTU Code: _____)

has satisfactorily completed the Tutorial work for the subject **Operation**

Research (3151910) for the academic year 20____-_____.

Place: _____

Date: _____

Name and Sign of Faculty member

Head of the Department

Preface

Main motto of any laboratory/practical/field work is for enhancing required skills as well as creating ability amongst students to solve real time problem by developing relevant competencies in psychomotor domain. By keeping in view, GTU has designed competency focused outcome-based curriculum for engineering degree programs where sufficient weightage is given to practical work or tutorials. It shows importance of enhancement of skills amongst the students and it pays attention to utilize every second of time allotted for practical amongst students, instructors and faculty members to achieve relevant outcomes by performing the real-world problem as tutorials rather than having merely study type \mp tutorials. It is must for effective implementation of competency focused outcome-based curriculum that every practical is keenly designed to serve as a tool to develop and enhance relevant competency required by the various industry among every student. These psychomotor skills are very difficult to develop through traditional chalk and board content delivery method in the classroom. Accordingly, this manual is designed to focus on the industry defined relevant outcomes, rather than old practice of conducting practical to prove concept and theory.

By using this ~~lab~~ manual students can go through the relevant theory and procedure in advance before the actual performance which creates an interest and students can have basic idea prior to performance. This in turn enhances pre-determined outcomes amongst students. Each tutorial in this manual begins with competency, industry relevant skills, course outcomes as well as practical outcomes (objectives).

This manual also provides guidelines to faculty members to facilitate student centric activities through each tutorial by arranging and managing necessary resources in order that the students follow the procedures with to achieve the outcomes. It also gives an idea that how students will be assessed by providing rubrics.

Operation research is the course which deals with to provide most optimal solution of various operational activities by converting problem into mathematical model and solve by adopting scientific approach/method. It provides a platform for students to develop problem solving skill based on scientific approach.

Utmost care has been taken while preparing this manual however always there is chances of improvement. Therefore, we welcome suggestions for improvement and removal of errors if any.

Tutorial – Course Outcome matrix

Course Outcomes (COs):						
1. Develop models for optimizing the management and production systems from the verbal description of the real system.						
2. Make use of LPP techniques for optimization of Production mix problem in industry.						
3. Evaluate transportation, transshipment, assignment and travelling salesman and Queuing problem.						
4. Apply quantitative techniques in machine replacement, game theory, business decision making under conditions of certainty, risk and uncertainty						
5. Demonstrate Project management Problem.						
Sr. No.	Tutorial on	CO1	CO2	CO3	CO4	CO5
1.	Operations Research and LPP	√	√			
2.	Transportation and Assignment			√		
3.	Queuing Theory			√		
4.	Replacement theory				√	
5.	Game Theory				√	
6.	Decision Theory				√	
7.	Project Management					√

Industry Relevant Skills

The following industry relevant competency is expected to be developed in the student by undertaking the tutorial work of this course.

1. Students shall be able to convert industrial problem or real world situation in to mathematical model to solve the same. So it can be improve mathematical problem solving skill.
2. Students shall be learning the utilization of optimization techniques in various industrial fields.
3. Students shall be improving decision making capabilities under multi-optional and interdependent factors through scientific approach.

Guidelines for Faculty Members

1. Faculty should provide the guideline with demonstration of real life situation to the students with all features with respect to concerned topic.
2. Faculty shall explain basic concepts/theory related to the concerned tutorial before assigning to the students.
3. Faculty should frame rubrics and make students aware about it well in advanced.
4. Faculty is expected to share the skills and competencies to be developed in the students and ensure that the respective skills and competencies are developed in the students after the completion of the tutorials.
5. Faculty may provide additional knowledge and skills to the students even though not covered in the manual but are expected from the students by concerned industry.

Instructions for Students

1. Students are expected to carefully listen to all the theory classes delivered by the faculty members and understand the COs, content of the course, teaching and examination scheme, skill set to be developed etc.
2. Students shall organize the work and develop the mathematical modeling as well as analyzing skill.
3. Student shall attempt to develop related hand-on skills and build confidence.
4. Student shall develop the habits of evolving more ideas, innovations, skills etc. apart from those included in scope of manual.
5. Student shall refer technical magazines and data books.
6. Student should develop a habit of submitting the tutorial work as per the schedule and s/he should be well prepared for the same.

Index (Progressive Assessment Sheet)

Sr. No.	Tutorial on	Page No.	Date of submission	Tutorial Marks	Sign. of Teacher with date	Remarks
1.	Operations Research and LPP					
2.	Transportation and Assignment					
3.	Queuing Theory					
4.	Replacement Theory					
5.	Game Theory					
6.	Decision Theory					
7.	Project Management					
Total:						

Tutorial No: 01

Operations Research and LPP

Competency for Industry:

1. To propose most appropriate solution for industrial problem by applying LPP mathematical model.

Objectives:

1. To know the operation research characteristics.
2. To develop skill to convert textual described problem into mathematical model.
3. To develop analyzing capabilities for LPP problem to find optimum solution.

Relevant COs: CO1, CO2

1. Discuss the scope of OR in the various sectors.
2. A small fabrication industry is faced with a problem of scheduling production and subcontracting for three products A, B and C. Each product requires casting, machining and assembly operations. Casting operation for product A and B can be subcontracted but product C requires special tooling hence it cannot be subcontracted. Each unit of product A, B and C requires 6, 10 and 8 minutes of casting time in the foundry shop of a company. Machining times per unit of products A, B and C are 6, 3 and 8 minutes while assembly times are 3, 2 and 2 minutes respectively. The time available per week in foundry, machining and assembly shop are 8000, 12000 and 10000 minutes respectively. If product A, B and C are produced completely in the company, the overall profits per unit of product are Rs. 700, Rs. 1000 and Rs. 1100 respectively. When castings are obtained from subcontractors, the profit per unit of product A and B are Rs. 500 and 900 respectively. Formulate above problem as LPP so as to maximize the profit for company by scheduling its production and subcontracting.
3. Use the graphical method to solve the following LP problem,

$$\text{Maximize } Z = 5x_1 + 4x_2$$

$$\text{Subject to } 6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$\text{And } x_1, x_2 \geq 0$$

4. Solve the following LPP by simplex method,

$$\text{Maximize } Z = 4x_1 + 3x_2$$

$$\text{Subject to } 6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$\text{And } x_1, x_2 \geq 0$$

5. Solve the following LPP by Big-M method,

$$\text{Maximize } Z = -3x_1 + x_2 - 2x_3$$

$$\text{Subject to } x_1 + 3x_2 + x_3 \leq 5$$

$$2x_1 - x_2 + x_3 \geq 2$$

$$4x_1 + 3x_2 - 2x_3 = 5$$

$$\text{And } x_1, x_2 \geq 0$$

6. Solve the following LPP (Infeasible solution),

$$\text{Maximize } Z = 2x_1 + x_2$$

$$\text{Subject to } \frac{3}{2}x_1 + 3x_2 \leq 6$$

$$x_1 \leq 2$$

$$x_1 + x_2 \geq 7$$

$$\text{And } x_1, x_2 \geq 0$$

7. Find the dual of the following LPP,

$$\text{Minimize } Z = 5x_1 - 6x_2 + 4x_3$$

$$\text{Subject to } 3x_1 + 4x_2 + 6x_3 \geq 9$$

$$x_1 + 3x_2 + 2x_3 \geq 5$$

$$7x_1 - 2x_2 - x_3 \leq 10$$

$$x_1 - 2x_2 + 4x_3 \geq 4$$

$$2x_1 + 5x_2 - 3x_3 = 3$$

$$\text{And } x_1, x_2 \geq 0$$

Rubric wise marks obtained:

Rubric No.	1	2	3	4	5	Total
Marks						

Tutorial No: 02 Transportation and Assignment

Competency for Industry:

1. To propose most appropriate solution for industrial problem by applying Transportation and Assignment theory.

Objectives:

1. To develop skill to convert textual described problem into mathematical model.
2. To develop analyzing capabilities for transportation and assignment problem and provide the solution.

Relevant COs: CO3

1. A Manufacturer wants to ship 8 loads of his product as shown in following matrix. The matrix gives the mileage from origins, O to the destinations, D. The shipping cost is Rs. 10 per load per mile. What will be the optimal schedule and optimal cost? Use Vogel's approximation method to find initial basis feasible solution and MODI method to obtain optimal solution.

	D₁	D₂	D₃	Supply
O₁	50	30	220	1
O₂	90	45	170	3
O₃	250	200	50	4
Demand	4	2	2	

2. Obtain an I.B.F.S. to the following transportation problem using N-W Corner method & Optimum solution by STEPPING STONE method.

	Q₁	Q₂	Q₃	Q₄	Supply
P₁	1	3	2	4	8
P₂	5	4	2	0	10
P₃	0	3	3	1	12
Demand	4	5	8	5	

3. Solve the following transportation problem for maximum profit. Use Vogel's approximation method to find out initial feasible solution.

		Market (Per unit profit in Rs.)			
		A	B	C	D
Warehouse	X	12	18	6	25
	Y	8	7	10	18
	Z	14	3	11	20

Availability in	Demand in the
X = 200 units	A = 180 units
Y = 500 units	B = 320 units
Z = 300 units	C = 100 units
	D = 400 units

4. The following is the cost matrix of assigning the 4 operators to 4 jobs. Each operator is assigned only one job so as to minimize the total cost of jobs. What will be the total minimum job cost?

	Operators				Job			
	J₁	J₂	J₃	J₄				
O₁	2	10	9	7				
O₂	15	4	14	8				
O₃	13	14	16	11				
O₄	4	15	13	9				

5. A solicitors' firm employs typists on hourly piece-rate basis for daily work. There are five typists available with hourly charges and speed mentioned in table below.

Typist	A	B	C	D	E
Rate per hour (Rs.)	5	6	3	4	4
No. pages typed/hour	12	14	8	10	11

There are five jobs available to the firm and it wishes to allocate one job to one typist only. The typist is paid for full hour even if he works for fraction of an hour. The details of job are given in table below.

Job	P	Q	R	S	T
No. of pages	199	175	145	298	178

Find least cost allocation for the firm using Assignment model.

6. The captain of cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows:

Batsman	Batting Position				
	I	II	III	IV	V
P	40	40	35	25	50
Q	42	30	16	25	27
R	50	48	40	60	50
S	20	19	20	18	25
T	58	60	59	55	53

Find the assignments of batsmen to positions which would give the maximum number of runs.

7. Consider the assignment problem shown in table below. In the problem 5 different jobs are to be assigned to 5 different operators such that the total processing time is minimized. The matrix entries represent processing times in hours. Develop a zero-one programming model and also solve with Hungarian method.

	1	2	3	4	5
1	20	24	30	24	16
2	14	32	28	28	22
3	26	28	14	18	18
4	24	20	22	26	20
5	16	26	30	22	30

Rubric wise marks obtained:

Rubric No.	1	2	3	4	5	Total
Marks						

Tutorial No: 03 **Queuing Theory**

Competency for Industry:

1. To propose most appropriate solution for industrial queue problem using Queuing theory.

Objectives:

1. To develop skill to convert textual described problem into mathematical model.
2. To develop analyzing capabilities and provide the solution by Queuing theory.

Relevant COs: CO3

1. A chemical company distributes its products by trucks loaded at its only loading station and loading station is working 24 hours, continuously. Both company's trucks and contractor's trucks are used for this purpose. It was find out that on an average 10 minutes one truck arrived and average loading time was 6 minute. If 50% trucks are contractor's trucks find (i) Traffic intensity factor, (ii) Weighting time of trucks in system, (iii) The expected weighting time of contractor's trucks per day.
2. A copy maker has one copy making machine and he operates as the order comes. The order arrival is poison distribution having interval time of 0.5 min. The average time to serve a copy is distributed with mean of 0.3 min. Determine the following:
(i) Utilization factor of the machine (ii) Idle time for machine in a day having working hours of 10 hours (iii) No of persons waiting in the system (iv) No of persons waiting in the queue (v) Average waiting time in the queue.
3. On an average 95 patients per 24 hrs day require the service of an emergency clinic. Also on the average, a patient requires 12 minutes of an active attention. Assume that the facility can handle only one emergency at a time. Suppose that it cost the clinic Rs.100 per patient treated to obtain an average servicing of 10 minutes & that minute of decreasing in this average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from one to one third patients to half a patient.

4. Customers arrive at one person barber shop according to Poisson process with a mean inter-arrival time 20 minutes. Customer spends on an average of 15 minutes in barber's chair.
- What is the probability that new arrival need not wait for the barber to be free?
 - What is the expected number of customers in barber shop?
 - How much time can a customer expect to wait for his turn?
 - How much time can a customer spend in the shop?
 - Management will put in another chair and hire another barber when customer's average time in shop exceeds 1.25 hours. How much must average rate of arrival increase to warrant a second barber.
5. A self-service store employs one cashier at its counter; Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find
- Average number of customers in the system
 - Average number of customers in queue
 - Average time a customer spends in the system
 - Average time a customer waits before being served.
6. Vehicles arrive at a toll-booth at a rate of 8 per minute. The serving capacity of operator at toll-booth is 10 vehicles per minute. Find out (i) utilization factor (ii) idle time for booth operator in an 8 hour shift (iii) number of vehicles waiting in the system (iv) number of persons waiting in the queue (v) average waiting time in the queue and (vi) total time spent by a vehicle in the system.

Rubric wise marks obtained:

Rubric No.	1	2	3	4	5	Total
Marks						

Tutorial No: 04 Replacement Theory

Competency for Industry:

1. To propose most appropriate solution for issues related to failures of items/parts etc for industries by applying Replacement theory.

Objectives:

1. To develop skill to convert textual described problem of failures into mathematical model.
2. To develop analyzing capabilities for replacement model to find out optimum solution.

Relevant COs: CO4

1. A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value Rs. 200. The running costs are found from experience to be as follows. When should the machine be replaced?

Year	1	2	3	4	5	6	7	8
Running Cost Rs.	200	500	800	1200	1800	2500	3200	4000

2. An electronic item contains 10000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 1 only. If all resistors are replaced at the same time, the cost per resistor reduces to 35 paise. The probability of failure of a resistor by the end of month is given in table below.

Month	1	2	3	4	5	6
Prob. of items failing by end of month	0.03	0.07	0.2	0.4	0.15	0.15

Two policies are being considered for replacement of resistors; First, replace all items simultaneously at fixed interval whether failed or not and do individual replacement in intermediate periods. Secondly, individual replacement of items as and when it fails. Determine optimum policy for replacement of bulbs based on above failure data and costs.

3. A machine cost Rs 500. Operation and maintenance cost are zero for the first year and increases by Rs. 100 every year. If money is worth 5% every year, determine the best age at which the machine should be replaced. The resale value of the machine is negligibly small. What is the weighted average cost of owning and operating the machine?

4. A person is planning to purchase a car. A new car is costing rupees 3 lacs. The resale value of the car at the end of the year is 85 % of the previous year. Maintenance and repair cost during the first year are rupees 10000 and they increase by 15 % every year. The minimum resale value of the car can be rupees 75000.
 - i. When should the car be replaced to minimize average annual cost?
 - ii. If interest rate of 12 % is assumed, calculate the average cost at the end of 10 years.

5. The value of the money is 10 % per year. Machine-1 is to be replaced every 3 years and Machine -2 is to be replaced for every 6 years with yearly expenditure as given below. Which machine costs less?

Expenditure (in rupees)		
Year	Machine:-1	Machine:2
	2000	3400
	400	200
	800	400
	---	600
	---	800
	---	1000

6. A vehicle having initial cost of Rs. 50,000 has the following data based on past experience:

Year	1	2	3	4	5	6	7
MaintenanceCost in Rs.	5000	6000	7000	9000	11500	16000	18000
Resale value in Rs.	30000	15000	7500	3750	2000	2000	2000

Rubric wise marks obtained:

Rubric No.	1	2	3	4	5	Total
Marks						

Tutorial No: 05 Game Theory

Competency for Industry:

1. To propose most appropriate solution to compete with competitor for sustaining the business using Game theory.

Objectives:

1. To develop skill to convert textual described problem into mathematical model.
2. To develop analyzing capabilities and provide optimum solution to sustain in business competition based on Game theory.

Relevant COs: CO4

1. A company management and the labour union are negotiating a new three year settlement.

Each of these has 4 strategies:

- I. Hard and aggressive bargaining
- II. Reasoning and logical approach
- III. Legalistic strategy
- IV. Conciliatory approach

The costs to the company are given in the following table for every pair of strategy choice.

What strategy will the two sides adopt? Also determine the value of the game.

	Union strategies		Company Strategies	
	I	II	III	IV
I	20	15	12	35
II	25	14	8	10
III	40	2	10	5
IV	-5	4	11	0

2. Solve the below game theory problem with the concept of dominance method

		Player B			
		I	II	III	IV
Player A	I	3	5	4	2
	II	5	6	2	4
	III	2	1	4	0
	IV	3	3	5	2

3. Two companies are thinking on selecting the advertising media. There are three medias available along with the pay of as shown in the pay of matrix Value is in gain sales in (1000 rupees) suggest optimal strategy for the marketing and find out the value of the game.

		TV	Radio	Internet
Player A	TV	150	200	-400
	Radio	0	75	-100
	Internet	450	100	250

4. The following is the pay-off matrix between player X and player Y. Find the optimal strategies, their frequencies and the value of the game. Use rule of dominance and oddment in calculations.

		Player Y			
		A	B	C	D
Player X	I	0.25	0.20	0.14	0.30
	II	0.27	0.16	0.12	0.14
	III	0.35	0.08	0.15	0.19
	IV	-	0.08	0.13	0.00

Rubric wise marks obtained:

Rubric No.	1	2	3	4	5	Total
Marks						

Tutorial No: 06 Decision Theory

Competency for Industry:

1. To propose most appropriate solution for industrial problem by applying Decision theory.

Objectives:

1. To develop skill to convert textual described problem into mathematical model.
2. To develop analyzing capabilities using Decision theory to find optimum solution.

Relevant COs: CO4

1. Consider a M/s ABC company, which is developing its annual plans in terms of three objectives: (i) Increased profits, (ii) Increased market share and (iii) increased sales. M/S ABC has formulated three different strategies for achieving the stated objectives. The table below gives relative weightage of objectives and scores project the strategy. Find the optimal strategy that yields maximum weighted or composite utility.

Measure of Performance of Three objectives	ROI (Profit)	% Increase (Market share)	% Increase (Sales growth)
Weights	0.3	0.6	0.4
Strategy			
S ₁	8	5	10
S ₂	4	7	8
S ₃	6	6	11

2. Jyoti CNC introduces three model of CNC machine, which can lead to increase, neutral and decrease in sales. Sales department workout the payoff in terms of yearly net profit for each type of sales action as tabulated below.

Estimated net profit (In Crores)			
Machine types	Increase	Neutral	Decrease
Model 1	60	30	30
Model 2	30	40	30
Model 3	15	10	25

help to take decision using Maximax and Laplace criterion.

3. A marketing manager of a mutual fund company has kept complete records of the sales effort of the sales personnel. These records contain data regarding the number of mutual fund policies sold and net revenues received by the company as a function of four different sales strategies. The manager has constructed the conditional payoff matrix given below, based on his records. (The state of nature refers to the number of policies sold). The number within the table represents utilities. Suppose you are a new salesperson and that you have access to the original records as well as the payoff matrix. Which strategy would you follow?

State of nature	N ₁	N ₂	N ₃
Probability	0.3	0.6	0.4
Strategy	Utility	Utility	Utility
S ₁ (1 call, 0 follow up)	6	3	8
S ₂ (1 call, one follow up)	2	5	6
S ₃ (1 call, two follow-ups)	4	4	9
S ₄ (1 call, three follow-ups)	9	2	6

4. The following matrix gives the payoff of different strategies (alternatives) A, B, and C against conditions (events) L, M, N and P. Identify the decision taken under the following approaches: (i) Pessimistic, (ii) Optimistic, (iii) Equal probability, (iv) Regret, (v) Hurwicz criterion. The decision maker's degree of optimism (α) being 0.7.

	Events			
	L (Rs.)	M (Rs.)	N (Rs.)	P (Rs.)
A	4000	-108	5000	16000
B	20000	4000	400	0
C	20000	14000	-2000	1000

Rubric wise marks obtained:

Rubric No.	1	2	3	4	5	Total
Marks						

Tutorial No: 07 Project Management

Competency for Industry:

1. To propose most appropriate solution for controlling the project activities and project duration.

Objectives:

1. To develop skill to convert textual described/data into mathematical model.
2. To develop analyzing capabilities for project information to optimize the duration.

Relevant COs: CO5

1. The time estimates and precedence relationships of different activities constituting a small construction project is given in following table;

Activity	A	B	C	D	E	F	G	H	I
Predecessor	-	-	B	B	A	A	F	C,E,G	F
Duration (days)	3	8	6	5	13	4	2	6	2

- i. Draw the project network.
 - ii. Determine project completion time.
 - iii. What is critical path?
2. Consider the following given data and based on that find out critical path for the given project:

Activity	Time (days)
1 - 2	6
1 - 3	9
2 - 4	3
3 - 4	4
3 - 5	8
2 - 6	12
4 - 6	7
5 - 6	1

3. Draw a network corresponding to following tabular information and calculate early and latest start & finish time also determine critical path.

Activity	1 - 2	1 - 3	2 - 6	3 - 4	3 - 5	4 - 6	5 - 6	5 - 7	6 - 7
Duration	4	6	8	7	4	6	5	19	10

4. Tasks A to I constitutes a project in which the precedence relationships are A < D; A < E; B < F; D < F; C < G; C < H, F < I; G < I.

Time in day for each task is as follows:

Task	A	B	C	D	E	F	G	H	I
Time	8	10	8	10	16	17	18	14	9

Draw the network to represent the project and find out total float of each activity and identify critical path

5. Determine the critical path for given project schedule and find out total float for each event.

Activity	Time (weeks)	Activity	Time (weeks)
1 - 2	10	4 - 6	9
1 - 3	6	4 - 8	8
1 - 4	7	5 - 7	7
2 - 5	3	6 - 7	15
2 - 6	3	7 - 9	4
3 - 8	12	8 - 9	6

Rubric wise marks obtained:

Rubric No.	1	2	3	4	5	Total
Marks						

Operation Research
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