Radiation

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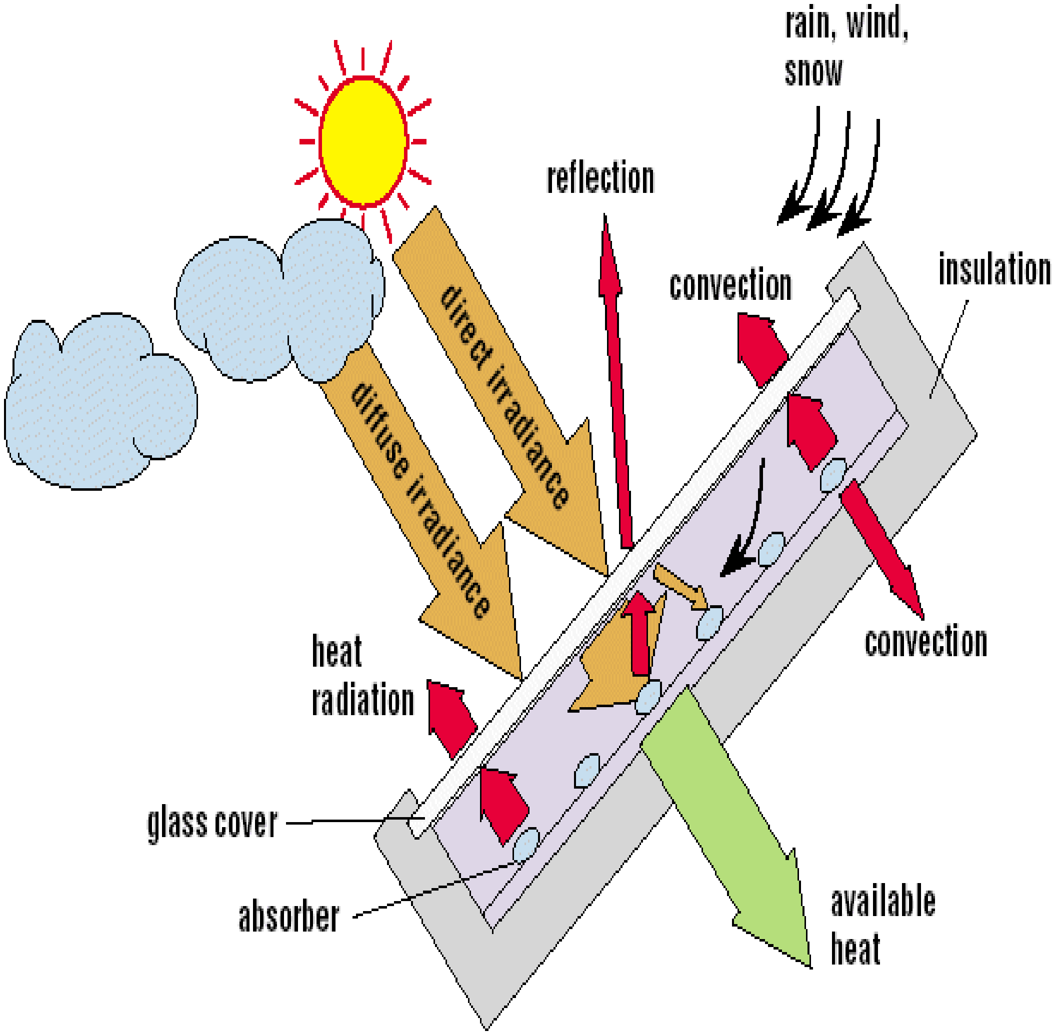
Flat Plate Solar Water Heater



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2. Radiation Terminologies
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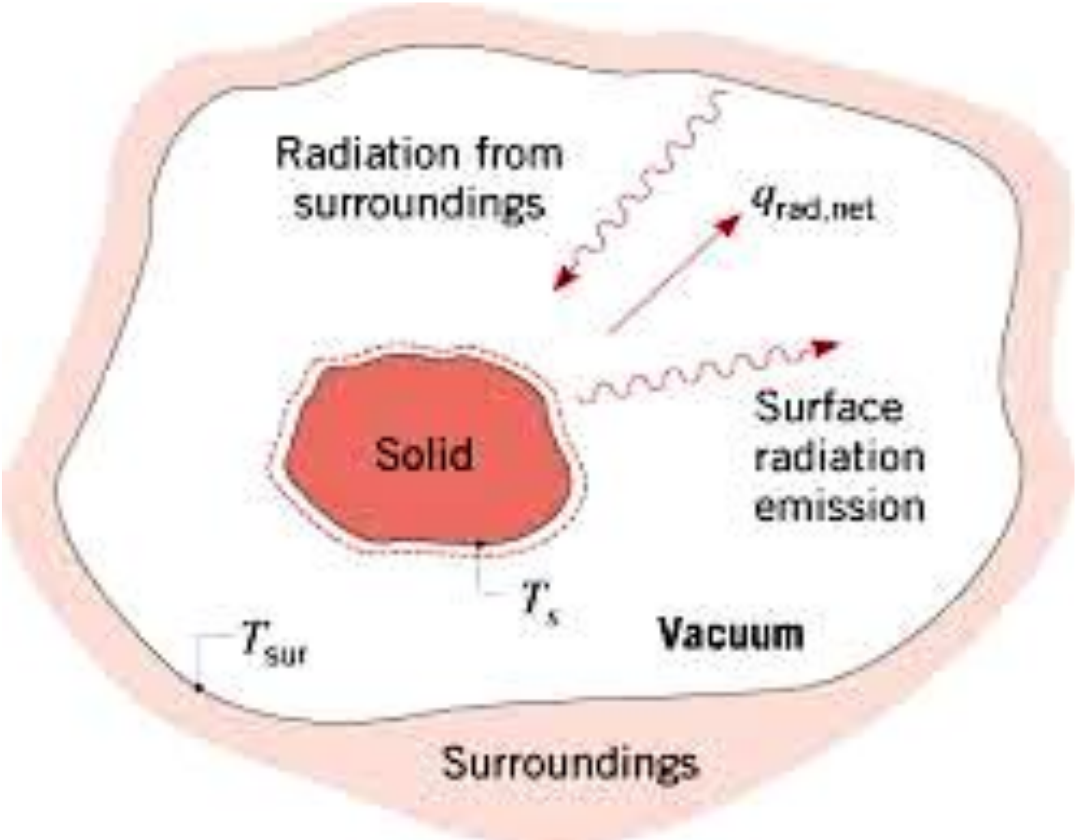
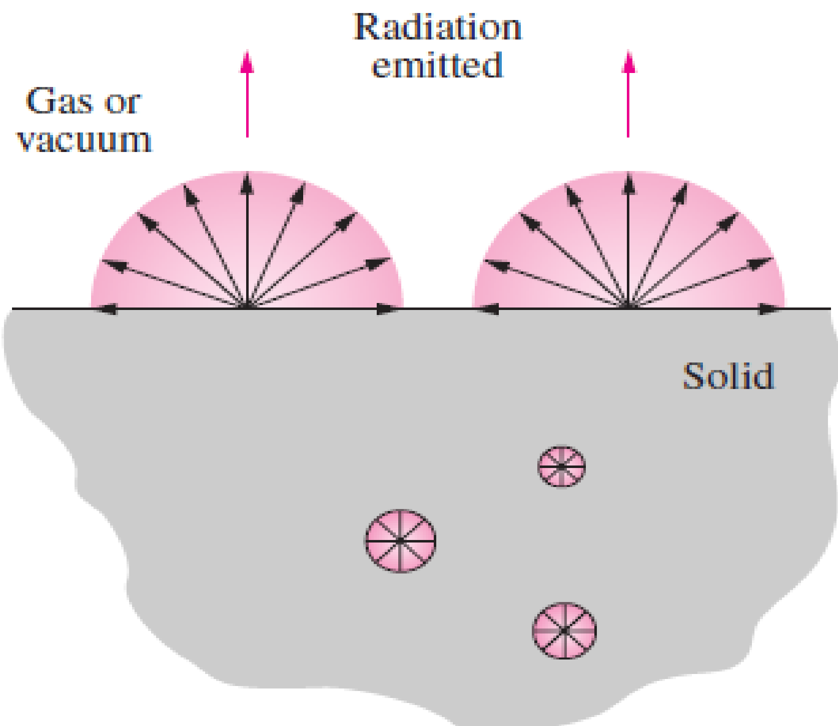
4. Heat Exchange between the surfaces(Black and Non Black Surfaces)
5. Problems

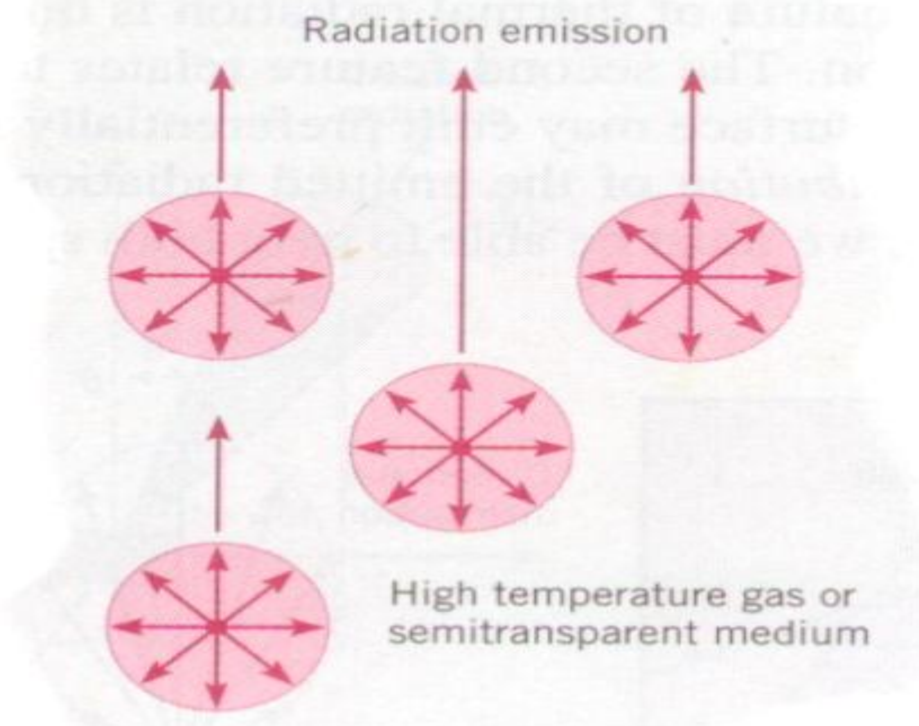
Thermal Radiation



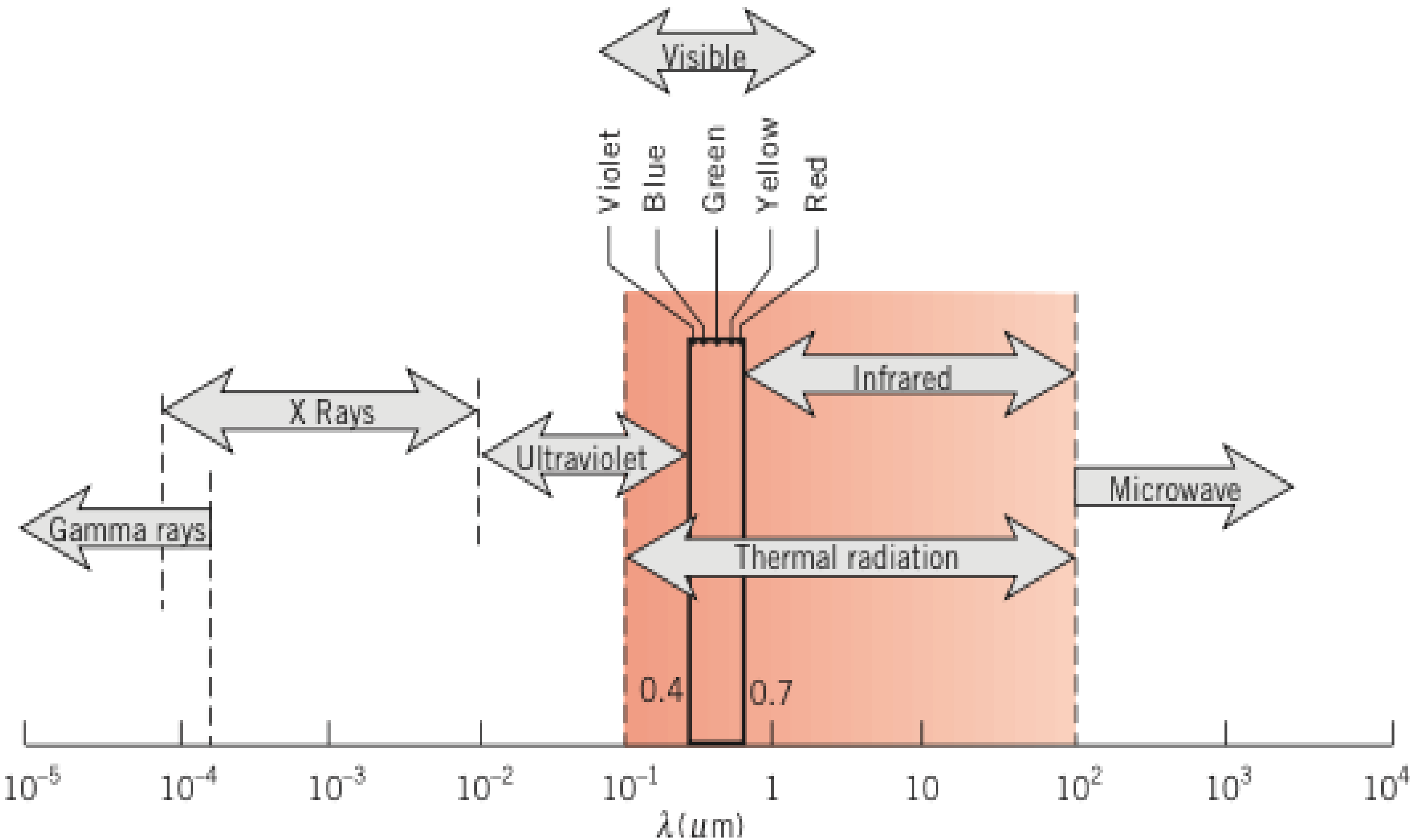
Any matter with temperature above absolute zero (0 K) emits electromagnetic radiation.

Surface and Volumetric Phenomenon





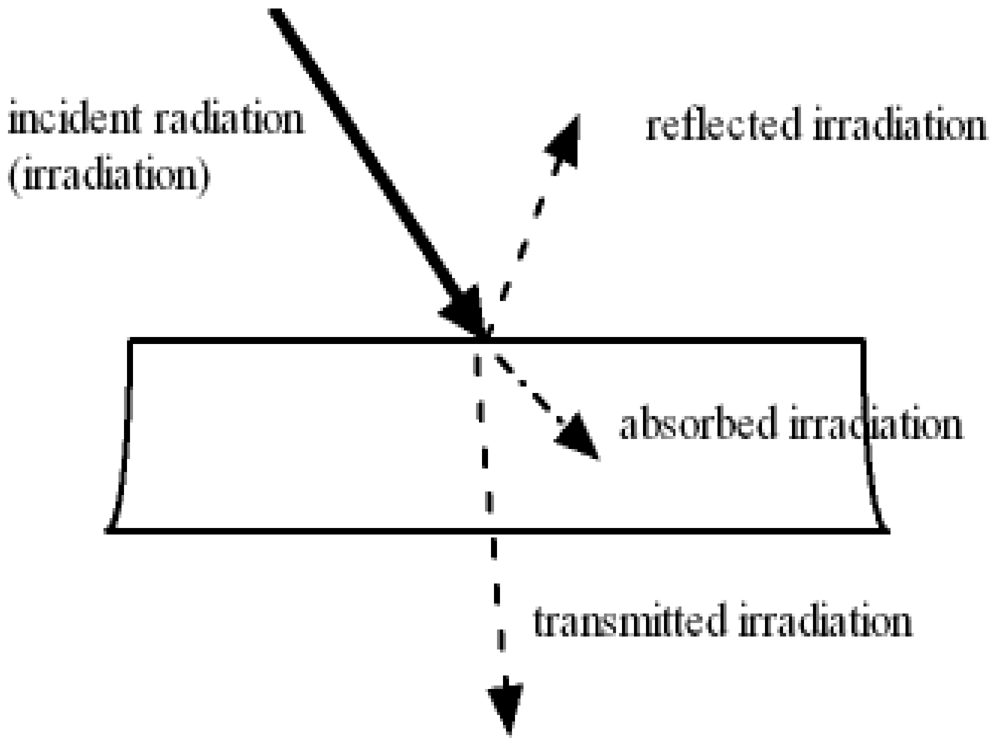
Spectrum of Electromagnetic Radiation



Irradiation Of Surface, *G*

Total Radiant energy received by per unit time per unit area from all directions at all wavelength.

It is not a surface property…….



Absorptivity: $\alpha = \dfrac{\text{Absorbed radiation}}{\text{Incident radiation}} = \dfrac{G_{\text{abs}}}{G}, \qquad 0 \le \alpha \le 1$

Reflectivity: $\rho = \dfrac{\text{Reflected radiation}}{\text{Incident radiation}} = \dfrac{G_{\text{ref}}}{G}, \qquad 0 \le \rho \le 1$

Transmissivity: $\tau = \dfrac{\text{Transmitted radiation}}{\text{Incident radiation}} = \dfrac{G_{\text{tr}}}{G}, \qquad 0 \le \tau \le 1$

$$G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} = G$$

$$\alpha + \rho + \tau = 1$$

Types Of Surface

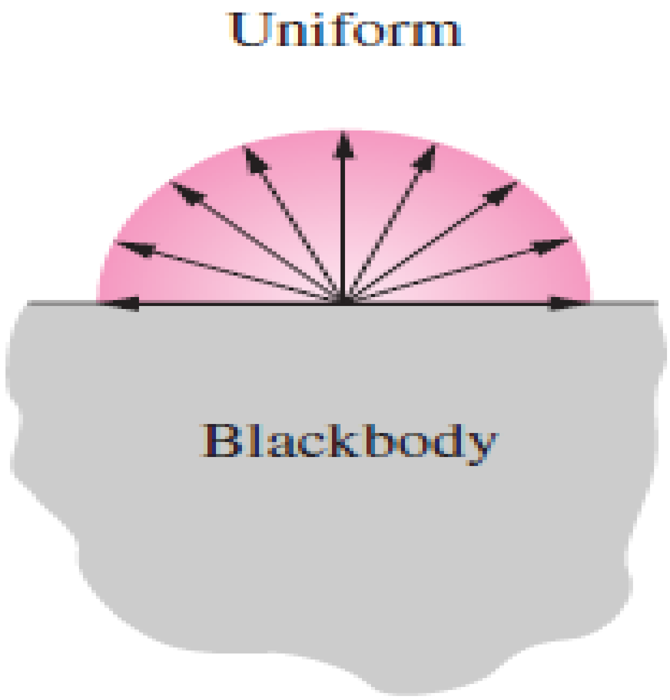
1. **Black Surface, ρ and $\tau = 0$ and $\alpha = 1$**
2. **Opaque Surface, $\tau = 0$ and $\alpha + \rho = 1$**
3. **White Surface, $\alpha = 0$, $\tau = 0$, $\rho = 1$**
4. **Grey Surface, $\alpha = \varepsilon$**
5. **Transparent Surface, $\alpha = 0$, $\rho = 0$, $\tau = 1$**

Emission Characteristics of Surfaces

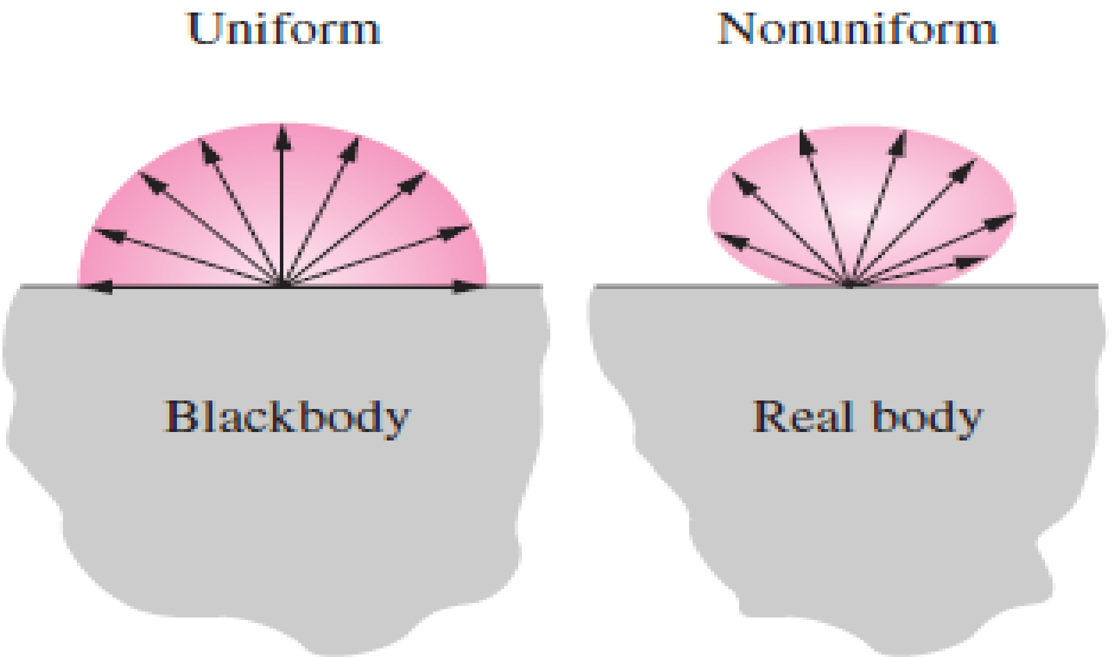
Total Hemispherical Emissive Power (e):

Radiant flux emitted from a surface of a body(W/m^2)

•Total Hemispherical Emissive Power of a black surface is denoted by e_b



Total Hemispherical Emissivity(ε)



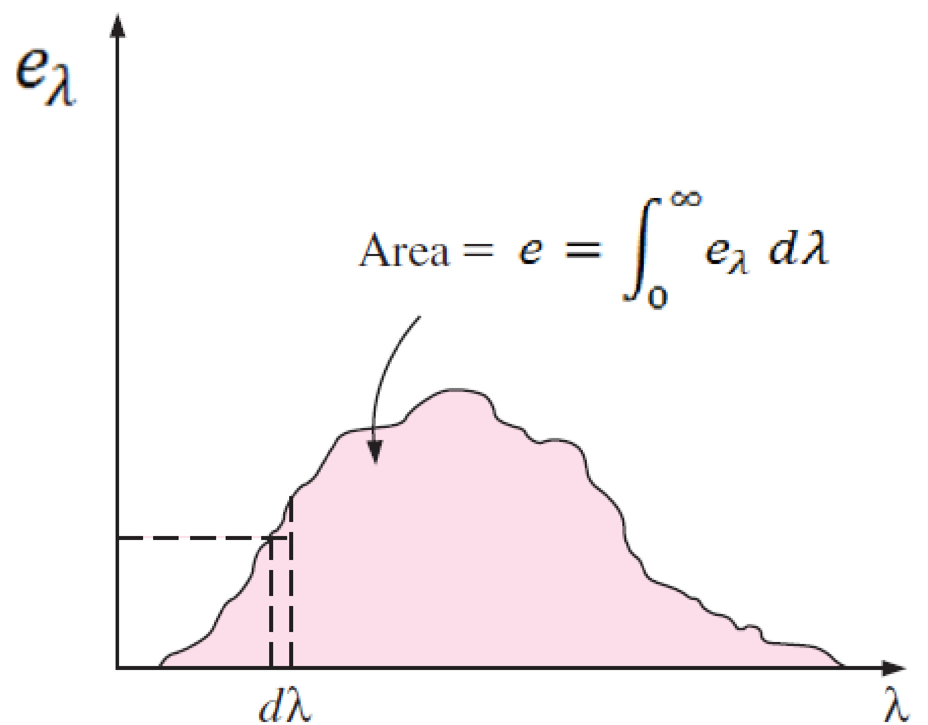
$$\varepsilon = \frac{e}{e_b}$$

Monochromatic (Spectral)Hemispherical Emissive Power (e_λ)

Radiant flux emitted by unit wavelength, **W/m²-μm**

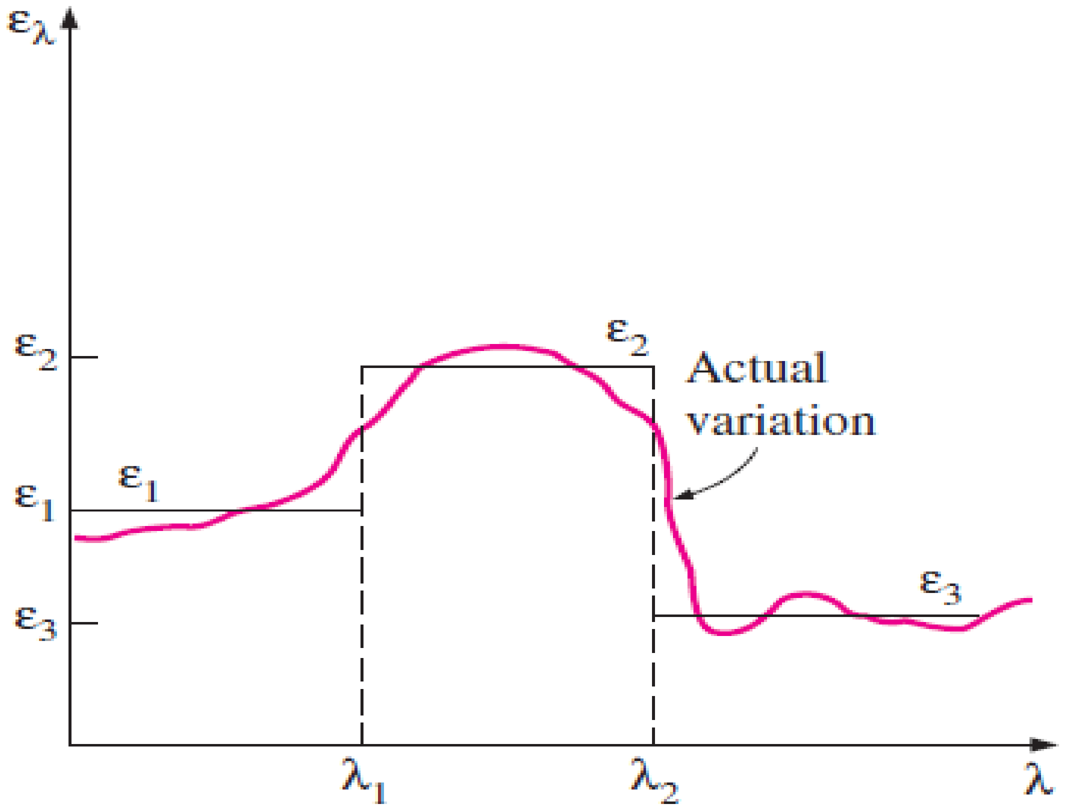
$$e_\lambda = \frac{de}{d\lambda} \quad \text{Or} \quad de = e_\lambda\, d\lambda$$

$$e = \int_0^\infty e_\lambda\, d\lambda$$



Monochromatic (Spectral)Hemispherical Emissivity (ε_λ)

$$\varepsilon_\lambda = \frac{e_\lambda}{e_{\lambda b}}$$



Laws Of Radiation

1. Planck’s Law

2. Wein’s Law

3. Stefan Boltsmann’s Law

4. Kirchoff’s Law

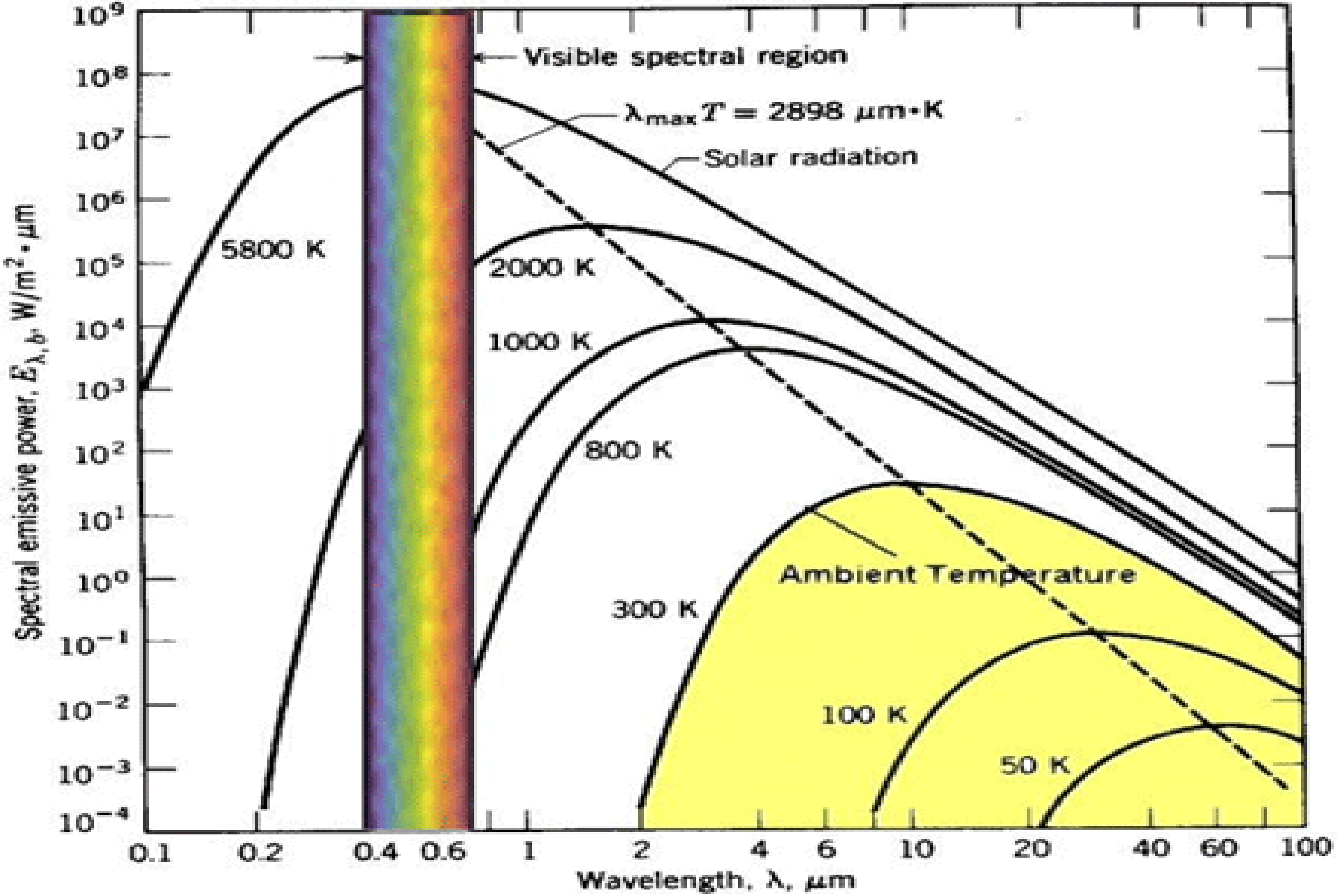
Planck's Law for Spectral Distribution

$$e_{b\lambda} = \frac{C1.\lambda^{-5}}{exp\left(\frac{C_2}{\lambda T}\right) - 1} \quad \text{W/(m}^2\ \mu\text{m)}$$

C_1=3.742 X 10^8 Wμm^4/m^2

C_2 =1.4387 X 10^4 μmK.

Spectral Blackbody Emissive Power



Wien's Displacement Law

$$E_{b\lambda} = \frac{C1.\lambda^{-5}}{exp\left(\frac{C_2}{\lambda T}\right) - 1}$$

$$\frac{dE_{b\lambda}}{d\lambda} = 0$$

$$\frac{C_2}{\lambda T} = 4.965$$

$$\lambda_{max}.T = 2898\mu mK.$$

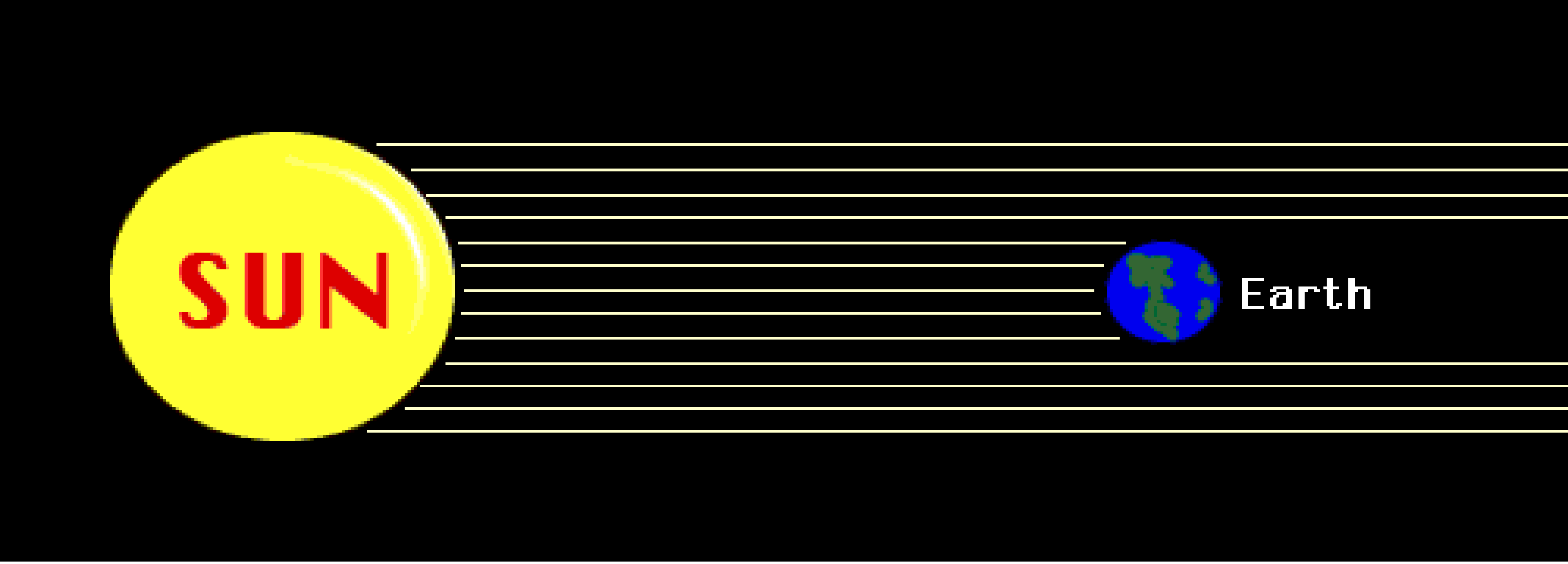
Stefan Boltsmann's Law

$$E_b = \int_0^\infty E_{b\lambda}\, d\lambda$$

$$= \int_0^\infty \frac{C1.\lambda^{-5}}{exp\left(\frac{C_2}{\lambda T}\right)-1}\, d\lambda$$

$$E_b = \sigma T^4 W/m^2$$

$\sigma = 5.67 \text{ X } 10^{-8} \text{ W/m}^2\text{K}^4$



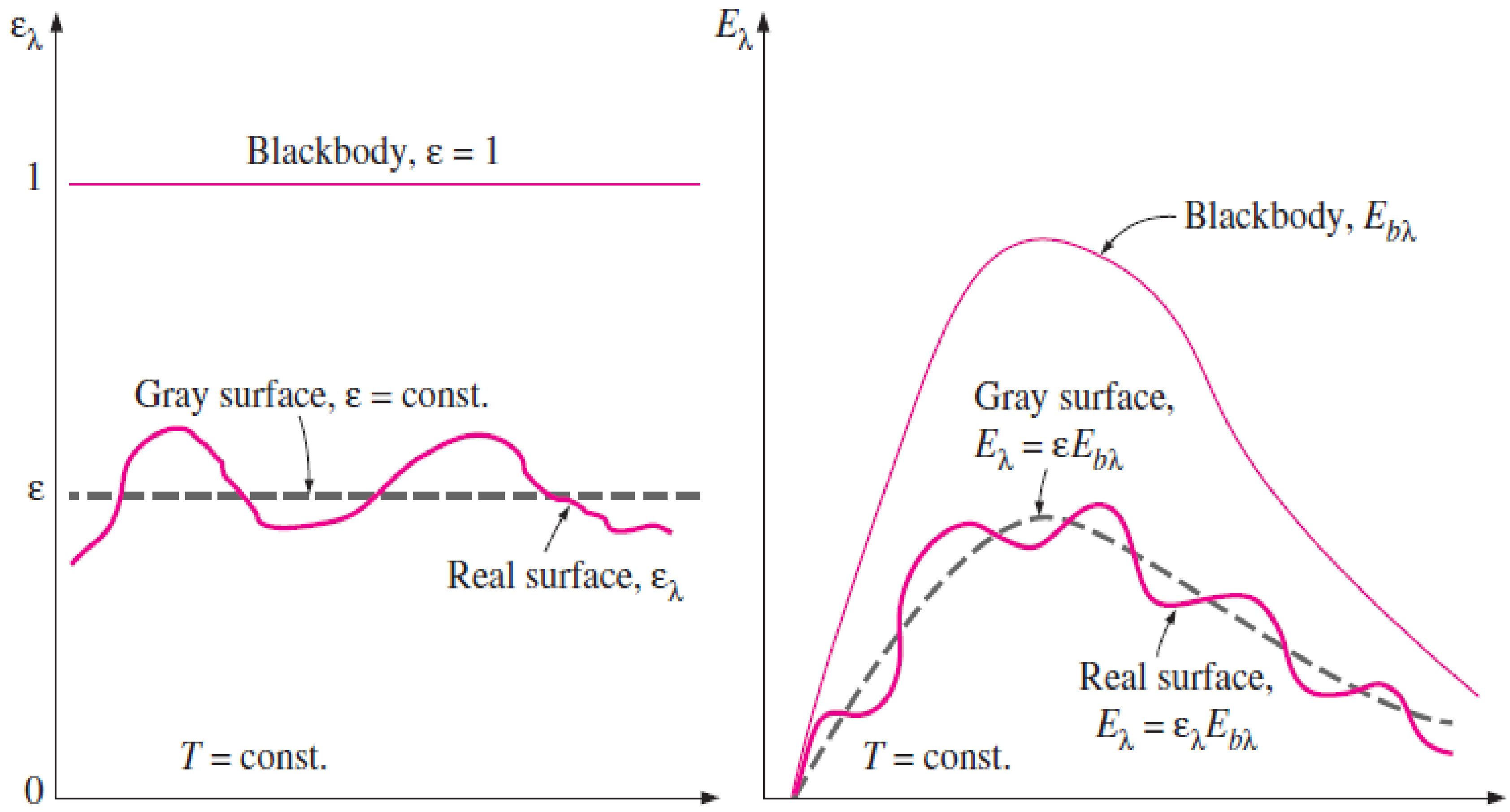
Temperature of sun surface is 5779 K. Surface may be assumed to be black surface .

- Calculate the values of λ_{m} ,$e_{b\lambda=\lambda m}$,e_{b} on the sun's surface.

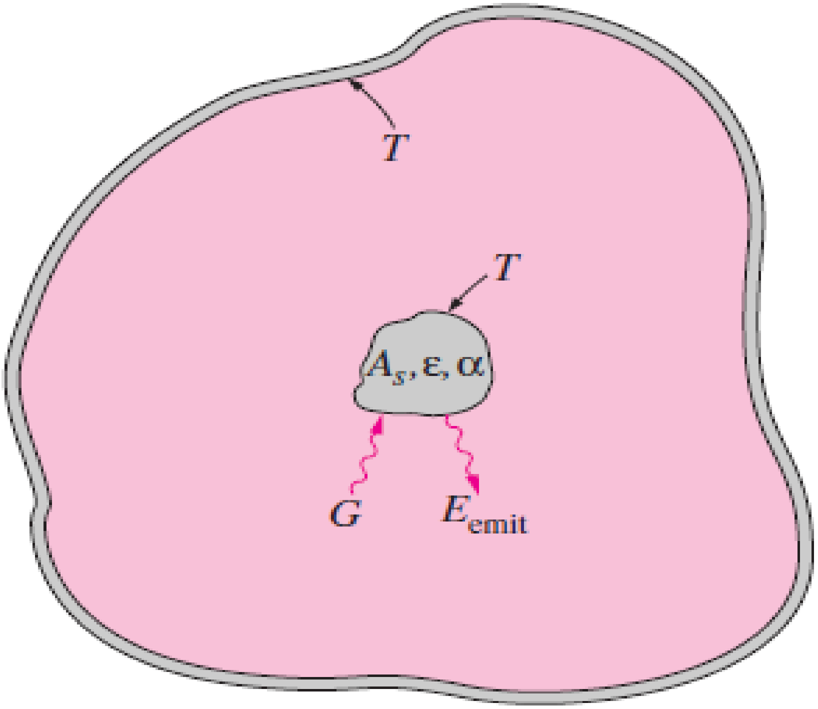
Also determine $e_{b\lambda=\lambda m}$,e_{b} as received at earth surface.

- Take distance between Sun and Earth is =1.496 x10^{8} km
- Radius of the sun is 0.695 x106 km

Emissivity Of Real and Grey Surface

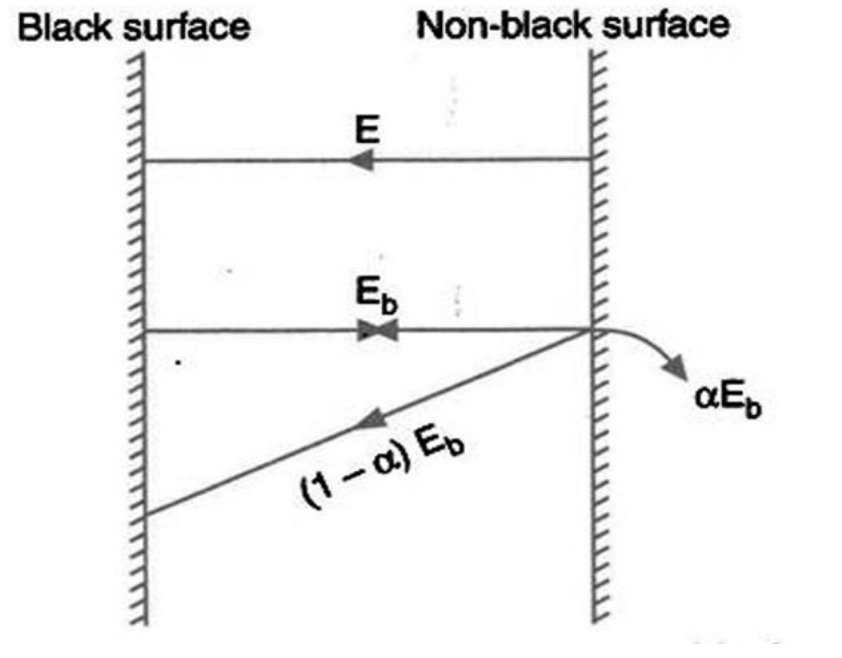


Kirchhoff's Law



The small body contained in a large isothermal enclosure used in the development of Kirchhoff's law.

$$\varepsilon(T) = \alpha(T)$$



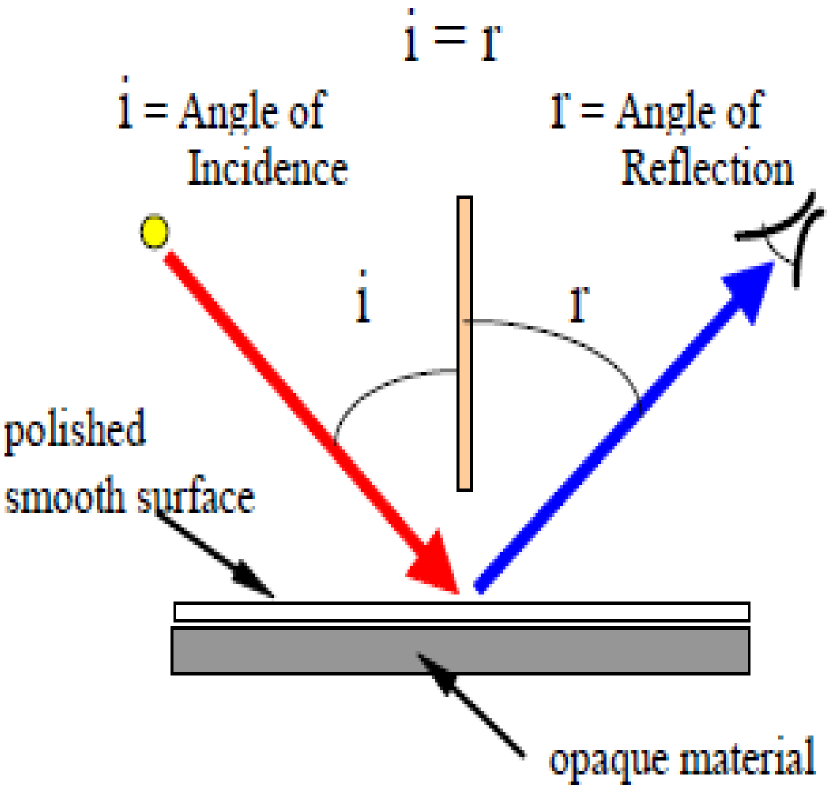
Net heat transfer for the non-black body is given as

$$E - \alpha E_b = 0$$

$$\frac{E}{E_b} = \alpha$$

$$\therefore \ \frac{E}{E_b} = \alpha = \epsilon$$

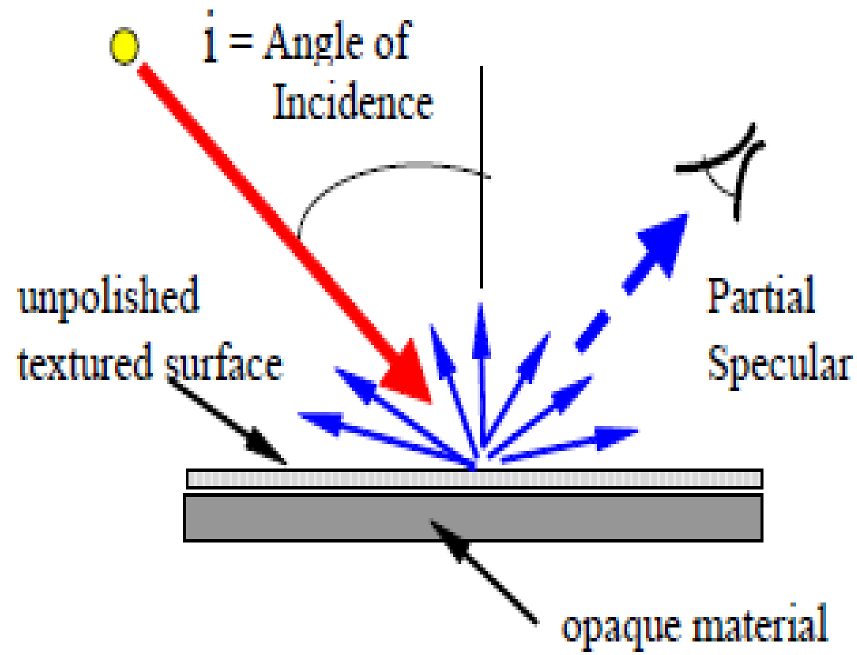
SPECULAR REFLECTION



EX: Mirrors

DIFFUSE REFLECTION

(combined diffuse and partial specular possible depending on surface)

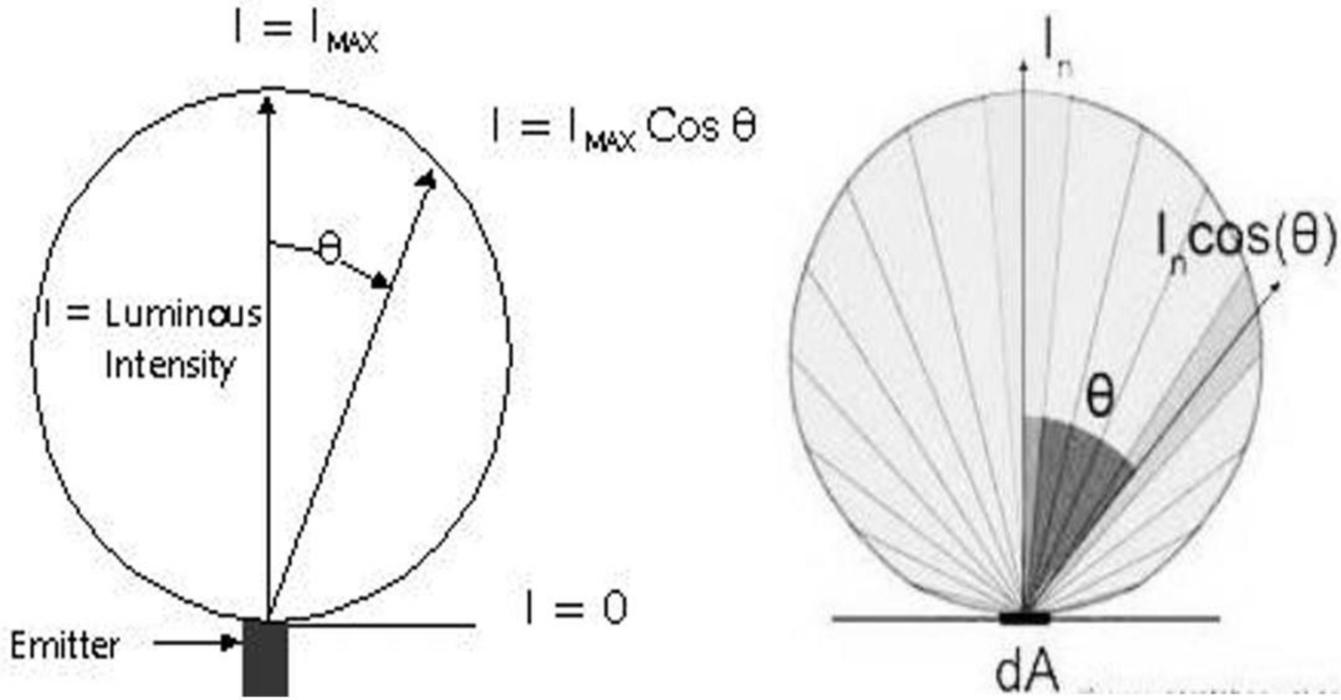


EX: Reflective Ceilings and Interior Walls

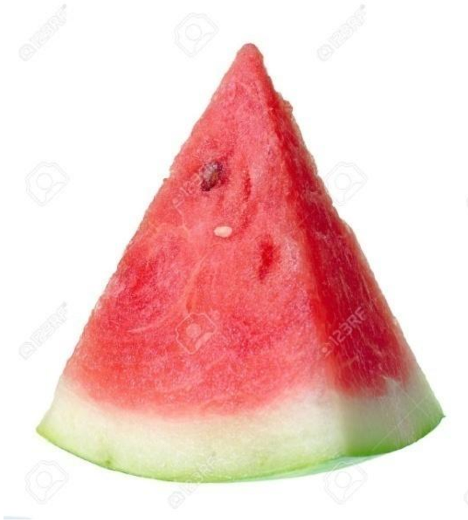
Total Hemispherical Emissive Power

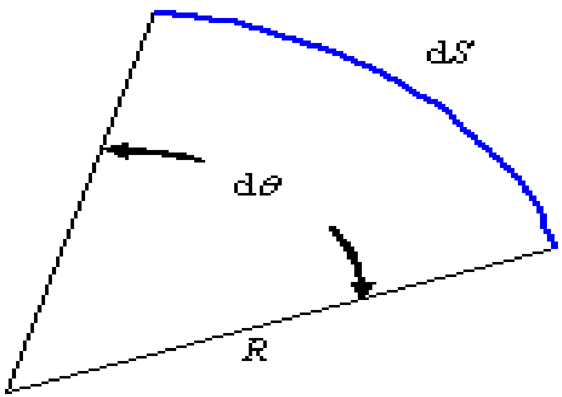
- Temperature
  - Kelvin , K
  - Emissive Power
- Spectral
  - Wavelength, λ
  - Monochromatic Emissive power
- Direction
  - Solid Angle, ω
  - Radiation Intensity

lambert's cosine law "the intensity of radiation in a direction θ from the normal to a black emitter is proportional to cosine of the angle θ".



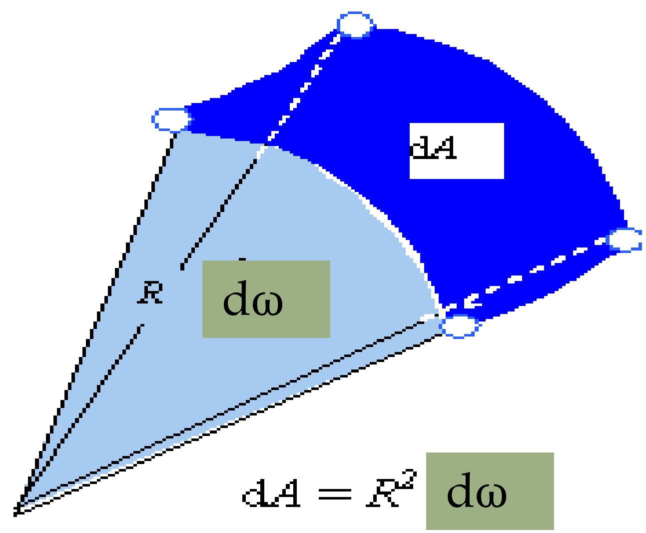
Lambert cosine law





$$dS = R\,d\theta$$

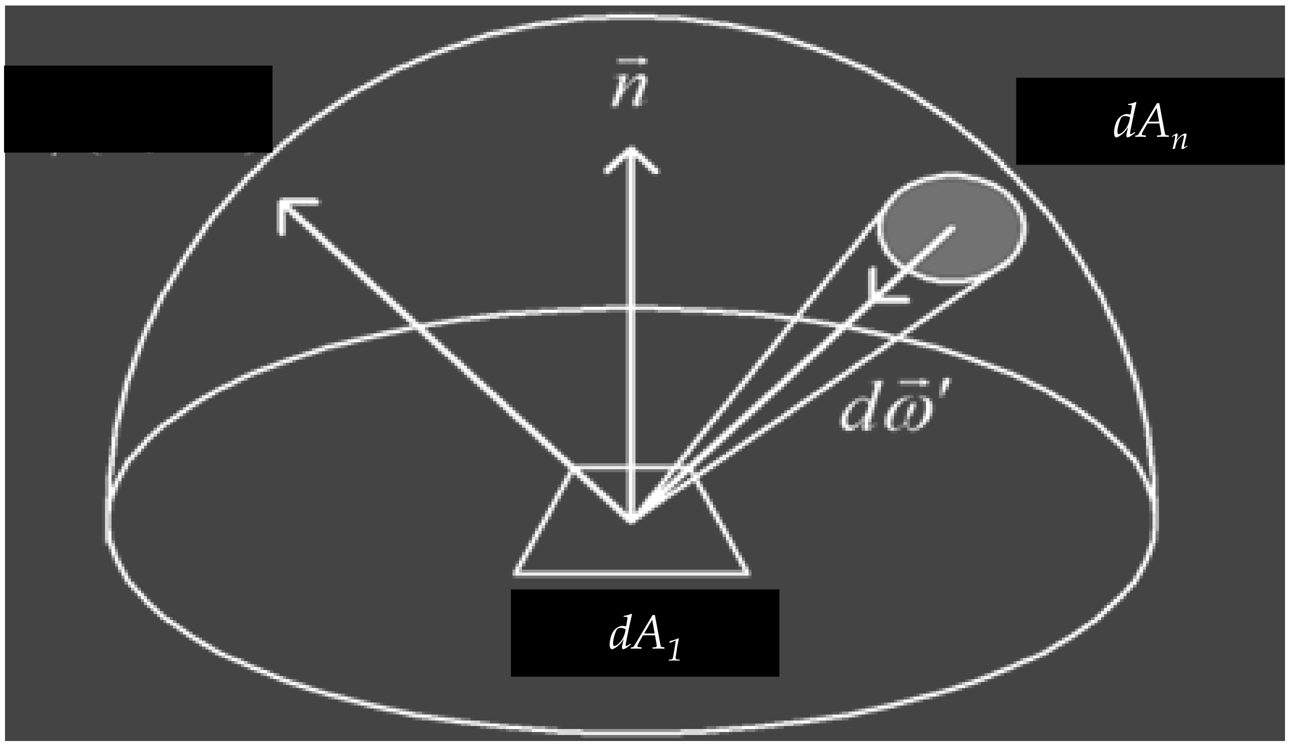
Planar angle



Solid angle

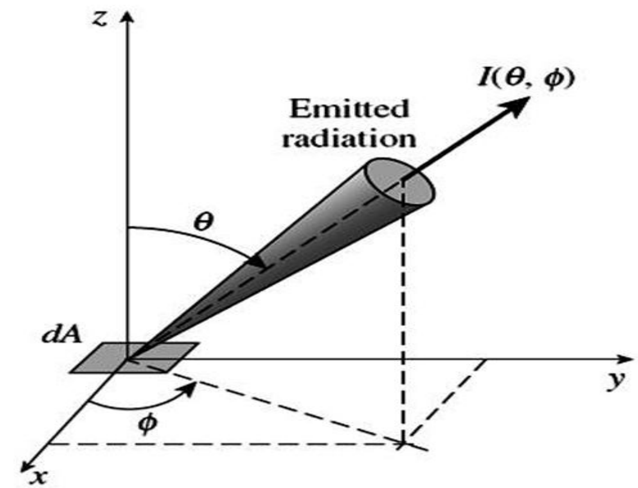
Solid Angle

Region Of sphere ,which is enclosed by a conical surface with the vertex of the cone at the centre of the sphere.



$$d\omega = \frac{dA_n}{r^2}$$

Intensity Of Radiation



Radiation emitted at angle θ

- When the collector is oriented at an angle θ_1 from the normal to the emitter, then the radiations striking and being absorbed by the collector can be expressed as:

$$(dE_b)_{\theta_1} = I_{\theta_1} d\omega_1 dA$$

$$= I_n \cos\theta_1 d\omega_1 dA$$

- Where, $d\omega_1$ is the solid angle subtended by the collector at the surface of the emitter dA.

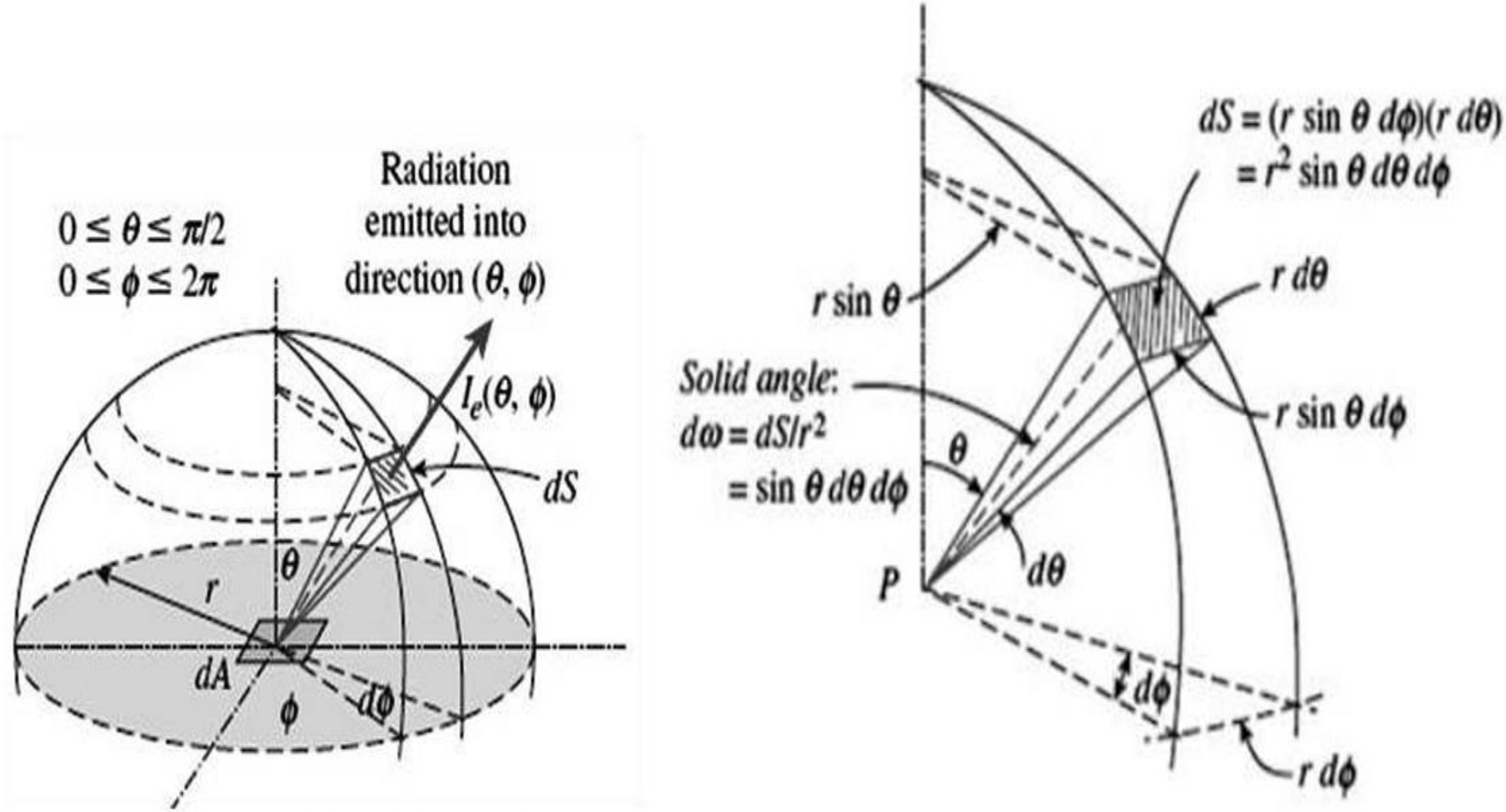


Fig. 5.17 Emission of radiation from differential element dA into hemispherical shape

$$Area of collector (dS) = (r d\theta)(r \sin\theta d\emptyset) = r^2 \sin\theta d\theta d\emptyset$$

$$solid angle (d\omega) = \frac{dS}{r^2} = \frac{r^2 \sin\theta d\theta d\emptyset}{r^2} = \sin\theta d\theta d\emptyset$$

$$solidangle(d\omega) = \frac{dS}{r^2} = \frac{r^2 \sin\theta d\theta d\emptyset}{r^2} = \sin\theta d\theta d\emptyset$$

- Then the radiations leaving the emitter and striking the collector is:

$$dE_b = I_\theta d\omega dA$$

- Substitute the value of I_θ and $d\omega$ in the above equation

$$dE_b = I_n \cos\theta \sin\theta d\theta d\emptyset\, dA - - - - - - - - (5.13)$$

- The total energy E_b radiated by the emitter and passing through a hemispherical region can be worked out by integrating the above equation over the limits

$$\theta = 0\ to \theta = \frac{\pi}{2} and \emptyset = 0\ to \emptyset = 2\pi$$

Thus,

$$\int_{hemisphere} dE_b = I_n\, dA \int_0^{\pi/2} \cos\theta \sin\theta d\theta \int_0^{2\pi} d\emptyset$$

$$E_b = I_n dA \cdot \frac{1}{2}(2\pi) = I_n dA\pi - - - - - - - - (5.14)$$

- But the total emissive power of the emitter with area dA and the temperature T is also given by:

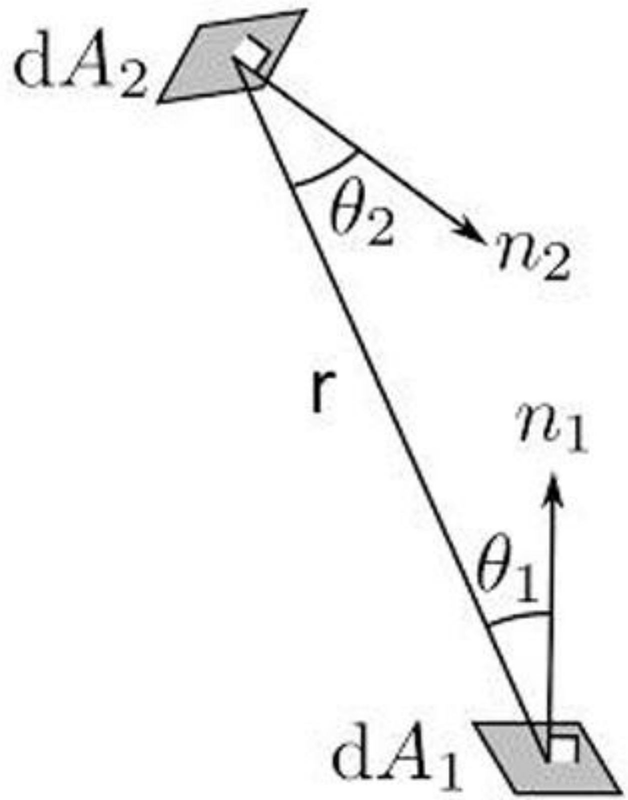
$$E_b = \sigma_b T^4 dA$$

- Combining the above equations, we get

$$I_n = \frac{\sigma_b T^4}{dA\pi} \cdot dA = \frac{\sigma_b T^4}{\pi} - - - - - - - - (5.15)$$

- Thus for a unit surface, the intensity of normal radiation I_n is the $1/\pi$ times the emissive power E_b.

Heat Exchange Between Two Black Surfaces: Shape Factor



Radiant heat exchange between two black surfaces

If the surface dA_2 subtends a solid angle $d\omega_1$ at the centre of the surface dA_1, then radiant energy emitted by dA_1 and impinging on (and absorbed by) the surface dA_2 is:

$$dQ_{12} = I_{\theta 1} d\omega_1 dA_1 = I_{n1} \cos\theta_1 \, d\omega_1 dA_1$$

Where,

$I_{\theta 1}$ = Intensity of radiation at an angle θ_1 with normal to the surface dA_1 and is given by $I_{n1} \cos\theta_1$

I_{n1} = Intensity of radiation normal to the surface dA_1

Projected area of dA_2 normal to the line joining dA_1 and dA_2 = $dA_2 \cos\theta_2$

$$\therefore dQ_{12} = I_{n1} \cos\theta_1 \frac{dA_2 \cos\theta_2}{r^2} dA_1 = I_{n1} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{r^2}$$

But

$$I_{n1} = \frac{\sigma_1 T_1^4}{\pi}$$

$$\therefore dQ_{12} = \frac{\sigma_1 T_1^4}{\pi} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{r^2} \text{-------- } (6.1)$$

– Integration of equation 6.1 over finite areas A_1 and A_2 gives:

$$Q_{12} = \frac{\sigma_1 T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2} - - - - - - - - (6.2)$$

"The fraction of the radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening relections."

The radiation shape factor is represented by the symbol F_{ij} which means the shape factor from a surface A_i to another surface A_j. Thus the radiation shape factor F_{12} of surface A_1 to surface A_2 is

$$F_{12} = \frac{direct\ radiation\ from\ surface\ 1\ incident\ upon\ surface\ 2}{total\ radiation\ from\ emitting\ surface\ 1}$$

$$= \frac{Q_{12}}{\sigma_1 A_1 T_1^4} = \frac{1}{\sigma_1 A_1 T_1^4} \frac{\sigma_1 T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

$$\frac{1}{A_1 \pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2} - - - - - - - - (6.3)$$

- From the equation no. 6.2 and 6.3, the radiation leaving A_1 and striking A_2 is given by

$$Q_{12} = A_1 F_{12} \sigma_1 T_1^4 \text{ --------- } (6.4)$$

- Similarly the energy leaving A_2 and striking A_1 is

$$Q_{21} = A_2 F_{21} \sigma_2 T_2^4 \text{ --------- } (6.5)$$

- and the net energy exchange from A_1 to A_2 is :

$$(Q_{12})_{net} = A_1 F_{12} \sigma_1 T_1^4 - A_2 F_{21} \sigma_2 T_2^4$$

- When the surfaces are maintained at the same temperatures, T_1 and T_2, there can be no heat exchange,

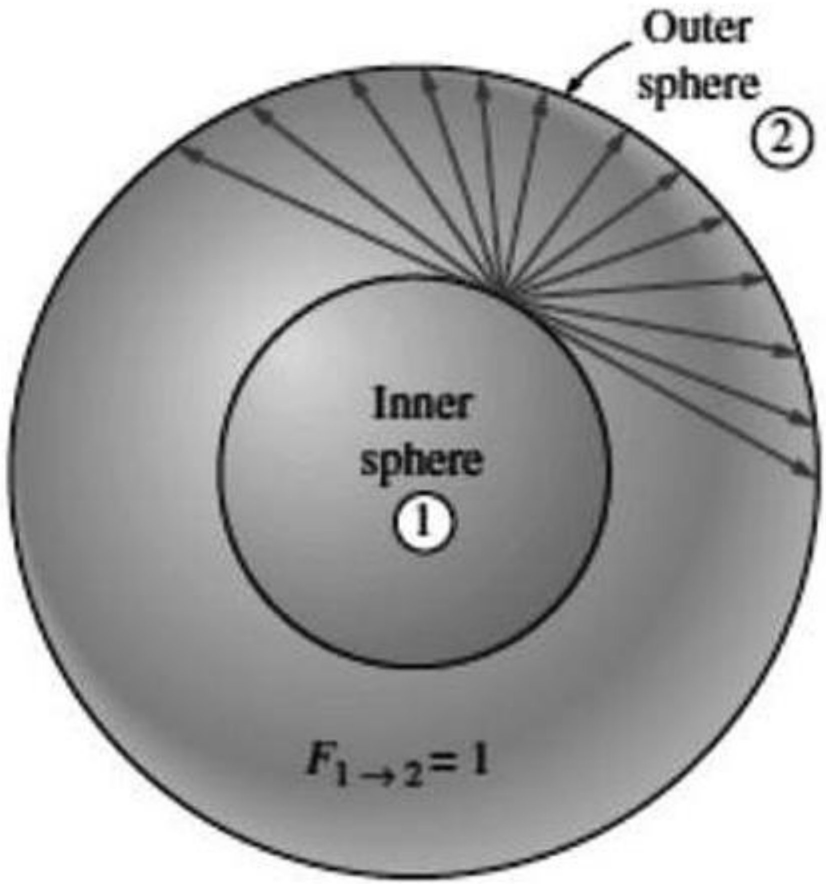
$$0 = (A_1 F_{12} - A_2 F_{21}) \sigma_b T_1^4 \{because\ \sigma_1 = \sigma_2 = \sigma_b\}$$

- Since σ_b and T_1 are non-zero quantities,

$$A_1 F_{12} - A_2 F_{21} = 0\ or\ A_1 F_{12} = A_2 F_{21} \text{ --------- } (6.6)$$

- The above result is known as a reciprocity theorem. It indicates that the net radiant interchange may be evaluated by computing one way configuration factor from either surface to the other. Thus net heat exchange between surfaces A_1 and A_2 is

$$(Q_{12})_{net} = A_1 F_{12} \sigma_b (T_1^4 - T_2^4) = A_2 F_{21} \sigma_b (T_1^4 - T_2^4) \text{ --------- } (6.7)$$



Two concentric spheres

$$A_1F_{12} = A_2F_{21} \; but \; F_{12} = 0$$

$$\therefore F_{21} = \frac{A_1}{A_2}$$

(a) Flat surface *(b) Convex surface* *(c) Concave surface*

Shape factor of surface with respect to itself

The Reciprocity Relation

The view factor F_{12} and F_{21} are not equal to each other unless the area of the two surfaces are. That is,

$$F_{12} = F_{21} when A_1 = A_2$$

$$F_{12} \neq F_{21} \; when \; A_1 \neq A_2$$

We have already discussed that the view factors are related to each other is given by

$$A_1 F_{12} = A_2 F_{21}$$

This relation is know as reciprocity relation or the reciprocity rule.

Summation Rule

Radiaton leaving the surface i of an enclosure intercepted by completely by the surface of enclosure

$$\sum_{j=1}^{N} F_{i-j} = F_{1-1} + F_{1-2} + F_{1-3} \ldots\ldots + F_{1-n} = 1$$

Where, N is the number of surfaces of the enclosure.

Applying the summation rule to surface 1 of a three-surface enclosure,

$$\sum_{j=1}^{3} F_{1-j} = F_{1-1} + F_{1-2} + F_{1-3} = 1$$

The Superposition Rule

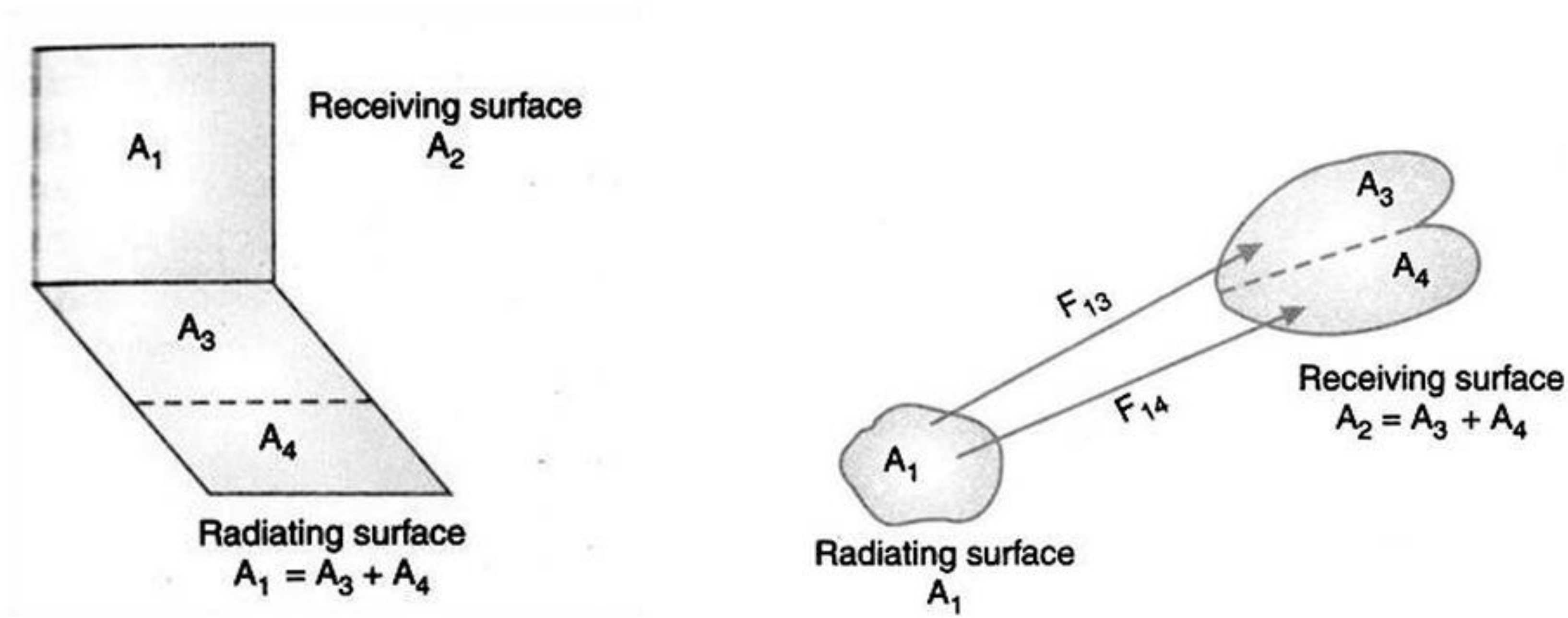


Fig. 6.5 Superposition rule

$$A_1F_{12} = A_3F_{32} + A_4F_{42}$$

$$A_1F_{12} = A_1F_{13} + A_1F_{14}$$

$$\mathrm{F}_{12} \neq \mathrm{F}_{32} + \mathrm{F}_{42}$$

$$F_{12} = F_{13} + F_{14}$$

The Symmetry Rule

Identical surfaces that are oriented in an identical manner with respect to another surface will intercept identical amounts of radiation leaving that surface.

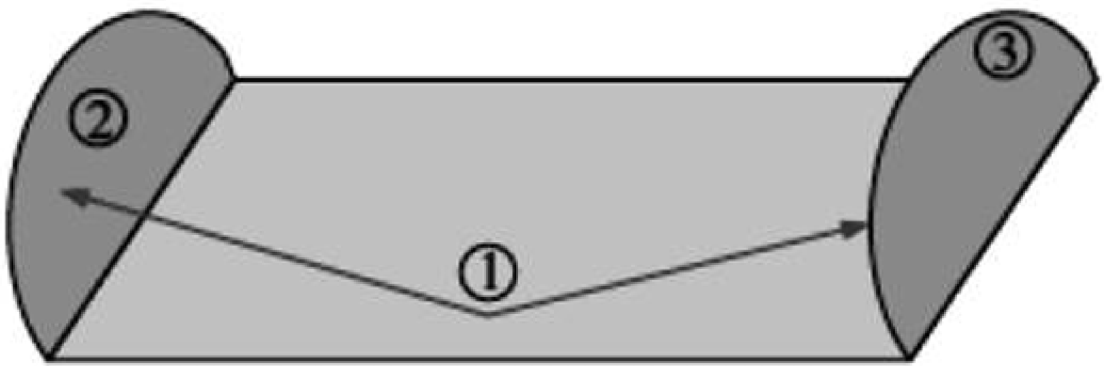
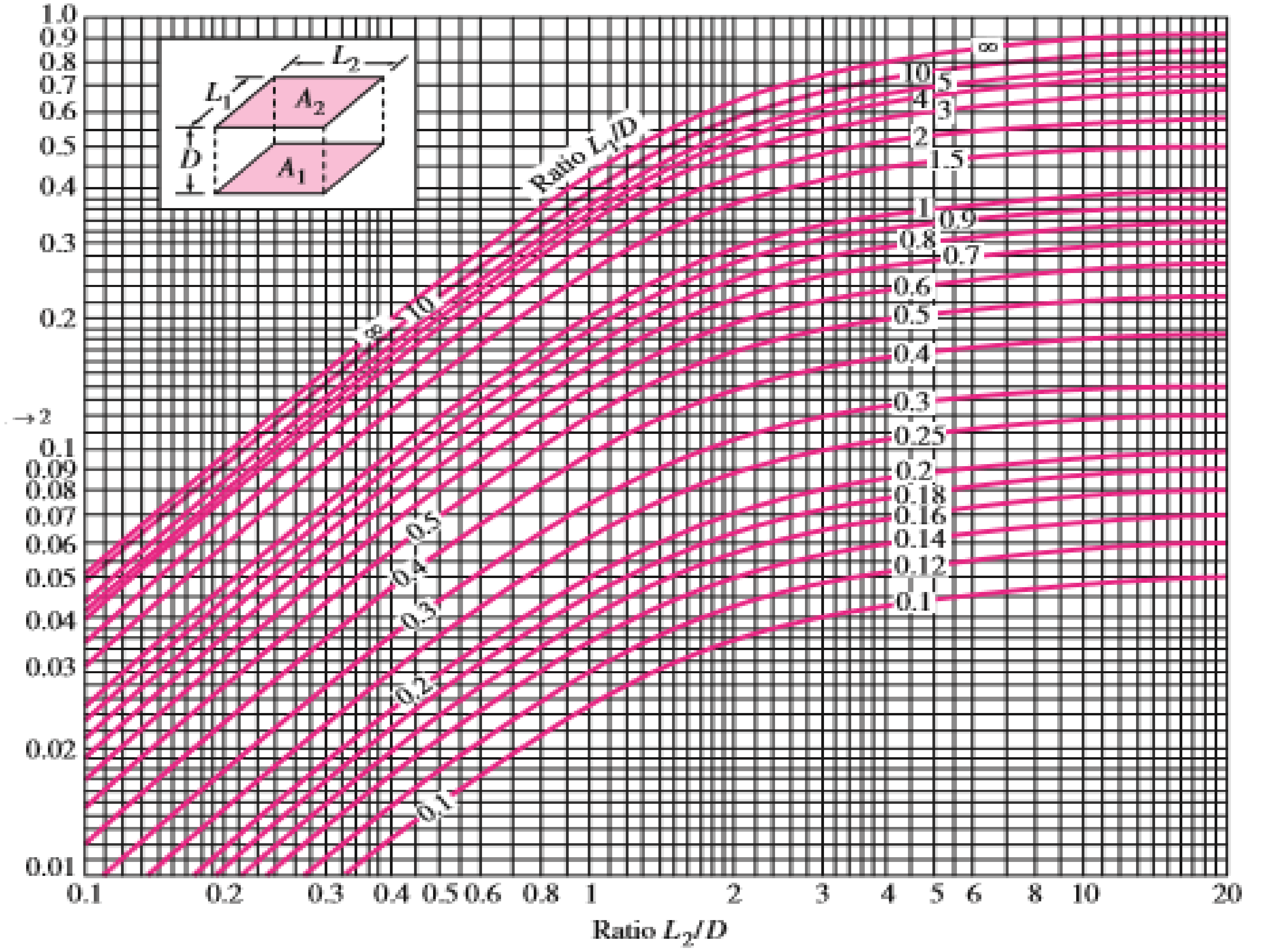


Fig. 6.6 Symmetry rule

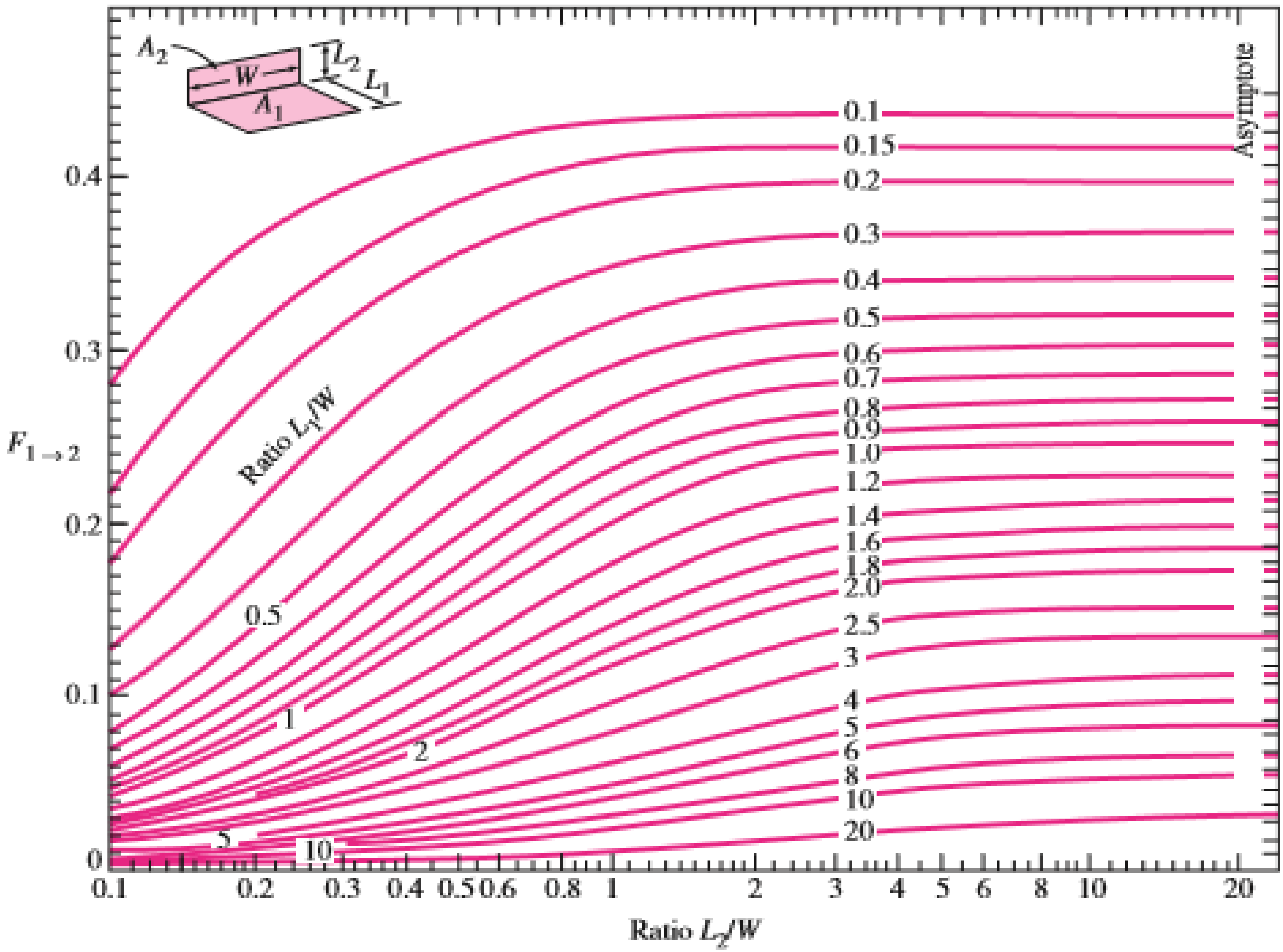
So, the symmetry rule can be expressed as two or more surfaces that posse symmetry about a third surface will have identical view factors from that surface.

From the figure 6.6

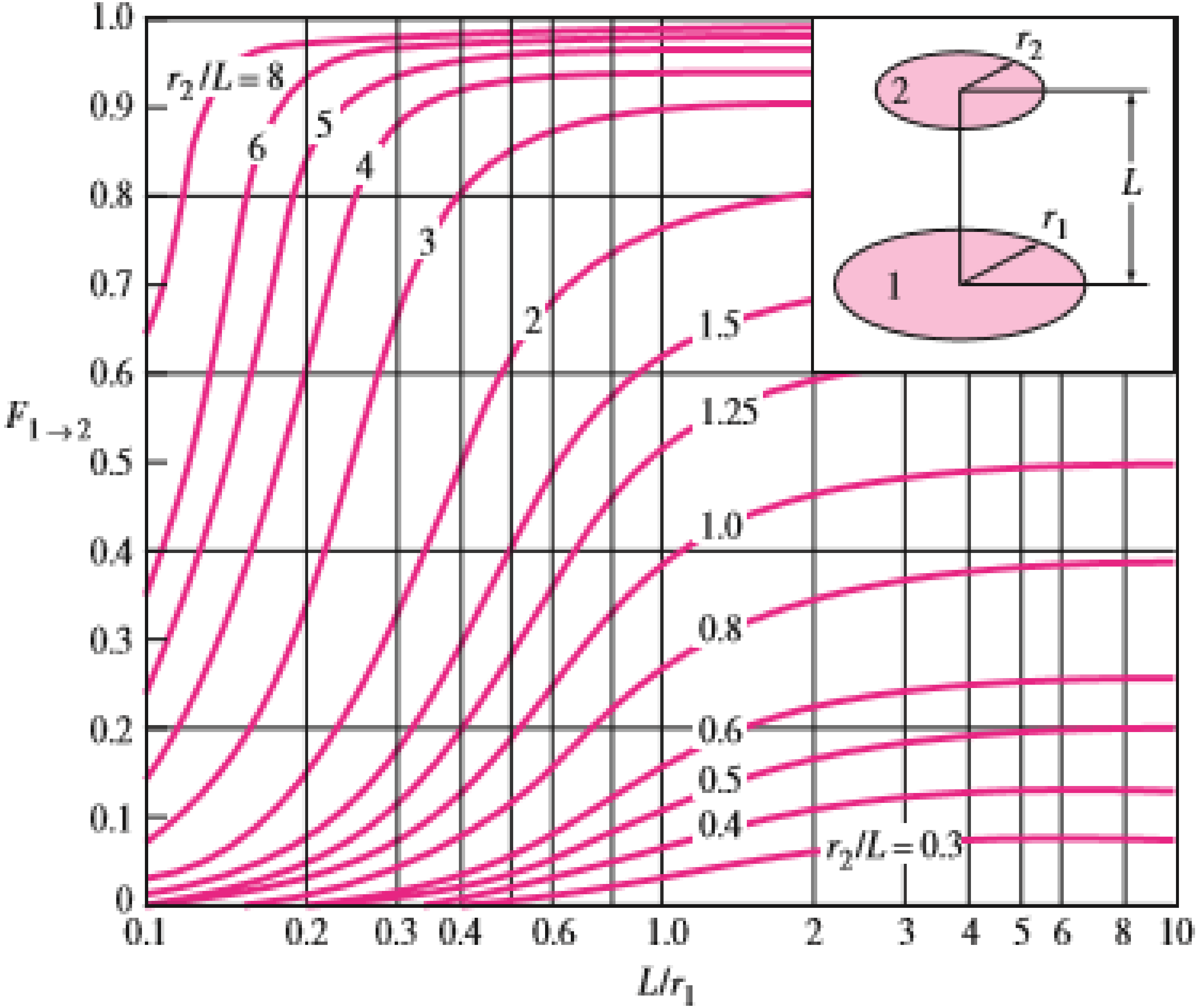
$$F_{12} = F_{13}$$



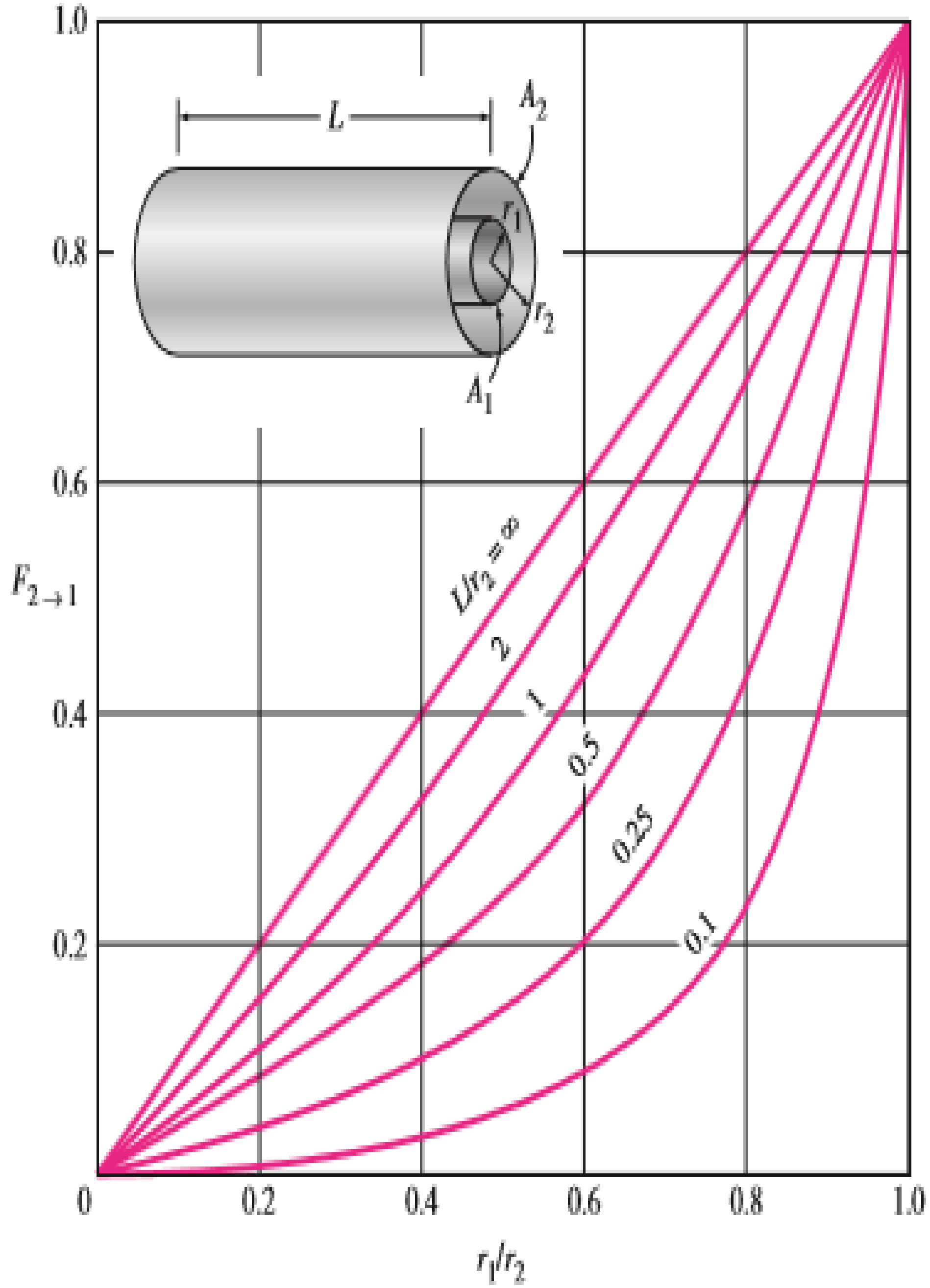
L_2
L_1
A_2
D
A_1
Ratio L_1/D
∞
10
5
4
3
2
1.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.25
0.2
0.18
0.16
0.14
0.12
0.1
→2
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.1 0.2 0.3 0.4 0.5 0.6 0.8 1 2 3 4 5 6 8 10 20
Ratio L_2/D



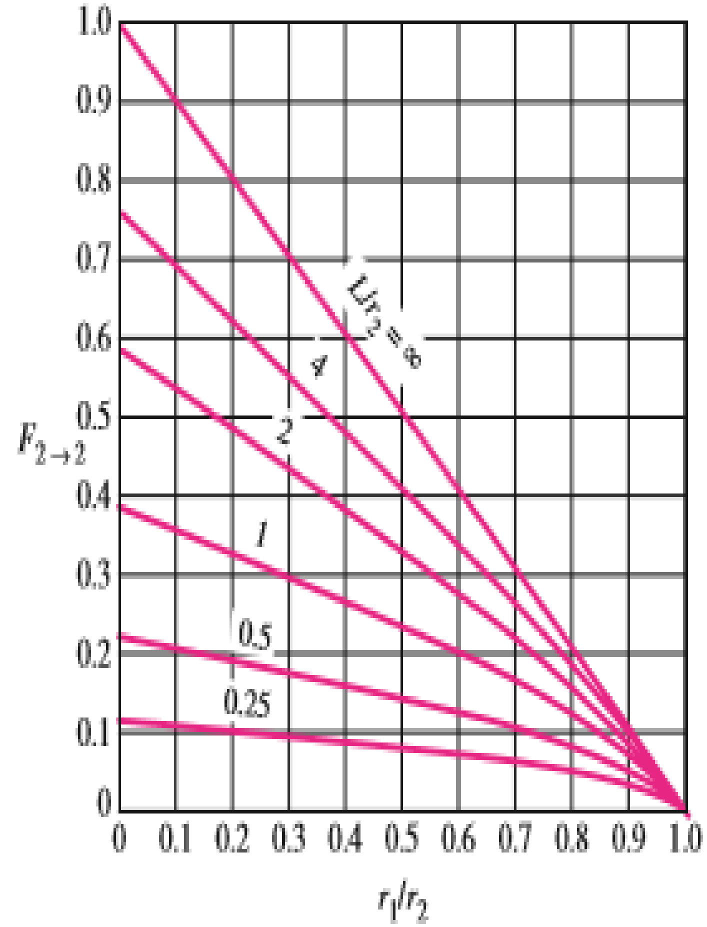
A_2
W
L_2
L_1
A_1
Asymptote
0.1
0.15
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0
2.5
3
4
5
6
8
10
20
Ratio L_1/W
0.5
1
2
5
10
$F_{1\to2}$
0
0.1
0.2
0.3
0.4
0.1
0.2
0.3
0.4
0.5
0.6
0.8
1
2
3
4
5
6
8
10
20
Ratio L_2/W



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$F_{1 \to 2}$
$r_2/L = 8$
6
5
4
3
2
1.5
1.25
1.0
0.8
0.6
0.5
0.4
$r_2/L = 0.3$
0.1
0.2
0.3
0.4
0.6
1.0
2
3
4
5
6
8
10
L/r_1
2
r_2
L
r_1
1

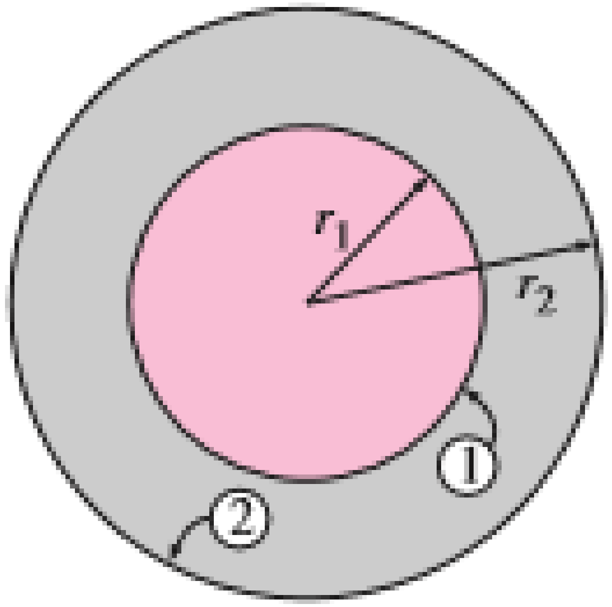


1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
L
A2
A1
r1
r2
F2→1
L/r2 = ∞
2
1
0.5
0.25
0.1
r1/r2



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
F2→2
L/r2 = ∞
4
2
1
0.5
0.25
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
r1/r2

r_1=3m and r_2 =6m

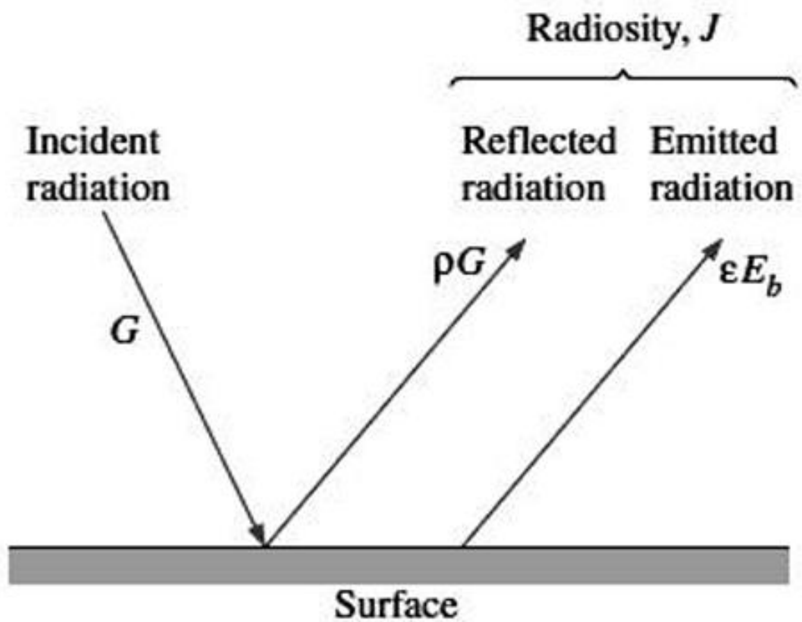


- Calculate associated View Factors.

A Furnace cavity, which in form of cylinder of 75mm diameter and 150 mm length is open at one end to large surroundings that are at 27 °C. The sides and bottom may be black surface and well insulated and maintained at 1350 °C and 1650 °C respectively.

How much power is required to maintain the furnace temperature.

RADIOSITY



Surface radiosity and irradation

$$J = E + \rho G = \varepsilon E_b + \rho G \cdot$$

- Where E_b is the emissive power of a perfect black body at the same temperature. As no energy is transmitted through the opaque body, $\alpha + \rho = 1$ and so

$$J = \varepsilon E_b + (1 - \alpha)G$$

- According to Kirchoff's law, the absorptivity α of the surface is equal to emissivity ε. Therefore,

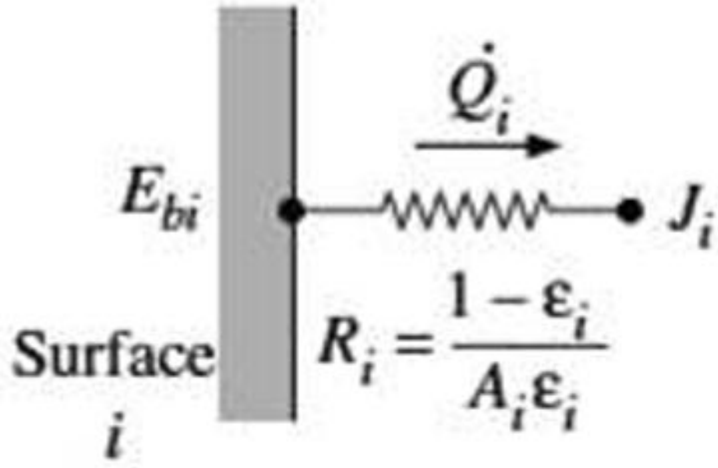
$$J = \varepsilon E_b + (1 - \varepsilon)G$$

$$\therefore G = \frac{J - \varepsilon E_b}{1 - \varepsilon}.$$

- The rate at which the radiation leaves the surface is given by the difference between its radiosity and irradiation.

$$\frac{Q_{net}}{A} = J - \frac{J - \varepsilon E_b}{1 - \varepsilon} = \frac{J - J\varepsilon - J + \varepsilon E_b}{1 - \varepsilon} = \frac{\varepsilon(E_b - J)}{1 - \varepsilon}$$

$$Q_{net} = \frac{A\varepsilon}{1 - \varepsilon}(E_b - J) = \frac{E_b - J}{(1 - \varepsilon)/A\varepsilon} - - - - - - - - - (6.21)$$



Electrical analogy of surface resistance to radiation

can be written in the form of electrical network as

$$Q_{net} = \frac{E_b - J}{(1-\varepsilon)/A\varepsilon} = \frac{E_b - J}{R_i}$$

Now consider the radiant heat exchange between two non-black surfaces. Out of total radiation J_1 leaving the surface 1, only a fraction $J_1A_1F_{12}$ is received by the other surface 2. Similarly the heat radiated by surface 2 and received by surface 1 is $J_2A_2F_{21}$. So net heat transfer between two surfaces is given by

$$Q_{12} = J_1A_1F_{12} - J_2A_2F_{21} --------(6.24)$$

From the recirpicity theorem : $A_1F_{12} = A_2F_{21}$

$$\therefore Q_{12} = (J_1 - J_2)A_1F_{12} = \frac{(J_1 - J_2)}{1/A_1F_{12}} --------(6.25)$$

Equation 6.25 can be represented by an electrical circuit as shown in figure 6.9. The factor $1/A_1F_{12}$ is related to distance between two bodies and its geometry, and is called space resistance to radiation heat transfer.

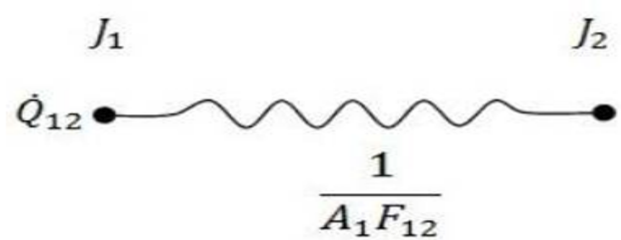


Fig. 6.9 Electrical analogy of space resistance to radiation

Equation 6.9 can be written in the form of electrical network as

$$Q_{12} = \frac{(J_1 - J_2)}{1/A_1F_{12}} = \frac{(J_1 - J_2)}{R_{12}} --------(6.26)$$

Where, R_{12} is the space resistance and given by

$$R_{12} = \frac{1}{A_1F_{12}} --------(6.27)$$

Radiation heat transfer can be represented by electrical network, consisting of two surface resistances of two radiating bodies and the space resistance between them as shown in figure 6.10.

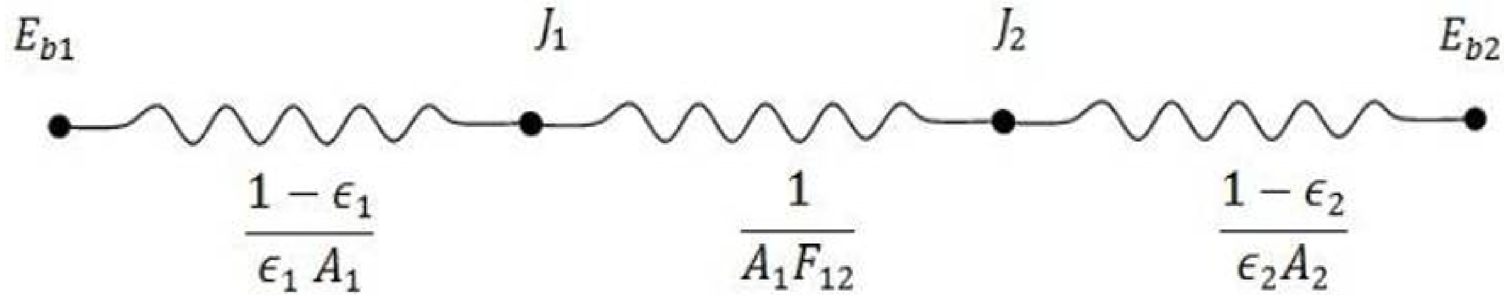


Fig. 6.10 Electrical analogy of radiation heat transfer between two surfaces

- The net heat exchange between two gray surfaces is given by

$$(Q_{12})_{net} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} - - - - - - - - - (6.28)$$

- Where R_1 and R_2 are surface resistances and R_{12} is space resistance, equation 6.28 can be written as,

$$(Q_{12})_{net} = \frac{E_{b1} - E_{b2}}{(1-\varepsilon_1)/A_1\varepsilon_1 + 1/A_1F_{12} + (1-\varepsilon_2)/A_2\varepsilon_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1-\varepsilon_1)/A_1\varepsilon_1 + 1/A_1F_{12} + (1-\varepsilon_2)/A_2\varepsilon_2}$$

$$= \frac{A_1\sigma(T_1^4 - T_2^4)}{(1-\varepsilon_1)/\varepsilon_1 + 1/F_{12} + (1-\varepsilon_2)/\varepsilon_2 \cdot A_1/A_2}$$

$$= \left(F_g\right)_{12}\sigma A_1(T_1^4 - T_2^4) - - - - - - - - - (6.29)$$

- Where, $\left(F_g\right)_{12}$ is called gray body factor and is given by

$$\left(F_g\right)_{12} = \frac{1}{(1-\varepsilon_1)/\varepsilon_1 + 1/F_{12} + (1-\varepsilon_2)/\varepsilon_2 \cdot A_1/A_2} - - - - - - - - - (6.30)$$