Module 5 – SLIDING CONTACT BEARINGS

Lecture 2 – HYDRODYNAMIC LUBRICATION OF JOURNAL BEARINGS THEORY AND PRACTICE

Contents

2.1 PETROFF'S EQUATION FOR BEARING FRICTION

In 1883, Petroff published his work on bearing friction based on simplified assumptions.

a. No eccentricity between bearings and journal and hence there is no "Wedging action" as in Fig.2.1.

b. Oil film is unable to support load.

c. No lubricant flow in the axial direction.

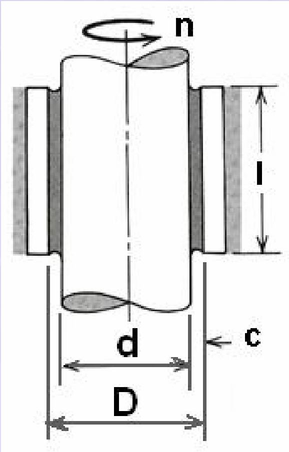

Fig. 2.1 Unloaded Journal bearing

With reference to Fig.2.1, an expression for viscous friction drag torque is derived by considering the entire cylindrical oil film as the “liquid block” acted upon by force F.

From Newton’s law of Viscosity:

$$F = \mu \frac{AU}{h} \qquad (2.1)$$

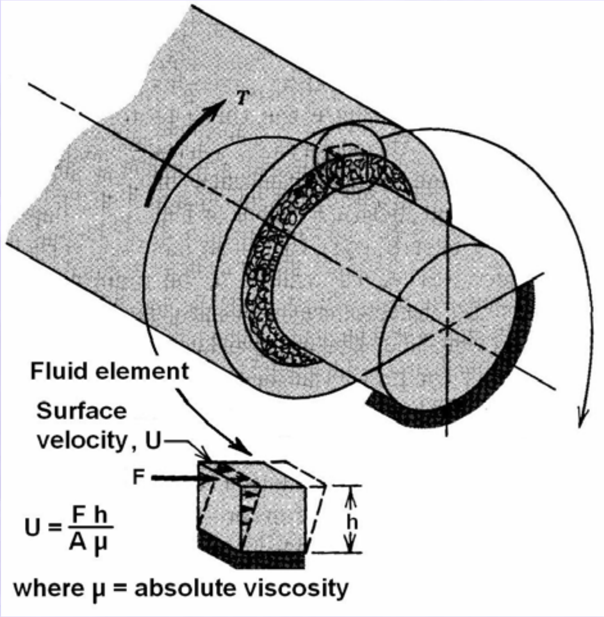

Fig. 2.2 Laminar flow of fluid in clearance space

Where F = friction torque/shaft radius = 2 T_f / d

A= π d l

U = π d n (Where n is in rps d is in m)

h = c (Where c = radial clearance = 0.5(D-d))

r = d /2

Substituting and solving for friction torque:

$$T_f = \frac{4\pi^2 \mu n l r^3}{c} \qquad (2.2)$$

If a small radial load W is applied to the shaft, Then the frictional drag force f w and the friction

Torque will be:

$$T_f = f\,w = 0.5\,f\,(d\,l\,p)\,d \qquad (2.\,3)$$

Equating eon. (2.2) and (2.3) and simplifying, we get

$$f = 2\pi^2\left(\frac{\mu n}{p}\right)\left(\frac{r}{c}\right) \qquad (2.4)$$

Where r = 0.5 d and u is Pa.

This is known as Petroff's equation for bearing friction. It gives reasonable estimate of co-efficient of friction of lightly loaded bearings.

The first quantity in the bracket stands for bearing modulus and second one stands for clearance ratio. Both are dimensionless parameters of the bearing.

Clearance ratio normally ranges from 500 to 1000 in bearings.

2.2 PETROFF'S EQUATION FOR BEARING FRICTION – Problem 1

A machine journal bearing has a journal diameter of 150 mm and length of 120 mm. The bearing diameter is 150.24 mm. It is operating with SAE 40 oil at 65^{o}C. The shaft is carrying a load of 8 kN and rotates at 960 rpm. Estimate the bearing coefficient of friction and power loss using Petroff's equation.

Data: d = 0.15m; D =0.15024m; l = 0.12 m; F=8kN;

SAE 40 oil T_o = 65^oC; n = 960/60 = 16 rps.

Q 1, f =? , N_{loss} =?

Solution:

r = 0.5d = 0.5 x 0.15 = 0.075 m

c = (D-d) /2 = 0.00012 m

p = F/dl = 8000/ 150x 120 = 0.44 MPa= 44X10^4 Pa

Viscosity of SAE 40 at 65°C, μ = 30 mPa.s = $30x10^{-3}$ Ns/m^2

(a) $$f = 2\pi^2\left(\frac{\mu n}{p}\right)\left(\frac{r}{c}\right) = 2\pi^2\left(\frac{30x10^{-3}x16}{44x10^4}\right)\left(\frac{0.075}{0.00012}\right) = 0.0134$$

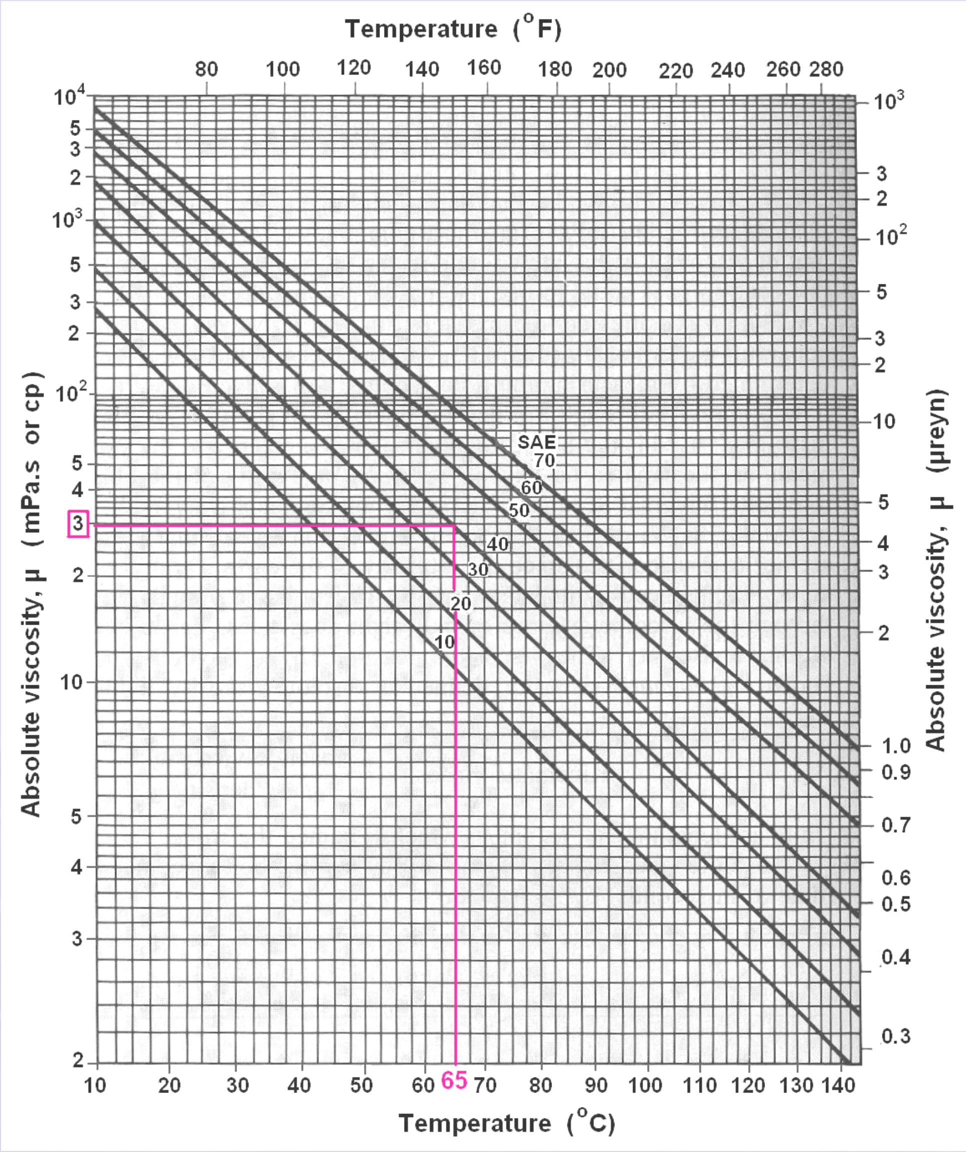

Fig.2.3a Viscosity – temperature curves of SAE graded oils

(b) Friction Torque T_f = f F r = 0.0134 x 8000 x 0.075 = 8.067 Nm

ω = 2πn /60 = 2 x 3.14 x 960 / 60 = 100.48 rad/s

Power loss: N_{loss} = T_f ω = 8.067 x 100.48 = 811 W

2.3 HYDRODYNAMIC LUBRICATION THEORY

Beauchamp Tower's exposition of hydrodynamic behavior of journal bearings in 1880s and his observations drew the attention of Osborne Reynolds to carryout theoretical analysis. This has resulted in a fundamental equation for hydrodynamic lubrication. This has provided a strong foundation and basis for the design of hydro-dynamic lubricated bearings.

In his theoretical analysis, Reynolds made the following assumptions:

a) The fluid is Newtonian.
b) The fluid is incompressible.
c) The viscosity is constant throughout the film.
d) The pressure does not vary in the axial direction.
e) The bearing and journal extend infinitely in the z direction. i.e., no lubricant flow in the z direction.
f) The film pressure is constant in the y direction. Thus the pressure depends on the x coordinate only.
g) The velocity of particle of lubricant in the film depends only on the coordinates x and y.
h) The effect of inertial and gravitational force is neglected.
i) The fluid experience laminar flow.

2.3.1 Reynolds' Equation

As shown in Fig.2.4, the Forces acting on a fluid element of height dy, width dx, velocity u, and top to bottom velocity gradient du is considered.

For the equilibrium of forces in the x direction acting on the fluid element acting on the fluid element shown in Fig. 2.5

$$-pdydz + \tau dxdz + \left(p + \frac{dp}{dx}dx\right)dydz - \left(\tau + \frac{\partial \tau}{\partial y}dy\right)dxdz = 0 \qquad (2.5)$$

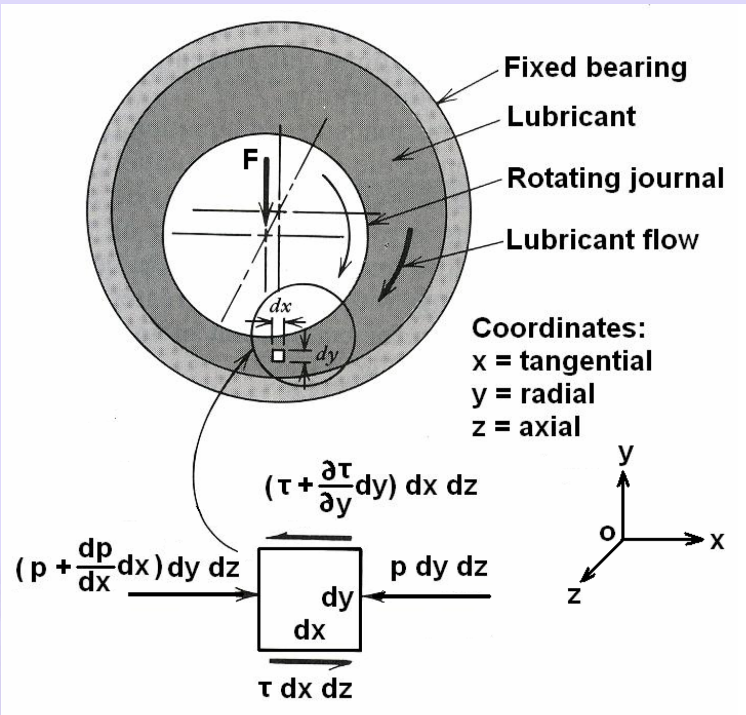

Fig.2.4 Pressure and viscous forces acting on an element of lubricant. Only X components are shown

which reduces to $$\frac{dp}{dx}=\frac{\partial \tau}{\partial y} \qquad (2.6)$$

The equation for absolute viscosity is given as

$$\mu = F\,h/(A\,U) \qquad (2.7)$$

In eqn. (7) is the shear stress.

$$\tau = \mu \frac{\partial u}{\partial y} \qquad (2.8)$$

In eqn. (2.7) $\tau=\frac{F}{A}$ is the shear stress.

where the partial derivatives is used since the velocity u depends upon both x and y. Substituting eqn (8) in (6), we get

$$\frac{dp}{dx}=\mu\frac{\partial^2 u}{\partial y^2} \qquad (2.9)$$

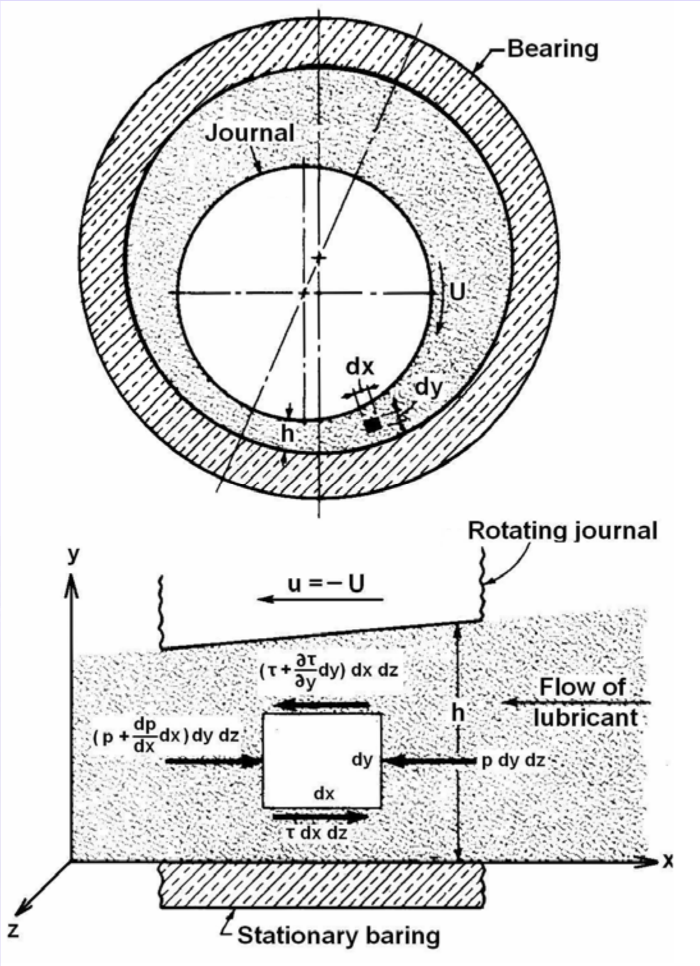

Fig.2.5 Pressure and viscous forces acting on an element of lubricant. Only X components are shown

Rearranging the terms, we get $$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dx} \quad (2.10)$$

Holding x constant and integrating twice with respect to y gives

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\left(\frac{dp}{dx} y + C_1\right) \quad (2.11)$$

$$u = \frac{1}{\mu}\left(\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2\right) \quad (2.12)$$

The assumption of no slip between the lubricants and the boundary surfaces gives boundary conditions enabling C_1 and C_2 to be evaluated:

u=0 at y=0, u=U at y=h

Hence,

$$C_1 = \frac{U\mu}{h} - \frac{h}{2}\frac{dp}{dx}\left(y^2 - hy\right) + \frac{U}{h}y \qquad (2.13)$$

$$\text{and } C_2 = 0 \qquad (2.14)$$

Substituting the values of C_1 and C_2 in Equation (2.12)

we get,

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - hy\right) + \frac{U}{h}y \qquad (2.15)$$

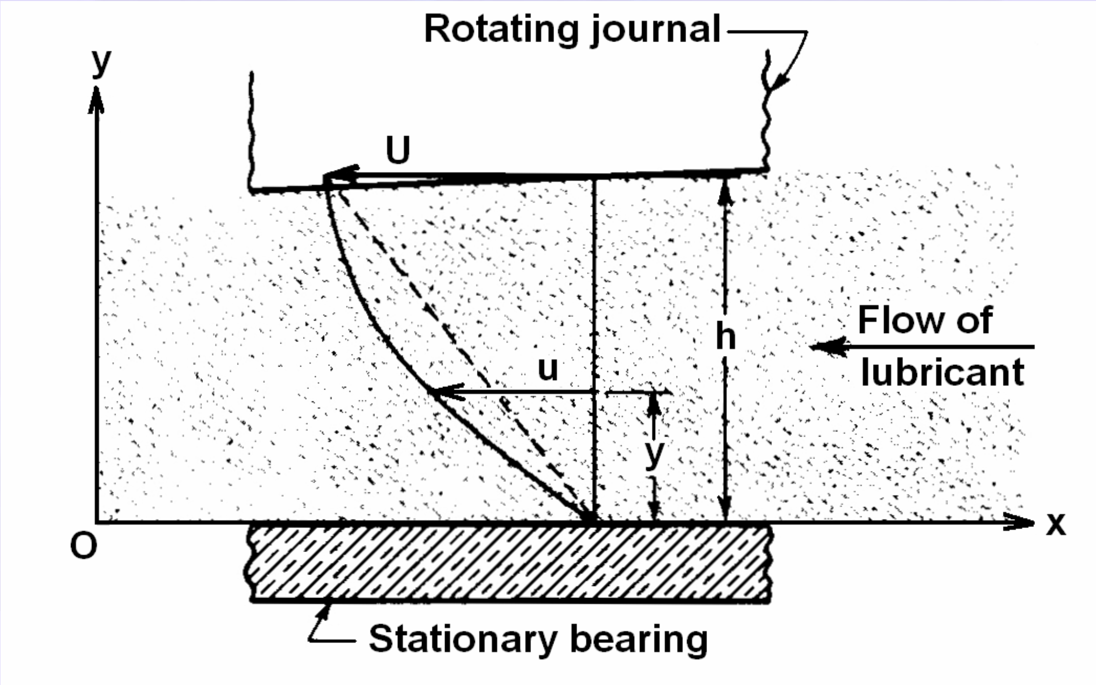

Fig. 2.6 Velocity distribution in the oil film

Velocity Distribution of the Lubricant Film shown in Fig.2.6 consists of two terms on the right hand side.

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - hy\right) + \frac{U}{h}y$$

Parabolic Linear – Dashed

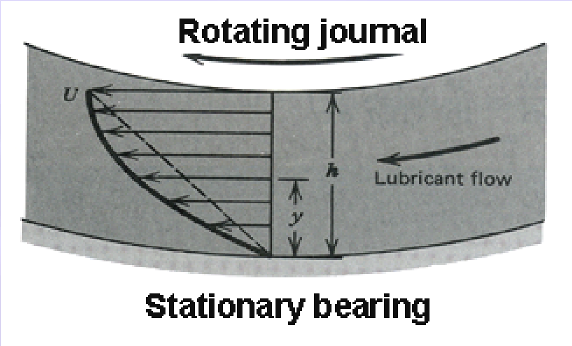

Fig. 2.7 Velocity gradient in the oil film

At the section when pressure is a maximum and the velocity gradient is linear.

$$\frac{dp}{dy} = 0$$

Let the volume of lubricant per-unit time flowing across the section containing the element in Fig. 2.6 be Q_f. For unit width in the Z direction,

$$Q_f = \int_o^h u\,dy = \frac{Uh}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} \qquad (2.16)$$

For an in-compressible liquid, the flow rate must be the same for all cross sections, which means that

$$\frac{dQ_f}{dx} = 0 \qquad (2.17)$$

Differentiating equation (2.16) with respect to x and equating to zero,

$$\frac{dQ_f}{dx} = \frac{U}{2}\frac{dh}{dx} - \frac{d}{dx}\left(\frac{h^3}{12\mu}\frac{dp}{dx}\right) = 0 \qquad (2.18)$$

Or

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx} \qquad (2.19)$$

This is the classical Reynolds' equation for one dimensional flow. This is valid for long bearings.

In short bearings, flow in the Z direction or end leakage has to be taken into account. A similar development gives the Reynolds' Equation for two dimensional flows:

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) + \frac{d}{dz}\left(\frac{h^3}{\mu}\frac{dp}{dz}\right) = 6U\frac{dh}{dx} \qquad (2.20)$$

Modern bearings are short and (l / d) ratio is in the range 0.25 to 0.75. This causes flow in the z direction (the end leakage) to a large extent of the total flow.

For short bearings, **Ockvirk** has neglected the x terms and simplified the Reynolds' equation as:

$$\frac{d}{dz}\left(\frac{h^3}{\mu}\frac{dp}{dz}\right) = 6U\frac{dh}{dx} \qquad (2.21)$$

Unlike previous equations (2.19) and (2. 20), equation (2. 21) can be readily integrated and used for design and analysis purpose. The procedure is known as **Ocvirk's short bearing approximation.**

2.4 DESIGN CHARTS FOR HYDRODYNAMIC BEARINGS

Solutions to eqn.2.19 were developed in first decade of 20th century and were applicable for long bearings and give reasonably good results for bearings with (l / d) ratios more than 1.5. Ocvirk's short bearing approximation on the other hand gives accurate results for bearings with (l /d) ratio up to 0.25 and often provides reasonable results for bearings with (l / d) ratios between 0.25 and 0.75.

Raimondi and Boyd have obtained computerized solutions for Reynolds eqn. (2.20) and reduced them to chart form which provide accurate solutions for bearings of all proportions. Selected charts are shown in Figs. 2.8 to 2.15. All these charts are plots of non-dimensional bearing parameters as functions of the bearing characteristic number, or the Sommerfeld variable S which itself is a dimensionless parameter.

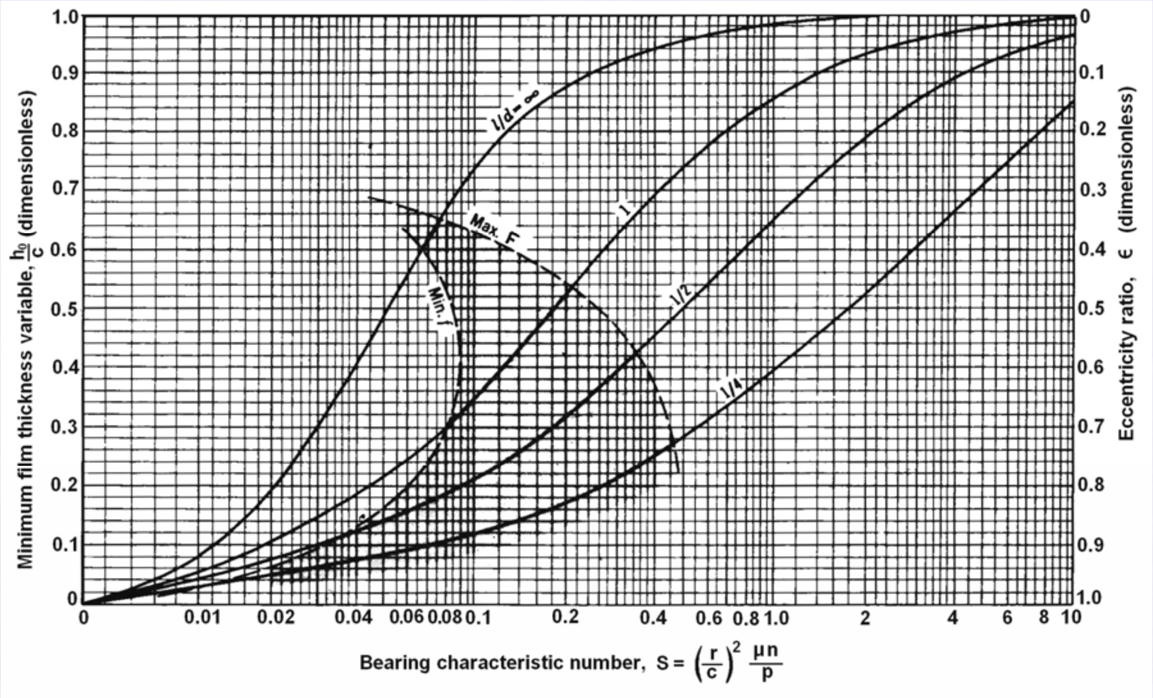

Fig.2.8 Chart for minimum film thickness variable and eccentricity ratio. The left shaded zone defines the optimum h_o for minimum friction; the right boundary is the optimum h_o for maximum load

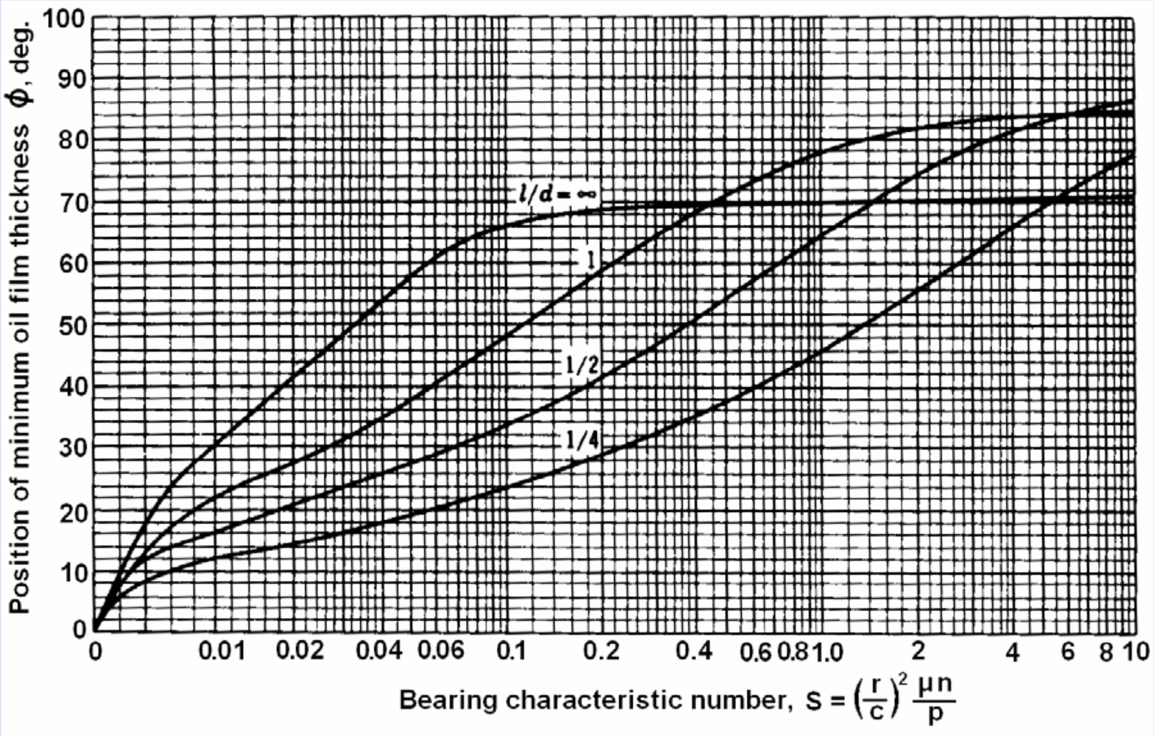

Fig.2.9 Chart for determining the position of the minimum film thickness h_o for location refer Fig.2.10

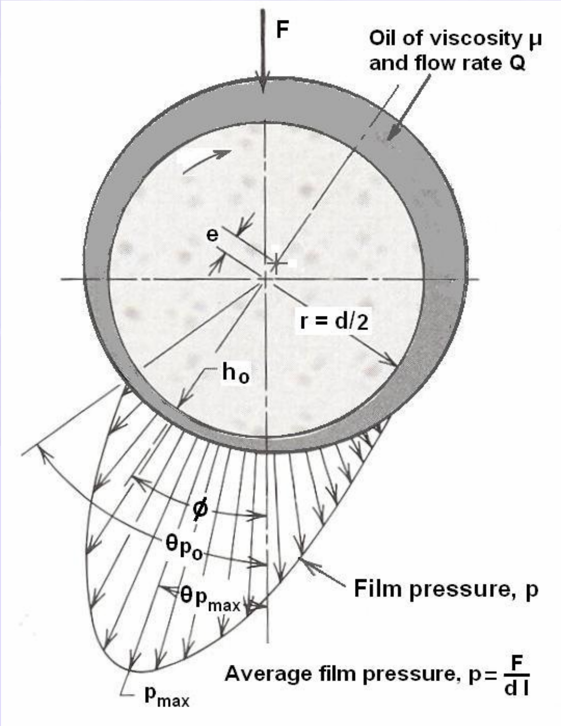

Fig.2.10 Stable hydrodynamic lubrication

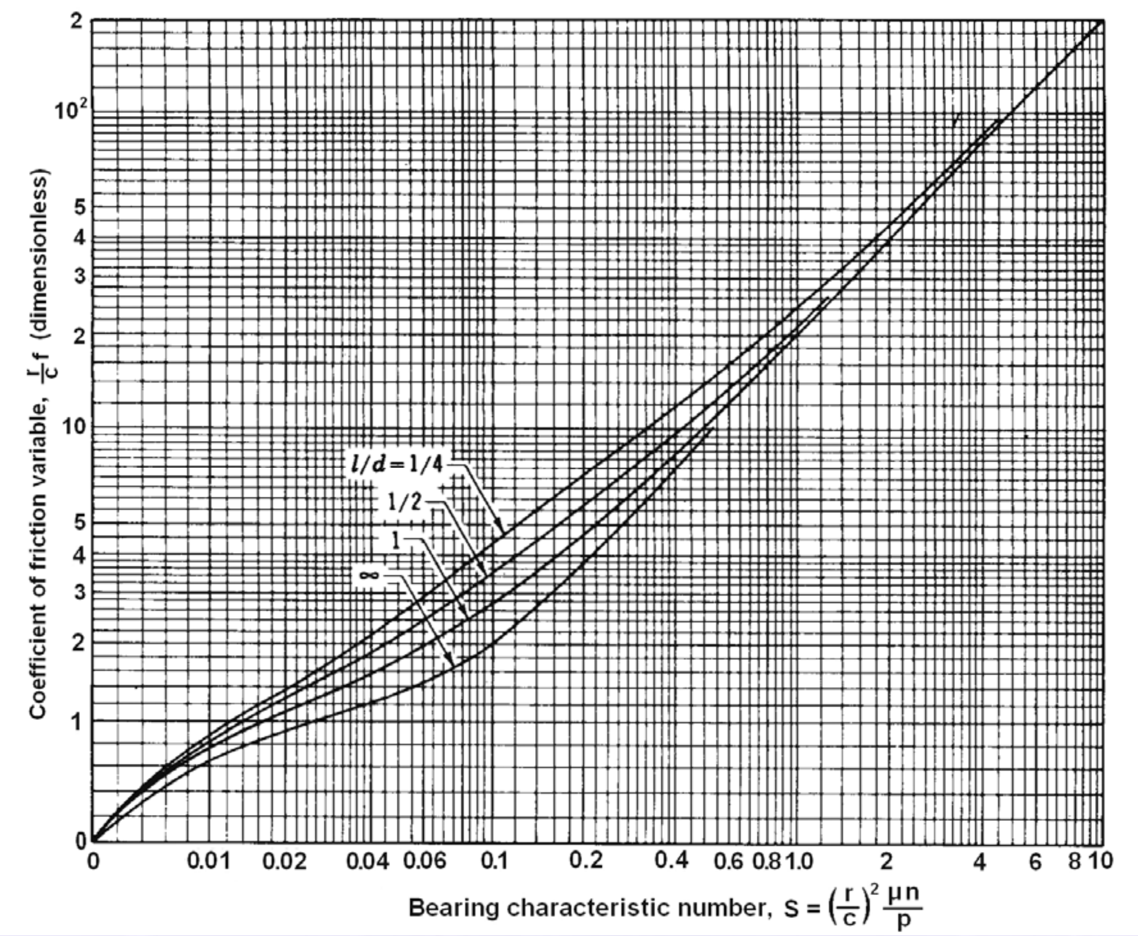

Fig. 2.11 Chart for coefficient of friction variable.

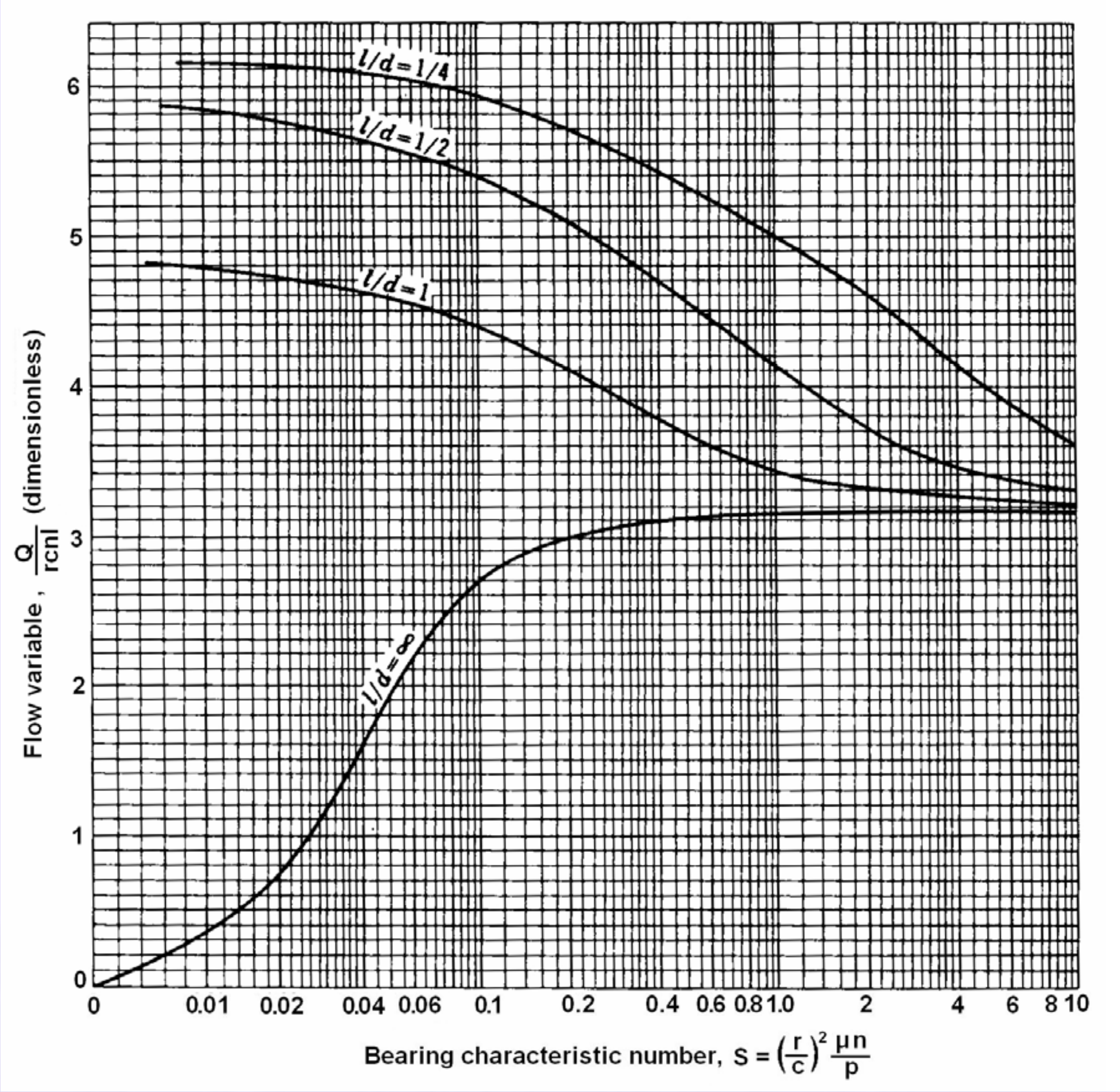

Fig. 2.12 Chart for flow variable.

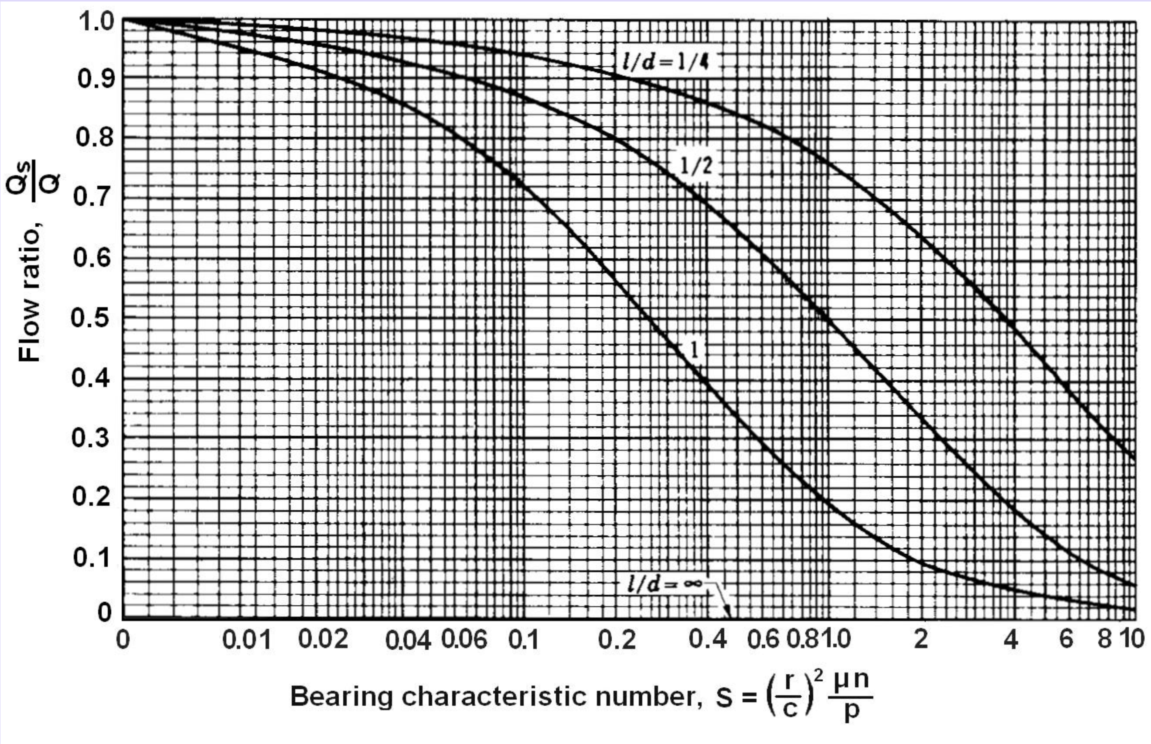

Fig.2.13 Chart for determining the ratio of side flow to total flow.

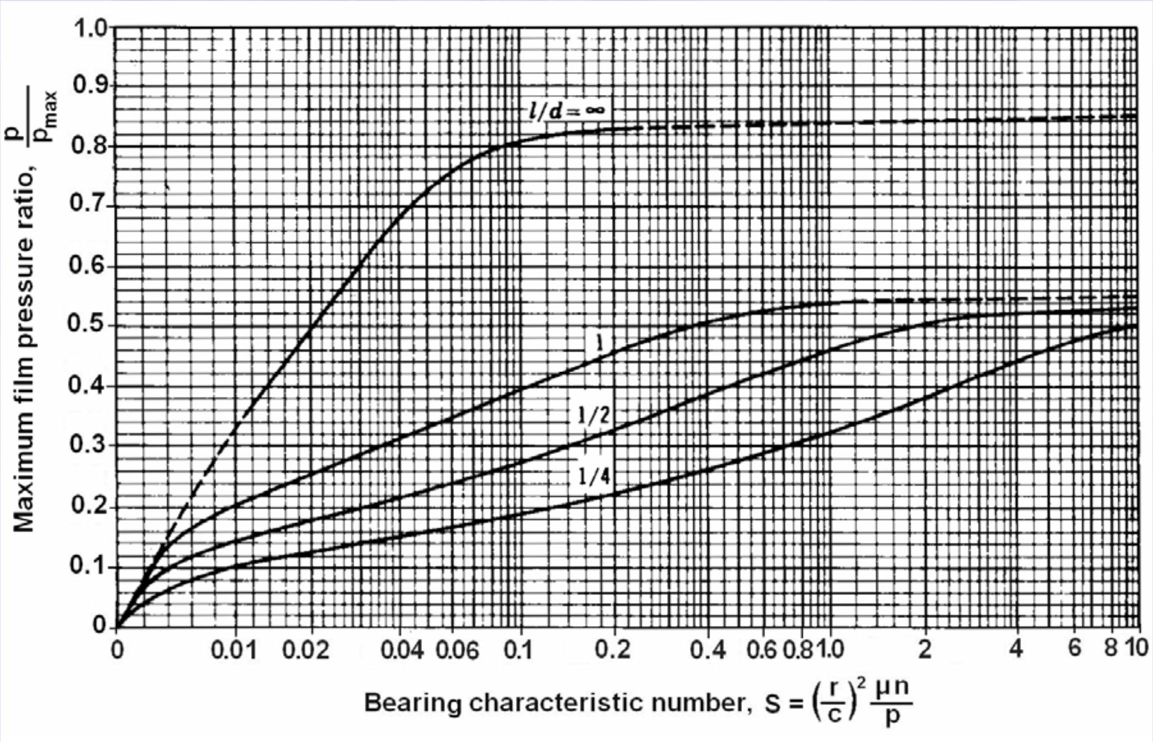

Fig. 2.14 Chart for determining the maximum film pressure.

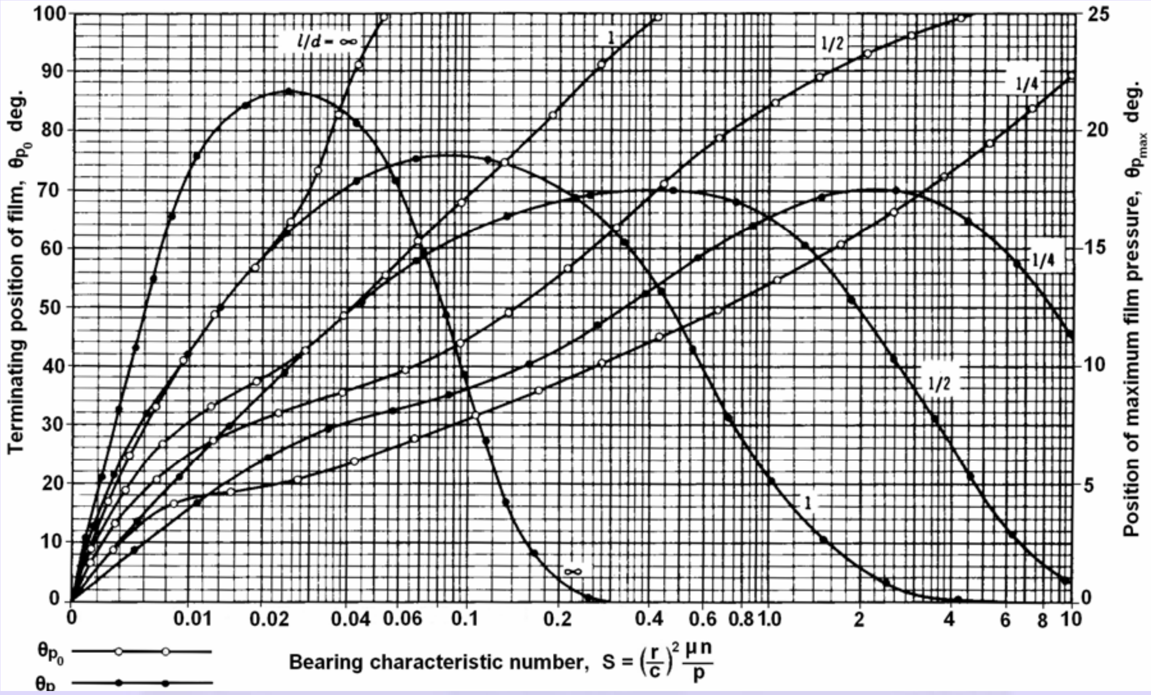

Fig. 2.15 Chart for finding the terminating position of oil film and position of maximum film pressure

2.5 DESIGN CHARTS FOR HYDRODYNAMIC BEARINGS – Problem 2

A journal of a stationary oil engine is 80 mm in diameter. and 40 mm long. The radial clearance is 0.060mm. It supports a load of 9 kN when the shaft is rotating at 3600 rpm. The bearing is lubricated with SAE 40oil supplied at atmospheric pressure and average operating temperature is about 65^{o}C. Using Raimondi-Boyd charts analyze the bearing assuming that it is working under steady state condition.

Data: d = 80 mm; l =40 mm; c = 0.06 mm; F = 9kN;
n = 3600rpm = 60 rps; SAE 40 oil; T_o = 65^{O}C;

Analysis:

1. p= F / ld = 9 x1000 /40 x 80 = 2.813 MPa

2. μ = 30 cP at 65^{o}C for SAE 40 oil from Fig. 2.3a.

3. $$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu n}{p}\right) = \left(\frac{40}{0.06}\right)^2 \left(\frac{30x10^{-3} \, x \, 60}{2.813 \, x \, 10^6}\right) = 0.284$$

4. For S = 0.284 and l/d = ½, h_o /c = 0.38 and

ε = e /c = 0.62 from Fig.6.

h_o = 0.38xc = 0.382x 0.06=0.023mm = 23μm

e = 0.62 x c = 0.62 x 0.06 = 0.037 mm

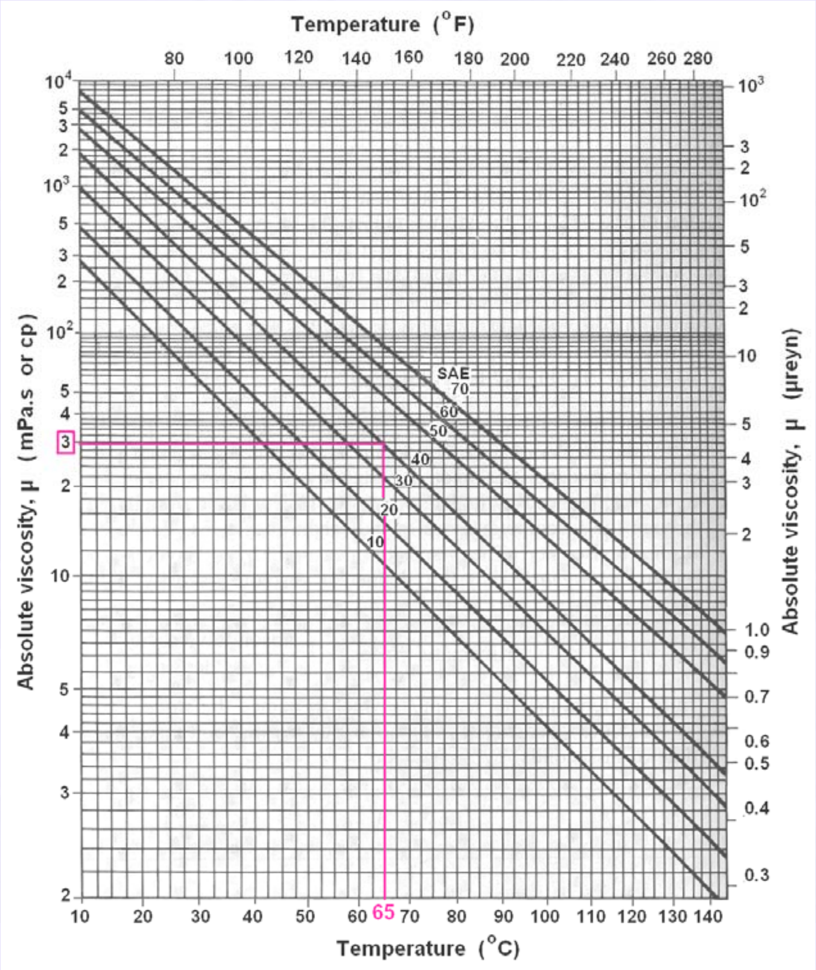

Fig.2.3a Viscosity – temperature curves of SAE graded oils

5. (r /c) f = 7.5, for S = 0.284 for l /d = ½ from Fig.2.11a.

f = 7.5 x (c / r) = 7.5x (0.06/40) = 0.0113

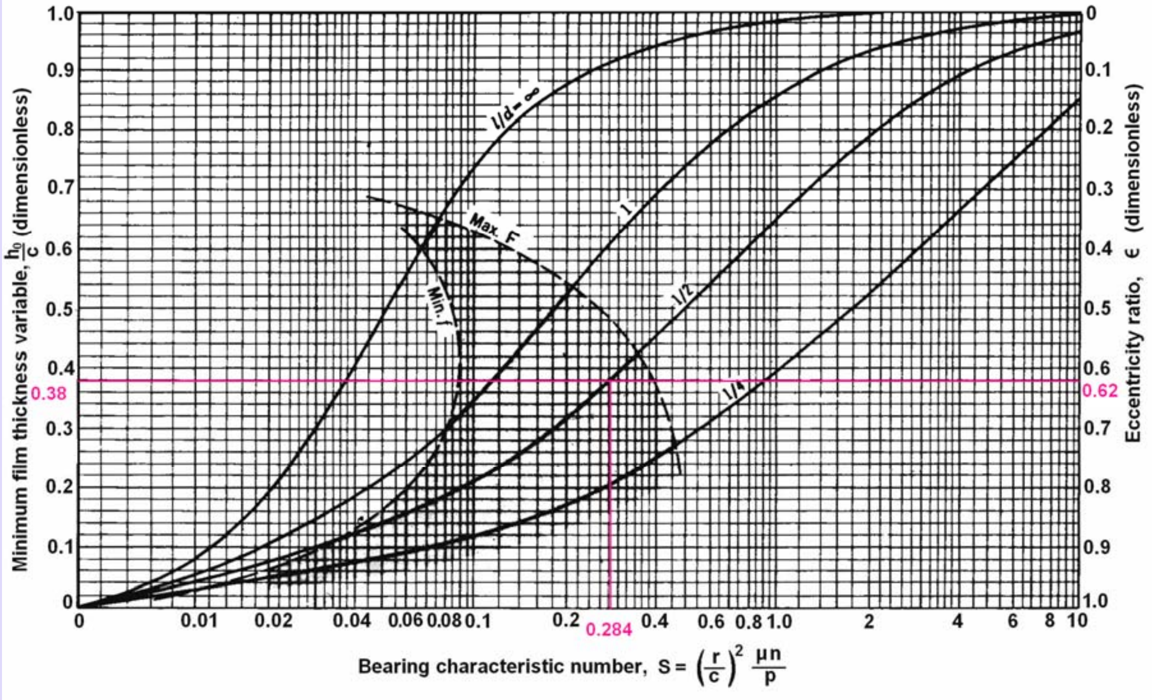

Fig.2.8a Chart for minimum film thickness variable and eccentricity ratio. The left shaded zone defines the optimum h_o for minimum friction; the right boundary is the optimum h_o for maximum load

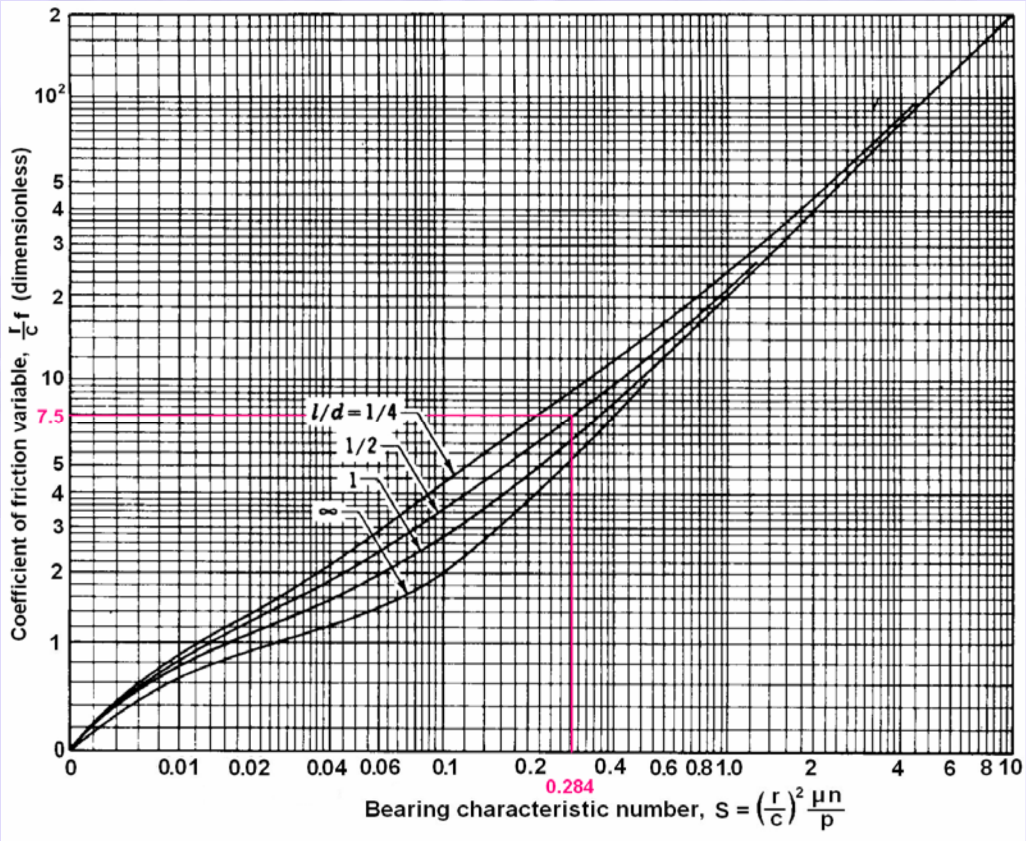

Fig. 2.11a Chart for coefficient of friction variable

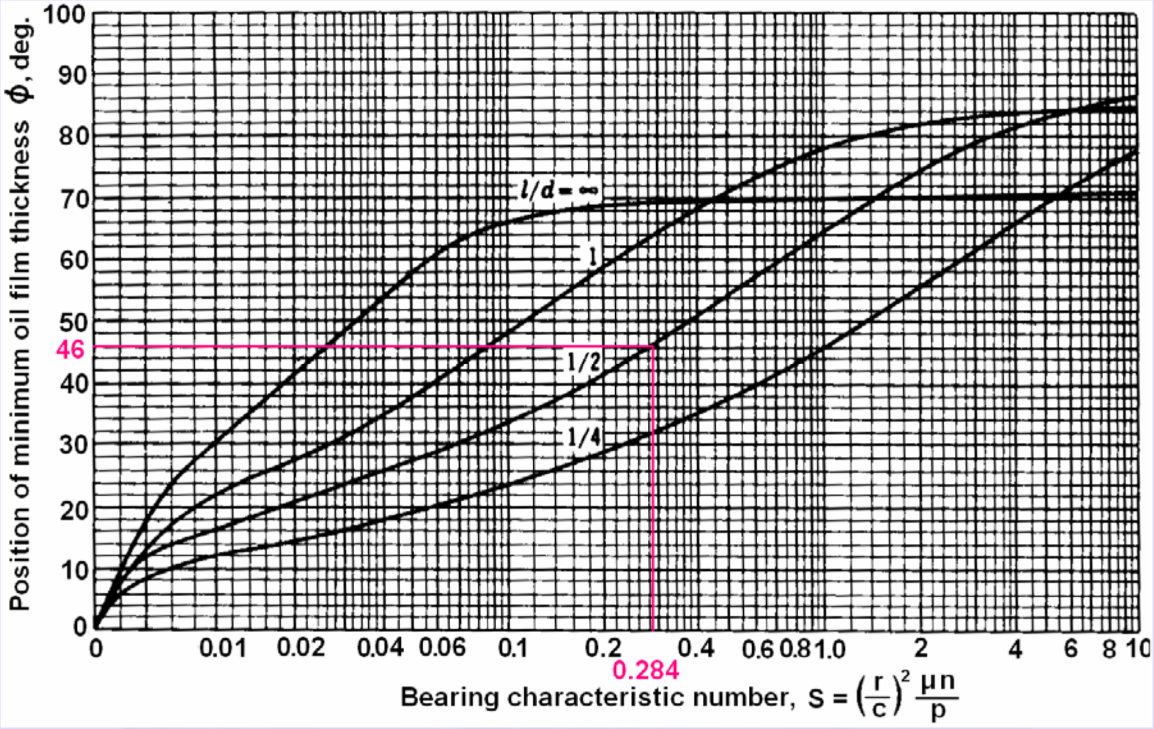

Fig.2.9a Chart for determining the position of minimum film thickness h_o

6. Φ = 46°, for S = 0.284 for l /d = ½ from Fig.2.9a.

7. (Q / r c n l) = 4.9, for S = 0.284 for l /d = ½ from Fig.2.12a.

   Q = 4.9 r c n l = 4.9 x 0.04 x 0.00006 x 60 x 0.04

   = $2.82\text{x}10^{-5}$ m^3/s = 28.2 cm^3 /s

8. (Q_s /Q) = 0.75, for S = 0.284 for l /d = ½ from Fig.2.13a.

   Q_s = 0.75 Q = 0.75 x 28.2 = 21.2 cm^3 /s

9. (p / p_{max}) = 0.36, for S = 0.284 for l /d = ½ from Fig.2.14a.

   p_{uma} = p /0.36 = 2.813 / 0.36 = 7.8 MPa

10. θ_{pox} = 61.5° and θ_{puma} = 17.5°, for S = 0.284 for l /d = ½ from Fig.2.15a.

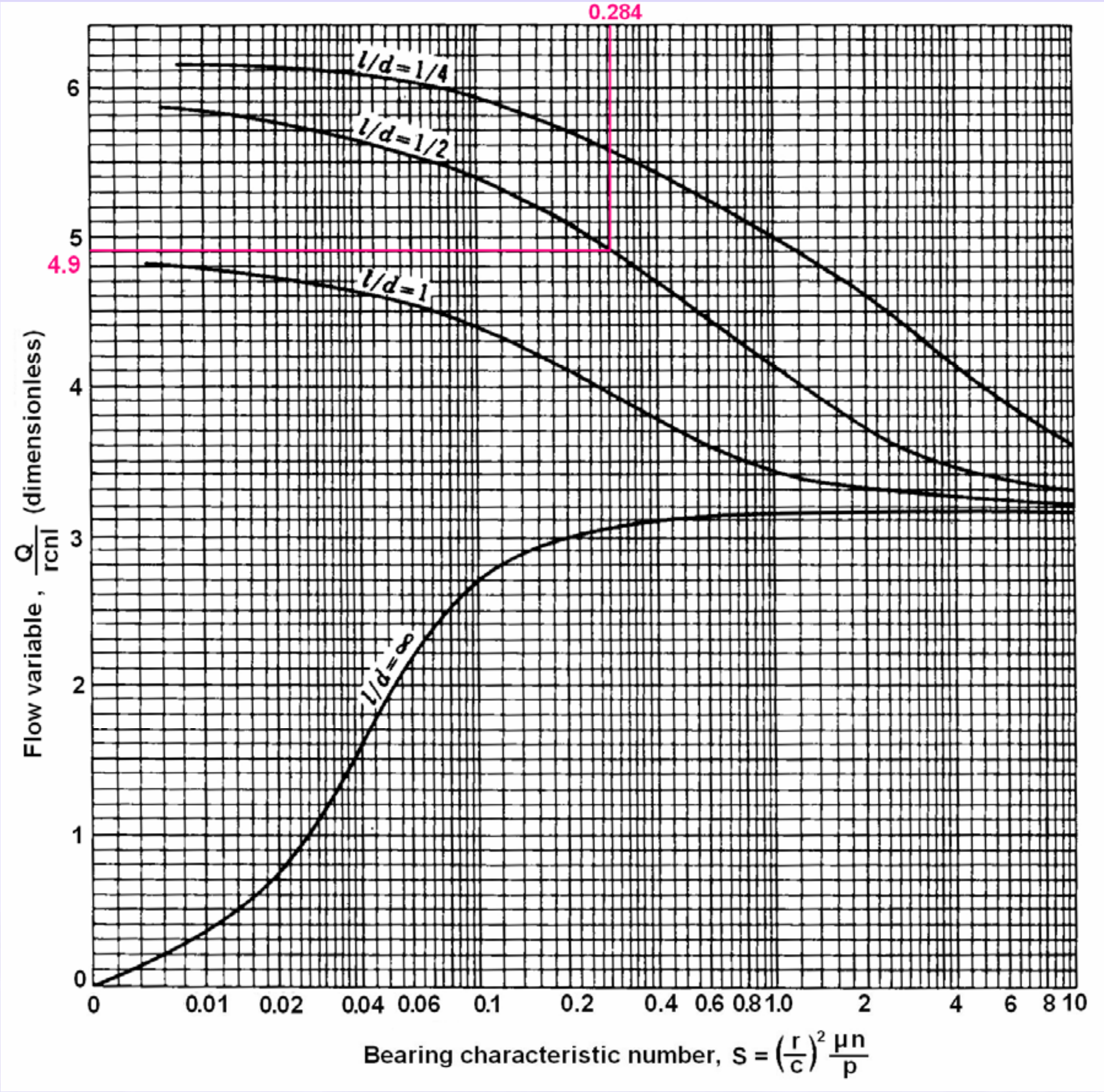

Fig. 2.12a Chart for flow variable

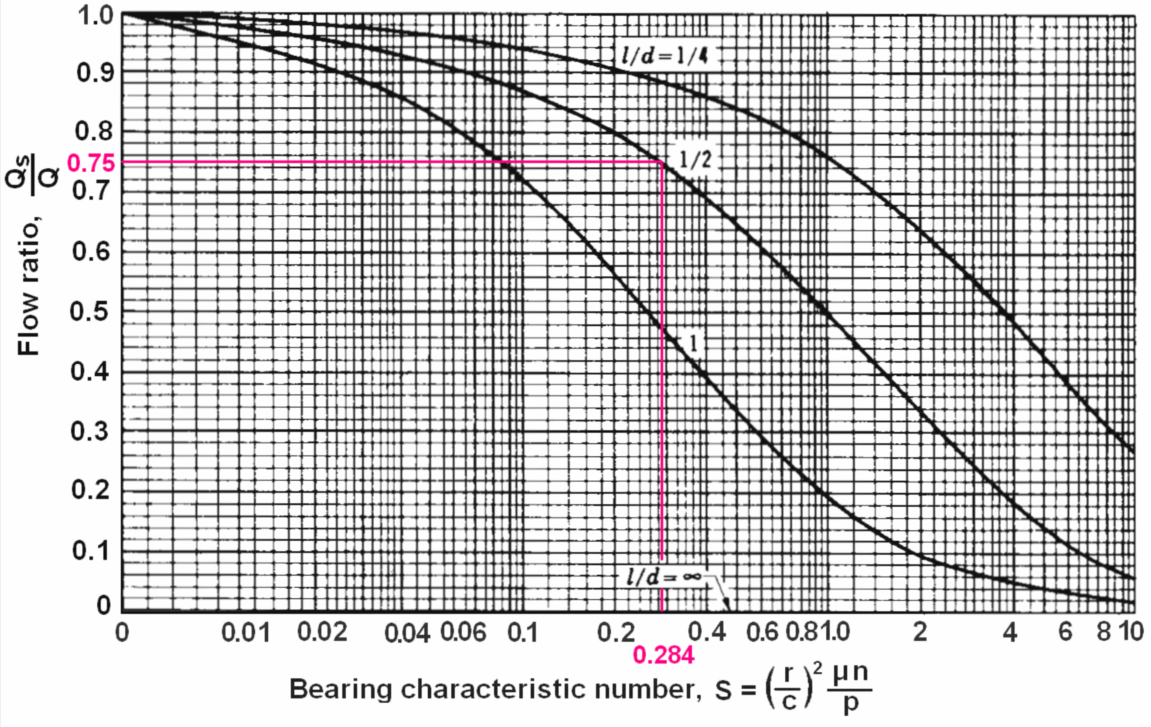

Fig.2.13a Chart for determining the ratio of side flow to total flow

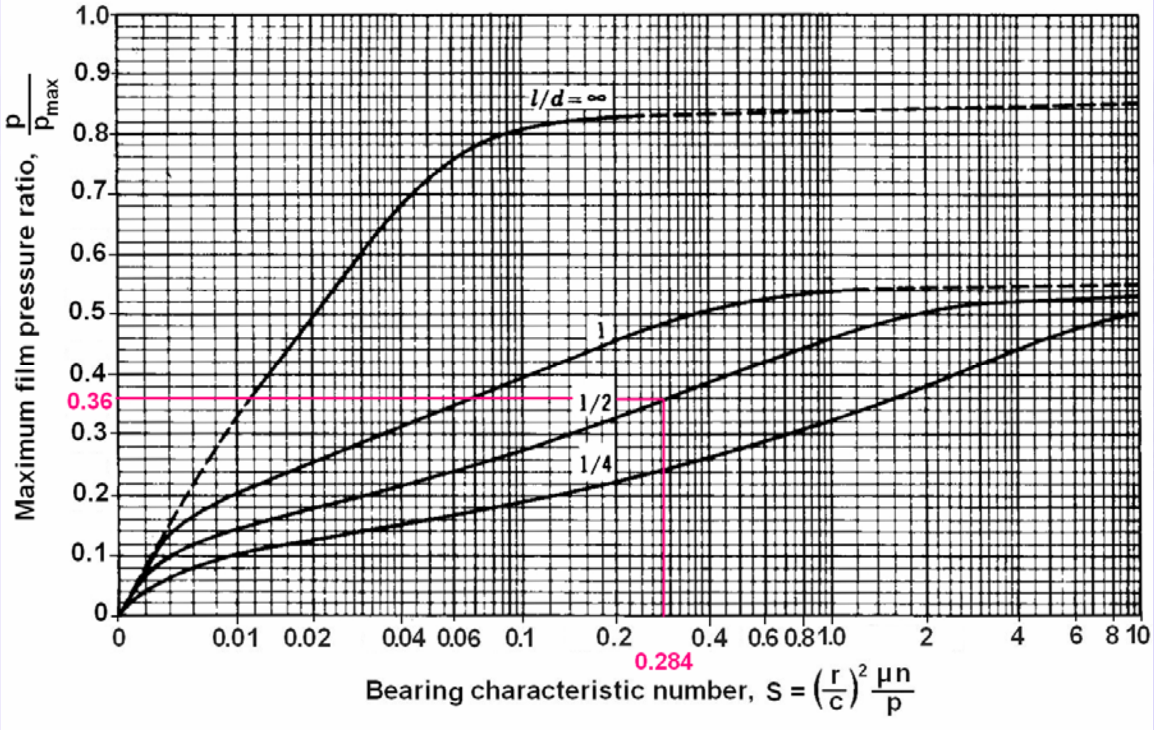

Fig. 2.14a Chart for determining the maximum film pressure

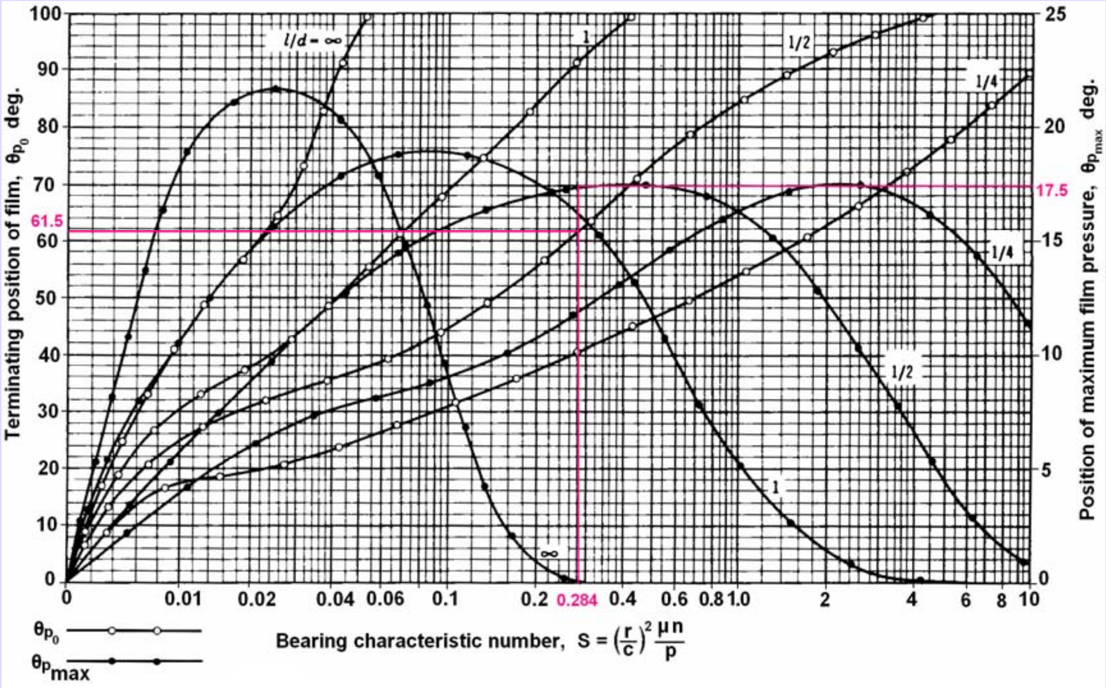

Fig. 2.15a Chart for finding the terminating position of oil film and position of maximum film pressure

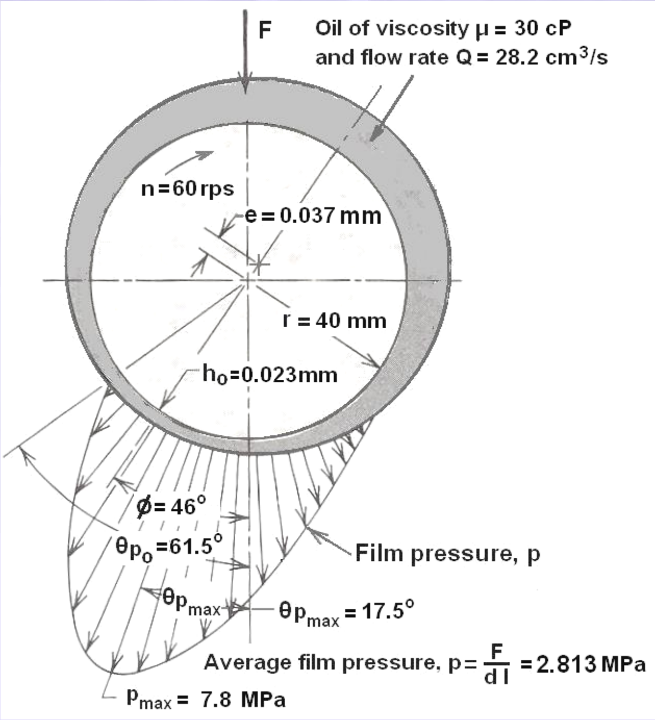

Fig.2.10a Stable hydrodynamic lubrication
