3rd SEM Mechanical Sub.: KTOM (3131906)

Introduction of Mechanisms and Machines

Chapter 1

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Outline

- ✓ Definitions Link or element, kinematic pairs, Degrees of freedom,
- ✓ Grubler's criterion (without derivation),
- ✓ Kinematic chain, Mechanism, Structure, Mobility of Mechanism, Inversion, Machine.

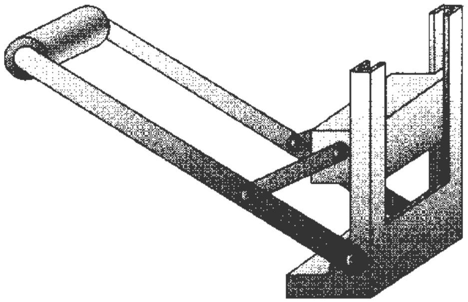
➢ **Kinematic Chains and Inversions:**

- ✓ Inversions of Four bar chain; Single slider crank chain and Double slider crank chain.

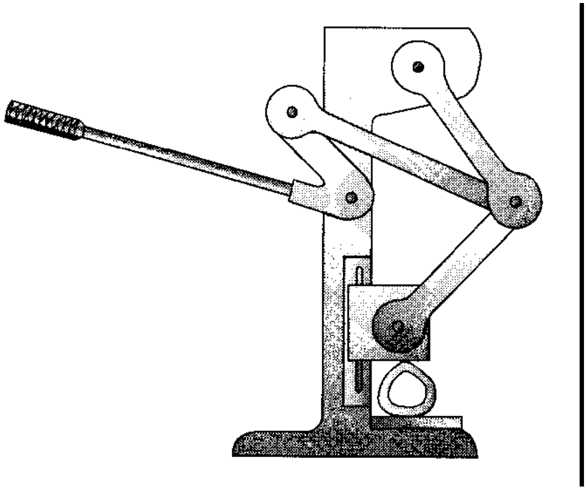
Machine and Mechanism

- ✓ **Machines** are mechanical devices used to **accomplish work**
- ✓ **Mechanism** is a **heart of a machine**. It is the mechanical portion of the machine that has the **function of transferring motion and forces** from a **power source** to an output
- ✓ **Mechanism** is a system of **rigid elements (linkages) arranged and connected** to **transmit motion** in a predetermined fashion
- ✓ ***Mechanism*** *consists of linkages and joints*

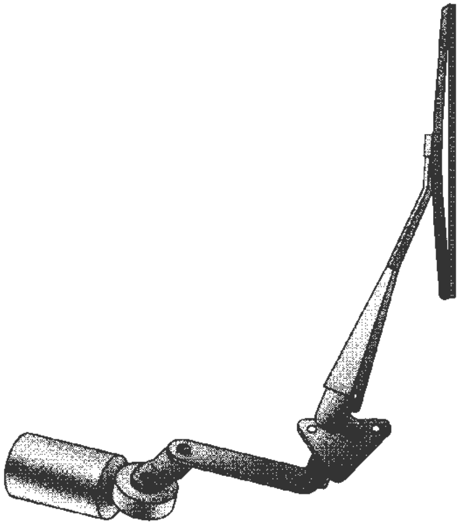
Example of Mechanism



Can crusher

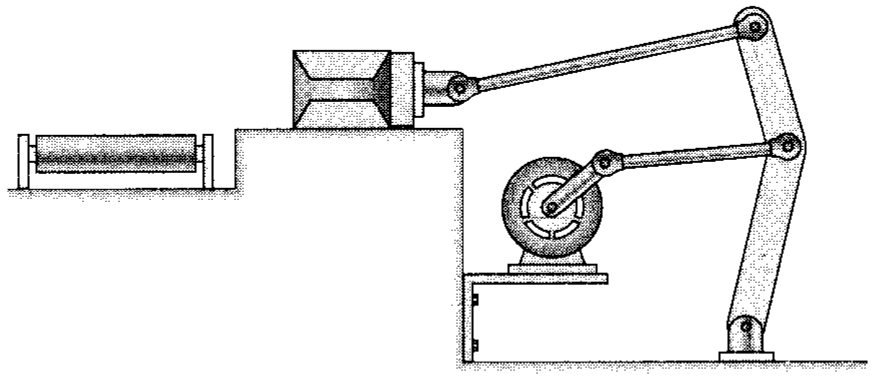


Simple press

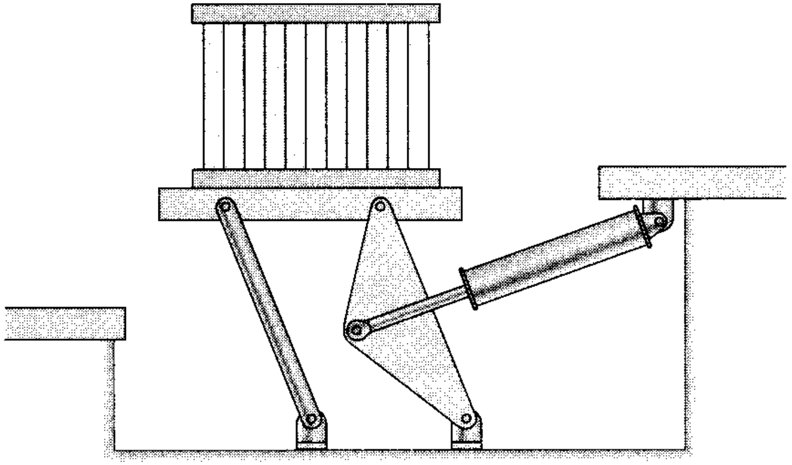


Rear-window wiper

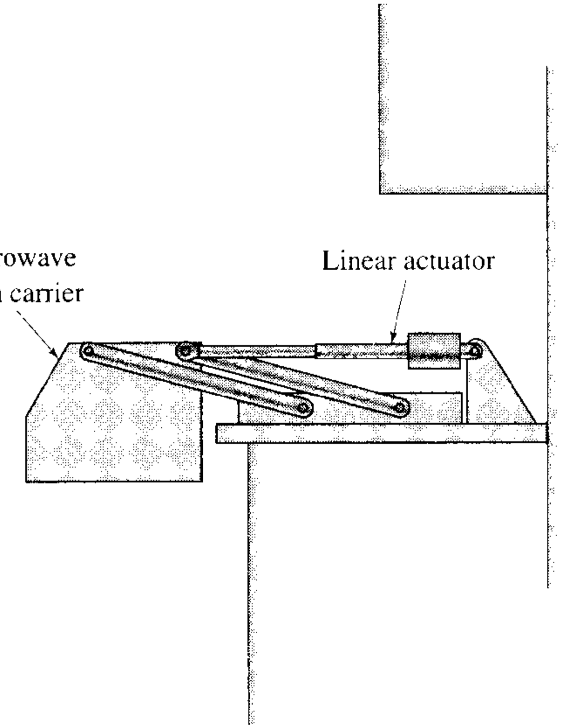
Example of Mechanism



Moves packages from an assembly bench to a conveyor

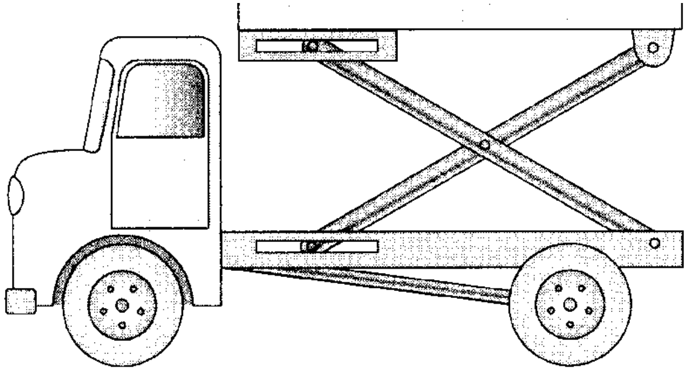


Lift platform

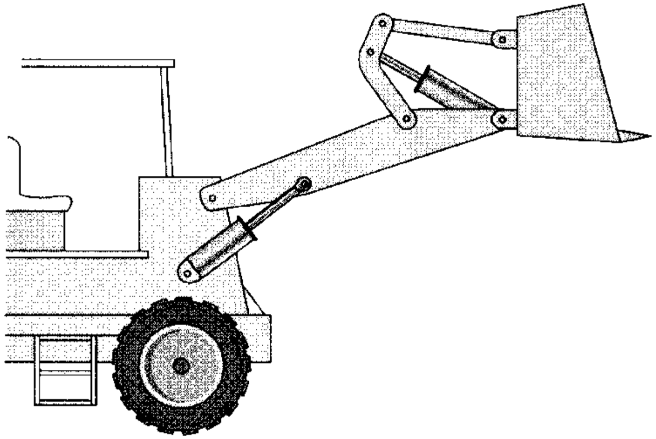


Microwave carrier to assist people on wheelchair

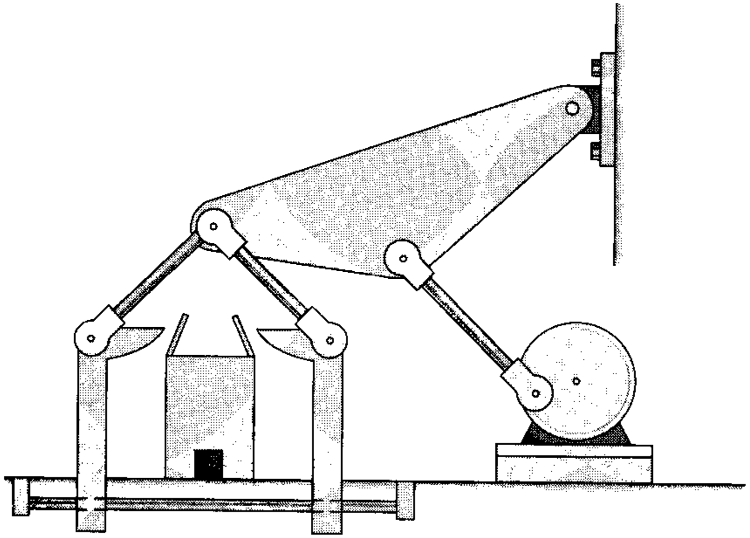
Example of Mechanism



Lift platform

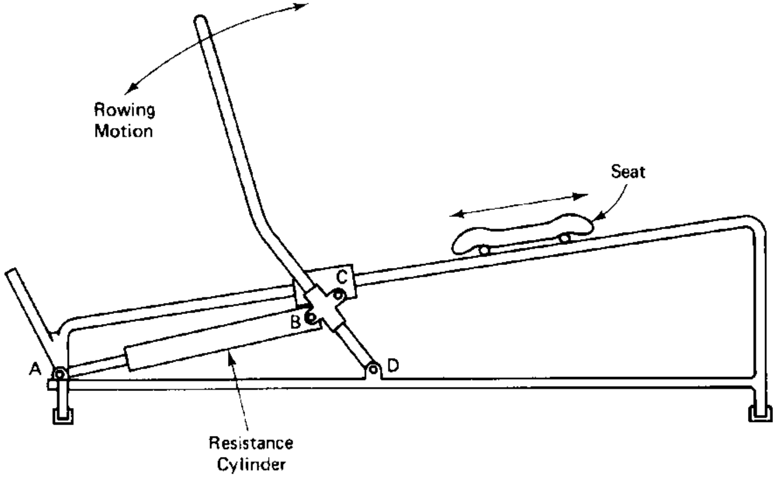


Front loader

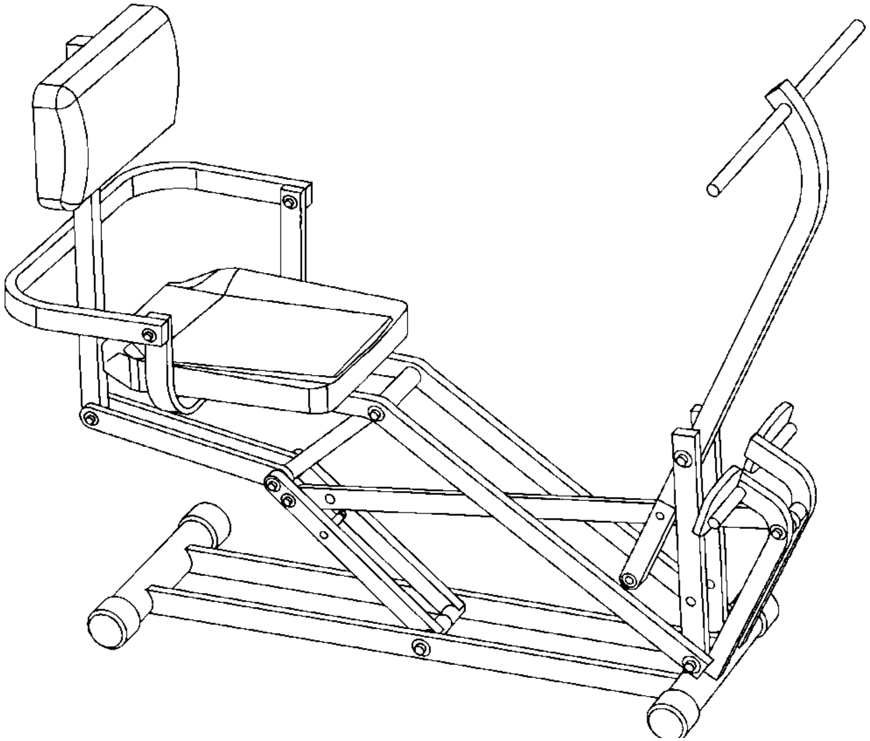


Device to close the top flap of boxes

Example of Mechanism



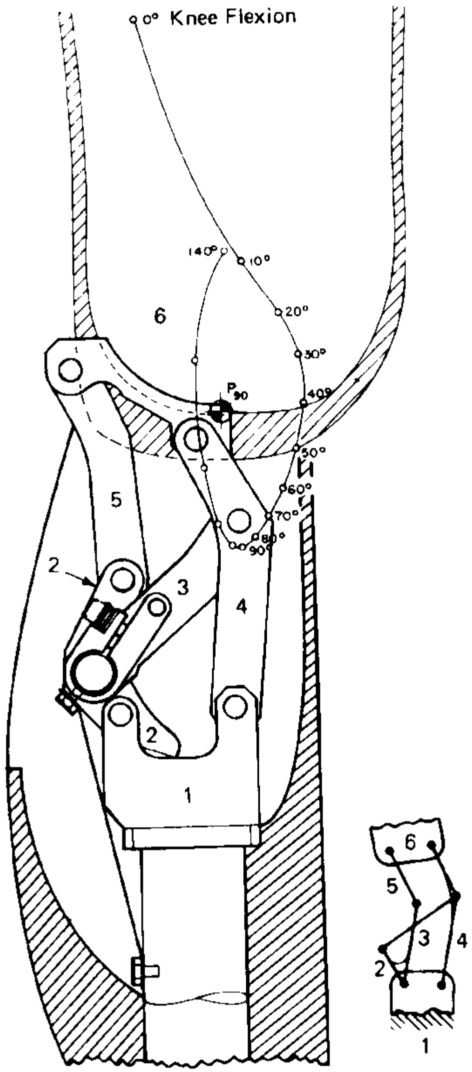
Rowing type exercise machine



Conceptual design for an exercise machine

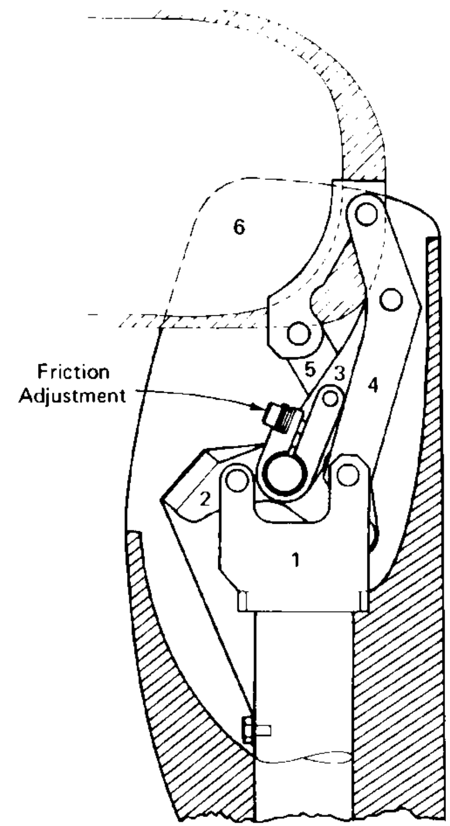
Example of Mechanism

Extension position



(a)

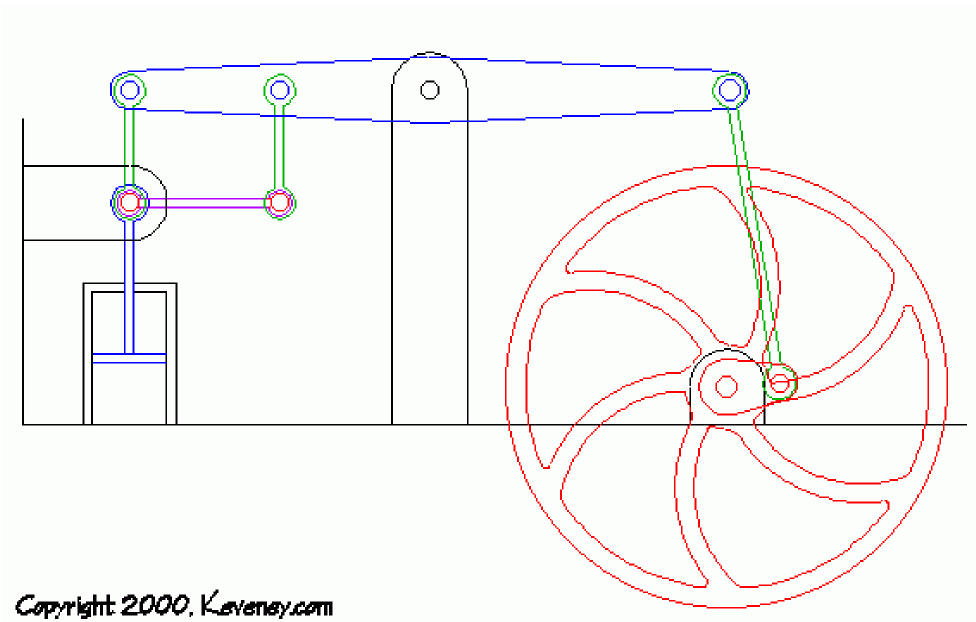
(b)

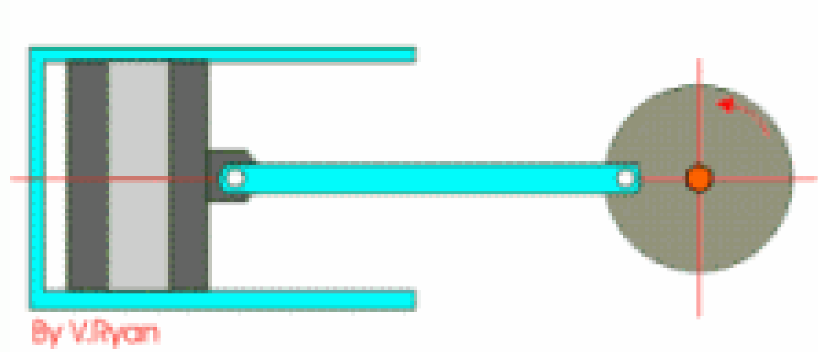


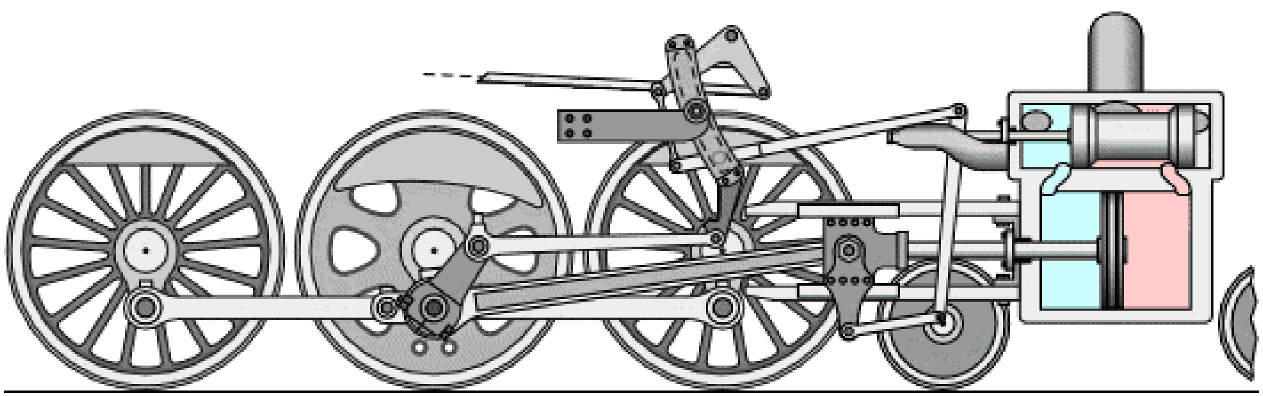
Flexed position

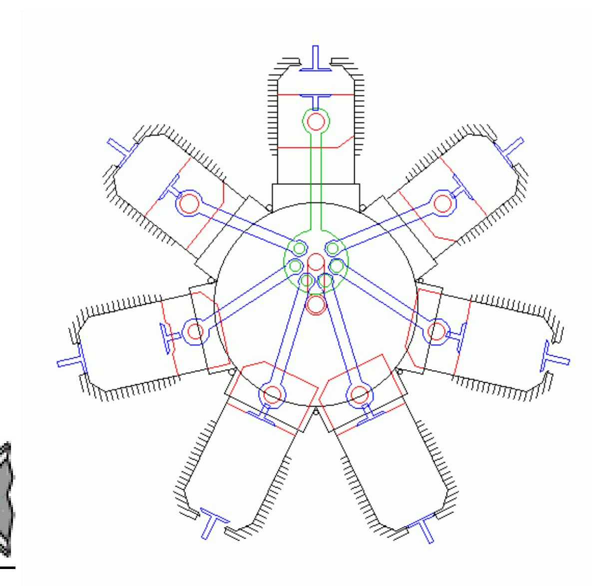
Six-bar linkage prosthetic knee mechanism

Example of Mechanism



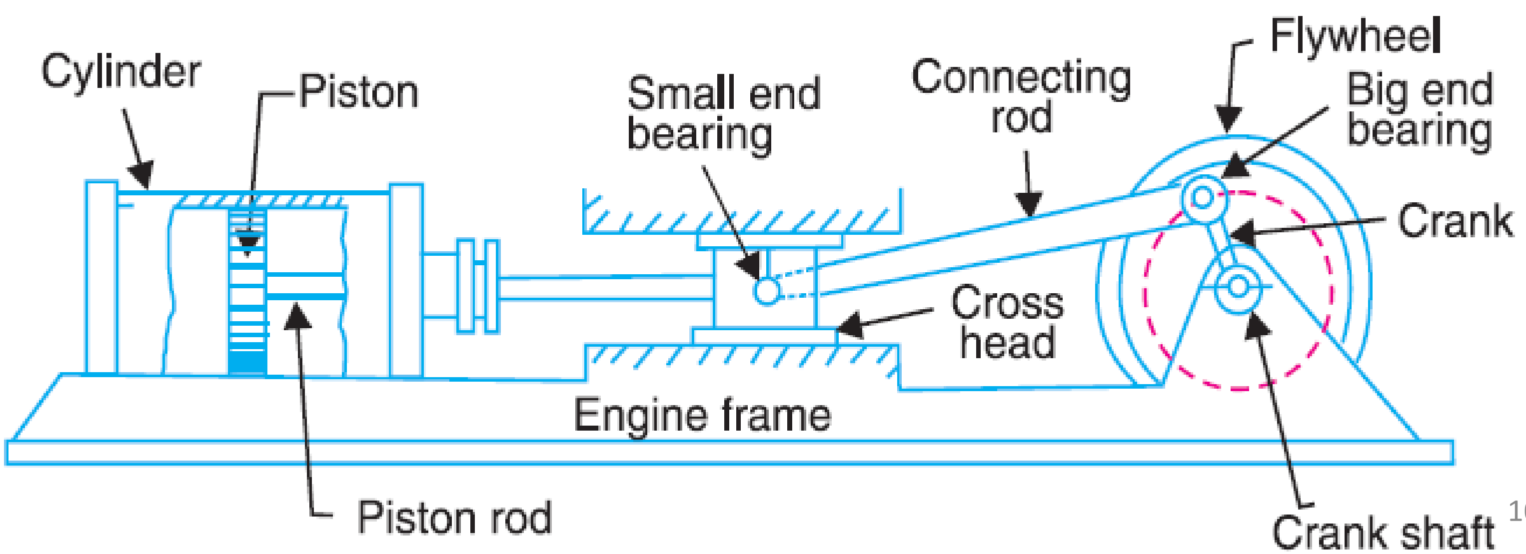






Kinematic Link or Element

- ✓ **Each part of a machine**, which **moves relative to some other part**, is known as a ***kinematic link (or simply link) or element***.
- ✓ *A link may consist of several parts, which are **rigidly*** **fastened together**, so that they do not move relative to one another.
- ✓ For **example**, in a ***reciprocating steam engine***, as shown in Fig. , piston, piston rod and crosshead constitute **one link** ; connecting rod with big and small end bearings constitute a **second link** ; crank, crank shaft and flywheel a **third link** and the cylinder, engine frame and main bearings a **fourth link**.



Types of Links

In order to **transmit motion**, the driver and the follower may be connected by the following **three types** of links :

1. ***Rigid link.***

   A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a **connecting rod, crank** etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. ***Flexible link.***

   A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, **belts, ropes, chains and wires** are flexible links and transmit tensile forces only.

3. ***Fluid link.***

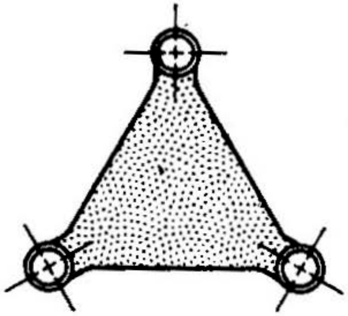
   A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of **hydraulic presses, jacks and brakes**.

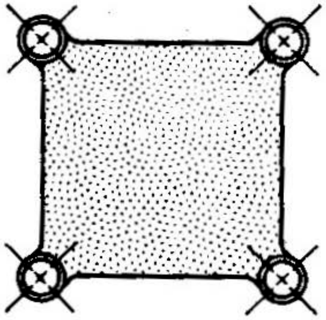
Link or Element

It is the name given to any body which has motion relative to another. All materials have some elasticity. A rigid link is one, whose deformations are so small that they can be neglected in determining the motion parameters of the link.

- ✓ **Binary link:** Link which is **connected** to other links at **two points**. (Fig. a)
- ✓ **Ternary link:** Link which is **connected** to other links at **three points**. (Fig. b)
- ✓ **Quaternary link:** Link which is **connected** to other links at **four points**. (Fig. c)

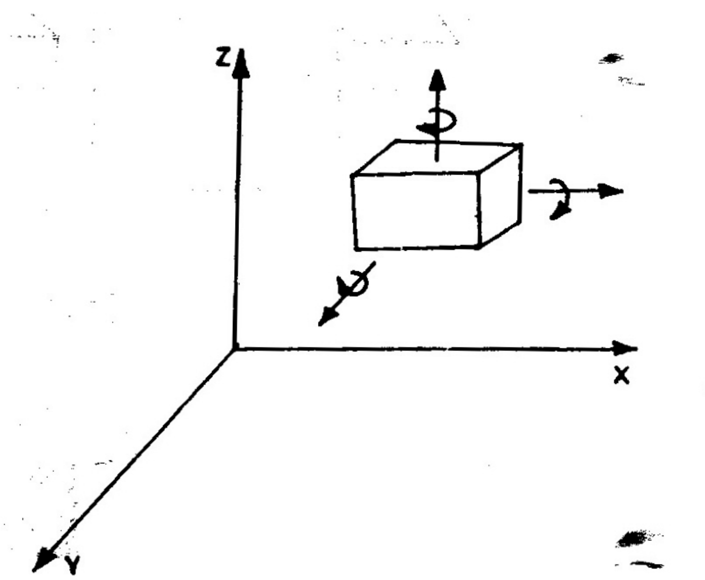

(a)


(b)


(c)

Degrees of freedom (DOF)

- ✓ It is the **number of independent coordinates** required to describe the **position** of a **body in space**.
- ✓ A free body in space (fig) can have **six degrees of freedom**. I.e., **linear positions** along x, y and z axes and **rotational/angular positions** with respect to x, y and z axes.
- ✓ In a **kinematic pair**, depending on the constraints imposed on the motion, the links may loose some of the **six degrees of freedom**.



Structure

- ✓ It is an **assemblage** of a **number of resistant bodies** (known as members) having **no relative motion** between them and meant for carrying loads having straining action. A **railway bridge, a roof truss, machine frames** etc., are the examples of a structure.

Difference Between a Machine and a Structure

The following differences between a machine and a structure are important from the subject point of view :

- ✓ The **parts** of a **machine** **move relative** to one another, whereas the members of a ***structure*** **do not move relative** to one another.
- ✓ A **machine** **transforms** the **available energy** into some **useful work**, whereas in a ***structure*** **no energy** is transformed into useful work.
- ✓ The links of a **machine** may **transmit both power and motion**, while the members of a **structure** **transmit forces only**.

Kinematic Pair

The **two links or elements** of a machine, when in **contact with each other**, are said to form a **pair**. If the **relative motion between them** is **completely or successfully constrained** (*i.e. in a definite* direction), the pair is known as ***kinematic pair***.

Types of Constrained Motions

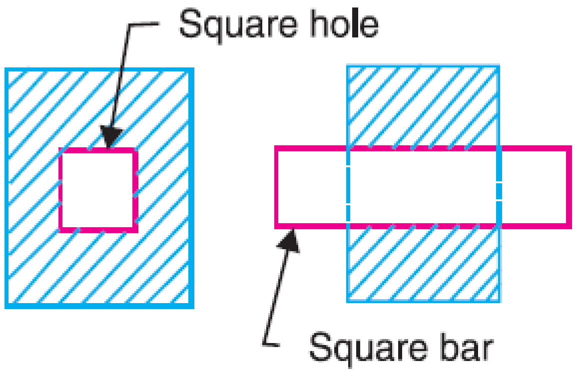
Following are **the three types** of constrained motions :

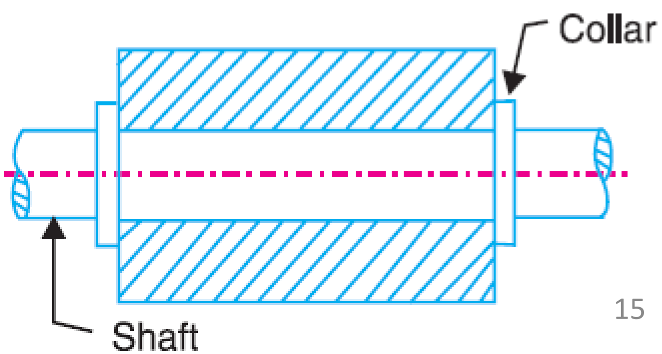
1. Completely constrained motion.

*When the motion between a pair is **limited to a definite*** **direction irrespective of the direction of force applied**, then the motion is said to be a completely constrained motion.

For **example**, **the piston and cylinder** (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (*i.e. it will only reciprocate) relative to the* cylinder irrespective of the direction of motion of the crank.

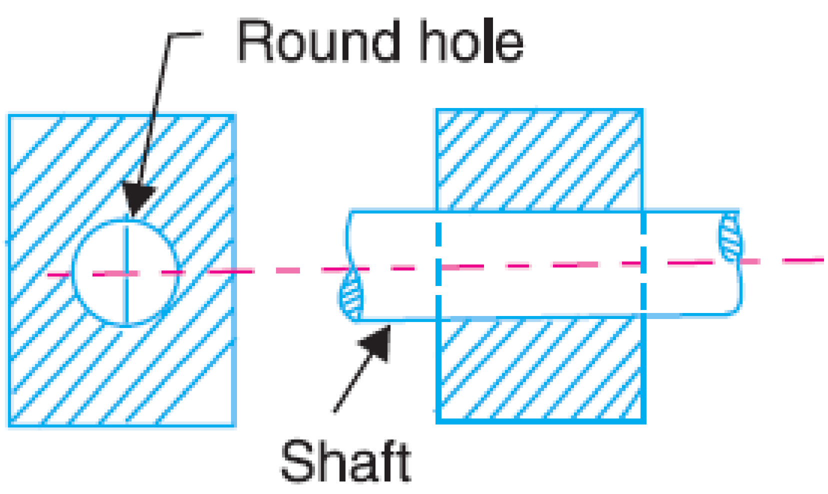
The motion of a square bar in a square hole, as shown in Fig. 2, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig. 3, are also examples of completely constrained motion.





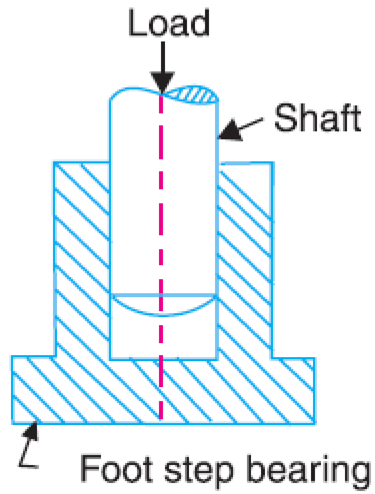
2. *Incompletely constrained motion.*

- ✓ *When the motion between a pair can* ***take place in more*** **than one direction**, then the motion is called an **incompletely constrained motion.**
- ✓ The change in the direction of impressed force may alter the direction of relative motion between the pair.
- ✓ A **circular bar or shaft in a circular hole**, as shown in Fig. 4, is an example of an incompletely constrained motion as it may **either rotate or slide in a hole**. These both motions have no relationship with the other.



3. *Successfully constrained motion*

- *When the motion between the elements, forming a pair, is* **such that the constrained motion is not completed by itself, but by some other means**, then the motion is said to be **successfully constrained motion.**
- Consider a **shaft in a foot-step bearing** as shown in Fig. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But **if the load is placed on the shaft to prevent axial upward movement of the shaft**, then the motion of the pair is said to be successfully constrained motion.
- The **motion of an I.C. engine valve and the piston reciprocating inside an engine cylinder** are also the examples of successfully constrained motion.



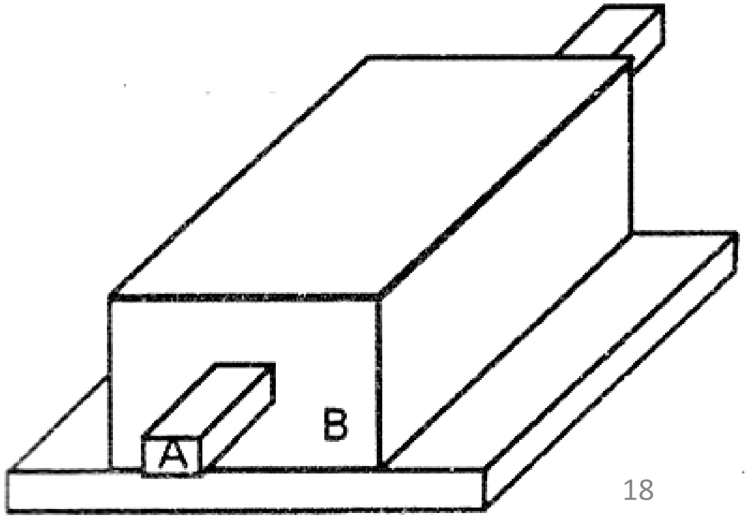
Classification of Kinematic Pairs

1. According to the type of relative motion between the elements.

(a) Sliding pair.

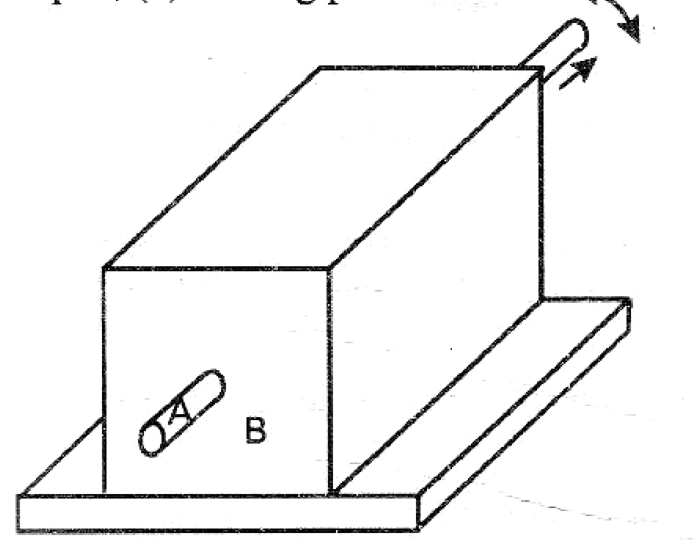
When the two elements of a pair are connected in such a way that ***one can* only slide relative to the other**, the pair is known as a sliding pair.

The **piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed** etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a **completely constrained motion**.



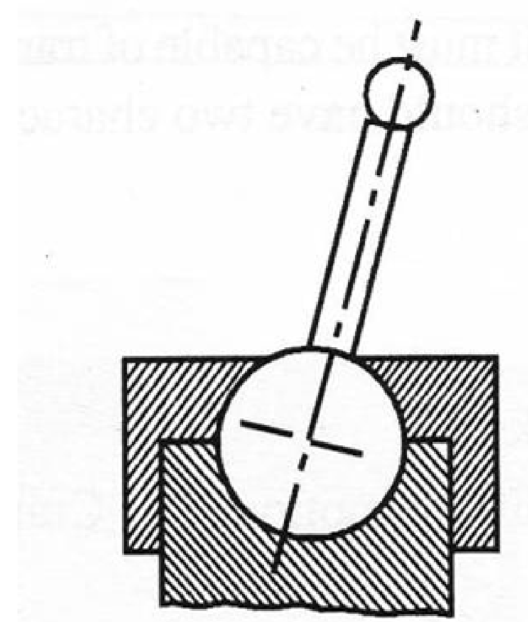
(*b*) *Turning pair.*

When the two elements of a pair are connected in such a way that ***one can* only turn or revolve about a fixed axis of another link**, the pair is known as turning pair.



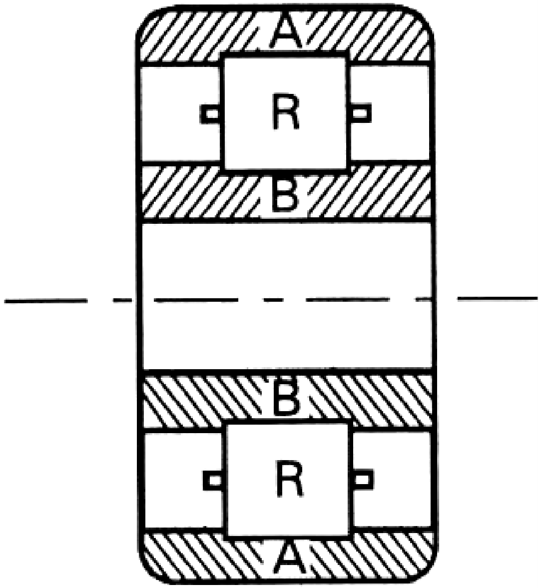
(*c*) *Spherical pair.*

When the two elements of a pair are connected in such a way that ***one* element (with spherical shape) turns or swivels about the other fixed element**, the pair formed is called a spherical pair. **The ball and socket joint, attachment of a car mirror, pen stand** etc., are the examples of a spherical pair.



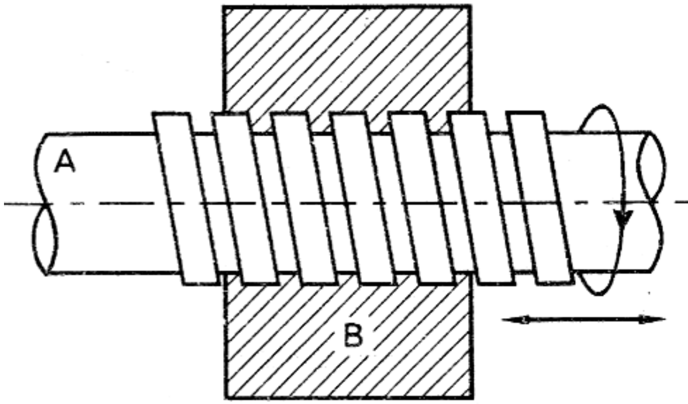
(d) Rolling pair.

When the two elements of a pair are connected in such a way that ***one rolls over*** **another fixed link**, the pair is known as rolling pair. **Ball and roller bearings** are examples of rolling pair.



(e) Screw pair.

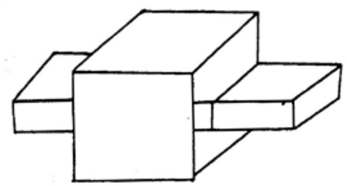
When the two elements of a pair are connected in such a way that ***one element*** **can turn about the other by screw threads**, the pair is known as screw pair. **The lead screw of a lathe with nut, and bolt with a nut** are examples of a screw pair.

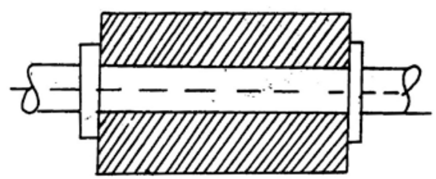


2. *According to the type of contact between the elements.*

(a) Lower pair.

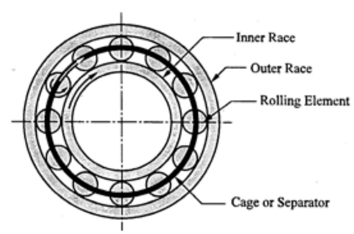
When the two elements of a pair have a ***surface contact*** *when* ***relative motion*** **takes place and the surface of one element slides over the surface of the other**, the pair formed is known as lower pair. It will be seen that **sliding pairs, turning pairs and screw pairs** form lower pairs.

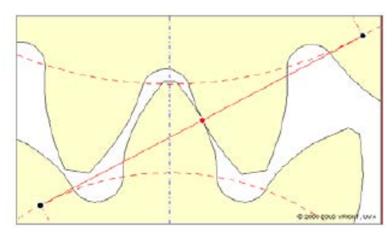




(b) Higher pair.

When the two elements of a pair have a ***line or point contact*** *when relative* motion takes place and the motion between the two elements is **partly turning and partly sliding**, then the pair is known as higher pair. A pair of **friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower** are the examples of higher pairs.





3. *According to the type of closure.*

(a) Self closed pair.

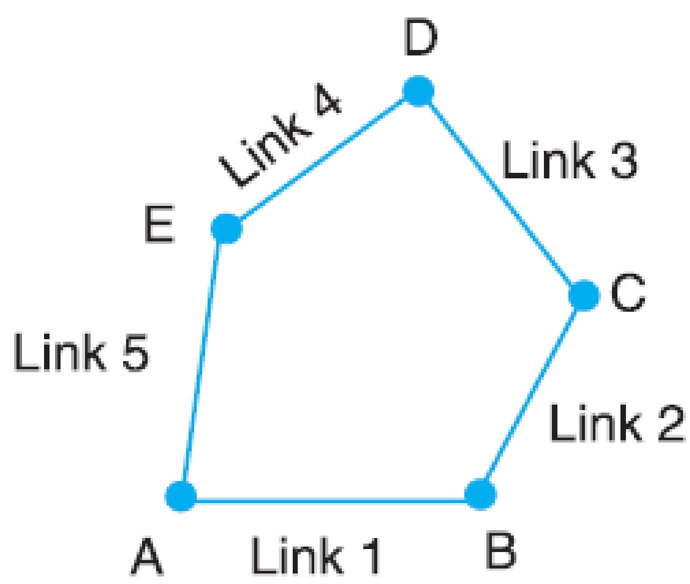
When the two elements of a pair are ***connected together mechanically*** *in* such a way that **only required kind of relative motion occurs**, it is then known as self closed pair. The **lower pairs** are self closed pair.

(b) Force - closed pair.

When the two elements of a pair ***are not connected mechanically but*** **are kept in contact by the action of external forces**, the pair is said to be a force-closed pair. The **cam and follower** is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

Kinematic Chain

- ✓ When the kinematic pairs are coupled in such a way that the **last link is joined to the first link to transmit definite motion** (*i.e. completely or* successfully constrained motion), it is called a ***kinematic chain.***
- ✓ *In other* words, a kinematic chain may be defined as **a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs** and **the relative motion between the links or elements is completely or successfully constrained.**



- ✓ If each link is assumed to form two pairs with two adjacent links, then the relation between the **number of pairs (*p*)** *forming a kinematic chain and the **number of links (l)** may be expressed in* the form of an equation :

$$l = 2p - 4$$

- ✓ Since in a kinematic chain each link forms a part of two pairs, therefore there will be as many links as the number of pairs.
- ✓ Another relation between the **number of links (*l*)** *and **the number of joints (j)** which* constitute a kinematic chain is given by the expression :

$$j = \frac{3}{2} l - 2$$

- ✓ The equations ***(i) and (ii) are applicable only to kinematic chains, in which lower pairs are*** used.

1. Consider the arrangement of three links AB, BC and CA with pin joints at A, B and C as shown in Fig. 5.6. In this case,

Number of links, $l = 3$

Number of pairs, $p = 3$

and number of joints, $j = 3$

From equation (*i*), $l = 2p - 4$

or $3 = 2 \times 3 - 4 = 2$

i.e. L.H.S. > R.H.S.

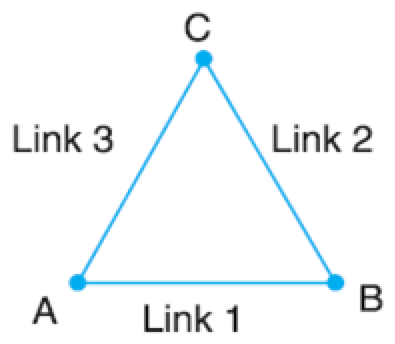


Fig. 5.6. Arrangement of three links.

Now from equation (*ii*),

$$j = \frac{3}{2}l - 2 \quad \text{or} \quad 3 = \frac{3}{2} \times 3 - 2 = 2.5$$

i.e. L.H.S. > R.H.S.

Since the arrangement of three links, as shown in Fig. 5.6, does not satisfy the equations (*i*) and (*ii*) and the left hand side is greater than the right hand side, therefore it is not a kinematic chain and hence no relative motion is possible. Such type of chain is called ***locked chain*** and forms a rigid frame or structure which is used in bridges and trusses.

2. Consider the arrangement of four links AB, BC, CD and DA as shown in Fig. 5.7. In this case

$$l = 4,\ p = 4,\ \text{and}\ j = 4$$

From equation *(i)*, $l = 2p - 4$

$$4 = 2 \times 4 - 4 = 4$$

i.e. L.H.S. = R.H.S.

From equation *(ii)*, $j = \frac{3}{2}l - 2$

$$4 = \frac{3}{2} \times 4 - 2 = 4$$

i.e. L.H.S. = R.H.S.

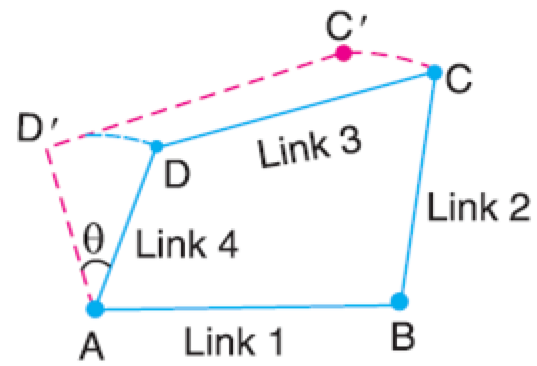


Fig. 5.7. Arrangement of four links.

Since the arrangement of four links, as shown in Fig. 5.7, satisfy the equations *(i)* and *(ii)*, therefore it is a ***kinematic chain of one degree of freedom***.

3. Consider an arrangement of five links, as shown in Fig. 5.8. In this case,

$$l = 5,\ p = 5, \text{ and } j = 5$$

From equation (*i*),

$$l = 2p - 4 \quad \text{or} \quad 5 = 2 \times 5 - 4 = 6$$

i.e. L.H.S. < R.H.S.

From equation (*ii*),

$$j = \frac{3}{2} l - 2 \quad \text{or} \quad 5 = \frac{3}{2} \times 5 - 2 = 5.5$$

i.e. L.H.S. < R.H.S.

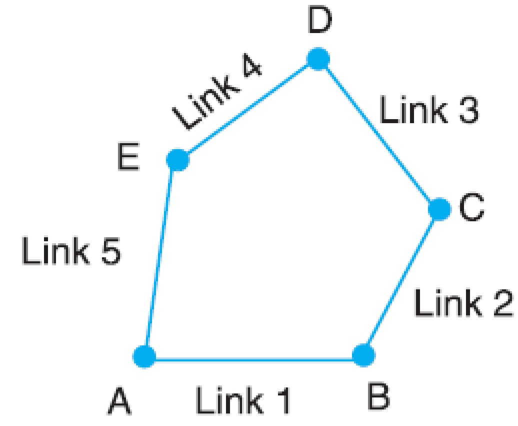


Fig. 5.8. Arrangement of five links.

Since the arrangement of five links, as shown in Fig. 5.8 does not satisfy the equations and left hand side is less than right hand side, therefore it is not a kinematic chain. Such a type of chain is called ***unconstrained chain*** *i.e.* the relative motion is not completely constrained. This type of chain is of little practical importance.

5.10. Types of Joints in a Chain

The following types of joints are usually found in a chain :

1. ***Binary joint***. When two links are joined at the same connection, the joint is known as binary joint. For example, a chain as shown in Fig. 5.10, has four links and four binary joins at *A* , *B*, *C* and *D*.

In order to determine the nature of chain, *i.e.* whether the chain is a locked chain (or structure) or kinematic chain or unconstrained chain, the following relation between the number of links and the number of binary joints, as given by A.W. Klein, may be used :

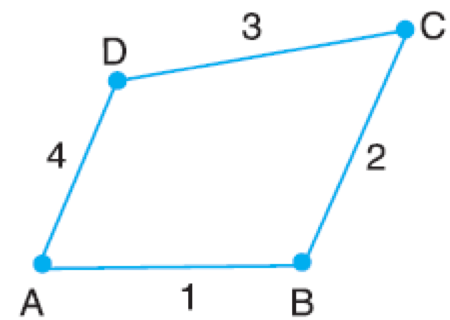


Fig. 5.10. Kinematic chain with all binary joints.

$$j + \frac{h}{2} = \frac{3}{2}l - 2 \qquad \text{... }(i)$$

where j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.

When $h = 0$, the equation (*i*), may be written as

$$j = \frac{3}{2}l - 2 \qquad \text{. . . }(ii)$$

Applying this equation to a chain, as shown in Fig. 5.10, where $l = 4$ and $j = 4$, we have

$$4 = \frac{3}{2} \times 4 - 2 = 4$$

Since the left hand side is equal to the right hand side, therefore the chain is a kinematic chain or constrained chain.

2. ***Ternary joint***. When three links are joined at the same connection, the joint is known as ternary joint. It is equivalent to two binary joints as one of the three links joined carry the pin for the other two links. For example, a chain, as shown in Fig. 5.11, has six links. It has three binary joints at *A*, *B* and *D* and two ternary joints at *C* and *E*. Since one ternary joint is equivalent to two binary joints, therefore equivalent binary joints in a chain, as shown in Fig. 5.11, are $3 + 2 \times 2 = 7$

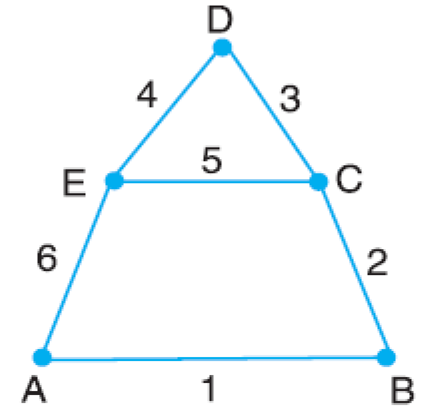


Fig. 5.11. Kinematic chain having binary and ternary joints.

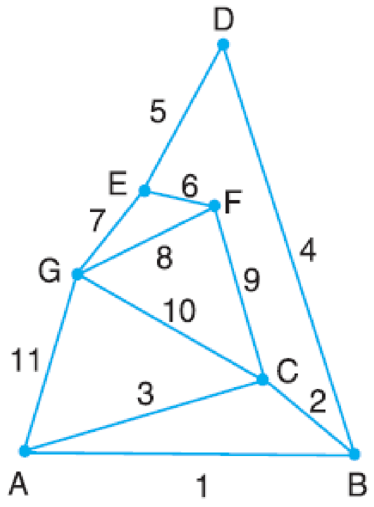
Let us now determine whether this chain is a kinematic chain or not. We know that $l = 6$ and $j = 7$, therefore from equation ***(ii)***,

$$j = \frac{3}{2} l - 2$$

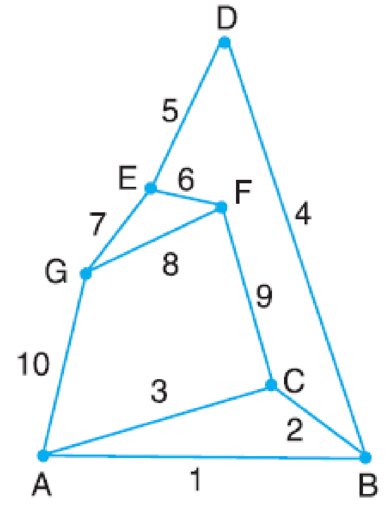
or

$$7 = \frac{3}{2} \times 6 - 2 = 7$$

Since left hand side is equal to right hand side, therefore the chain, as shown in Fig. 5.11, is a kinematic chain or constrained chain.



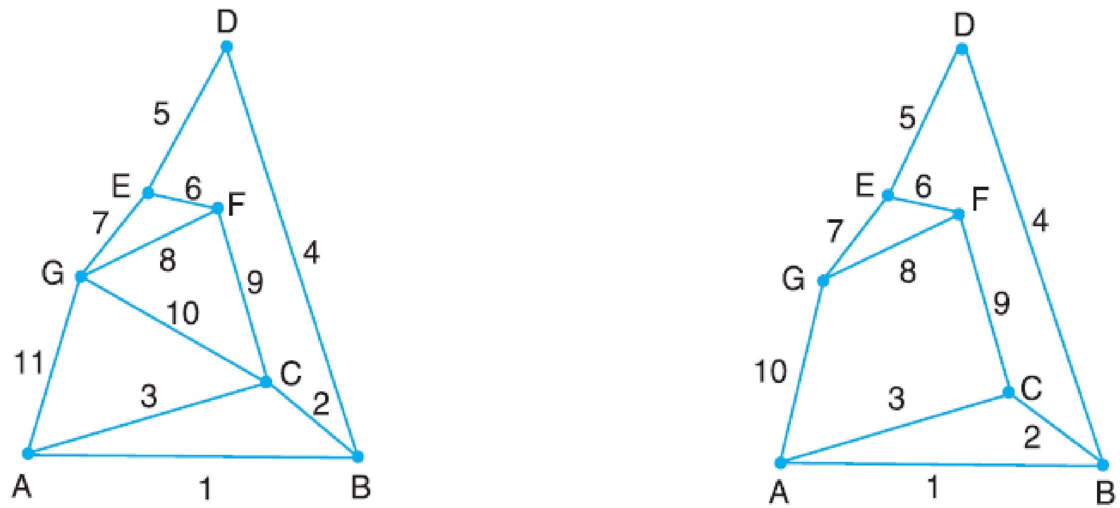
(*a*) Looked chain having binary, ternary and quaternary joints.



(*b*) Kinematic chain having binary and ternary joints.

3. *Quaternary joint.* When four links are joined at the same connection, the joint is called a quaternary joint. It is equivalent to three binary joints. In general, when l number of links are joined at the same connection, the joint is equivalent to $(l - 1)$ binary joints.

For example consider a chain having eleven links, as shown in Fig. 5.12 (*a*). It has one binary joint at *D*, four ternary joints at *A, B, E* and *F*, and two quaternary joints at *C* and *G*. Since one quaternary joint is equivalent to three binary joints and one ternary joint is equal to two binary joints, therefore total number of binary joints in a chain, as shown in Fig. 5.12 (*a*), are



(*a*) Looked chain having binary, ternary and quaternary joints.

(*b*) Kinematic chain having binary and ternary joints.

Fig. 5.12

$$1 + 4 \times 2 + 2 \times 3 = 15$$

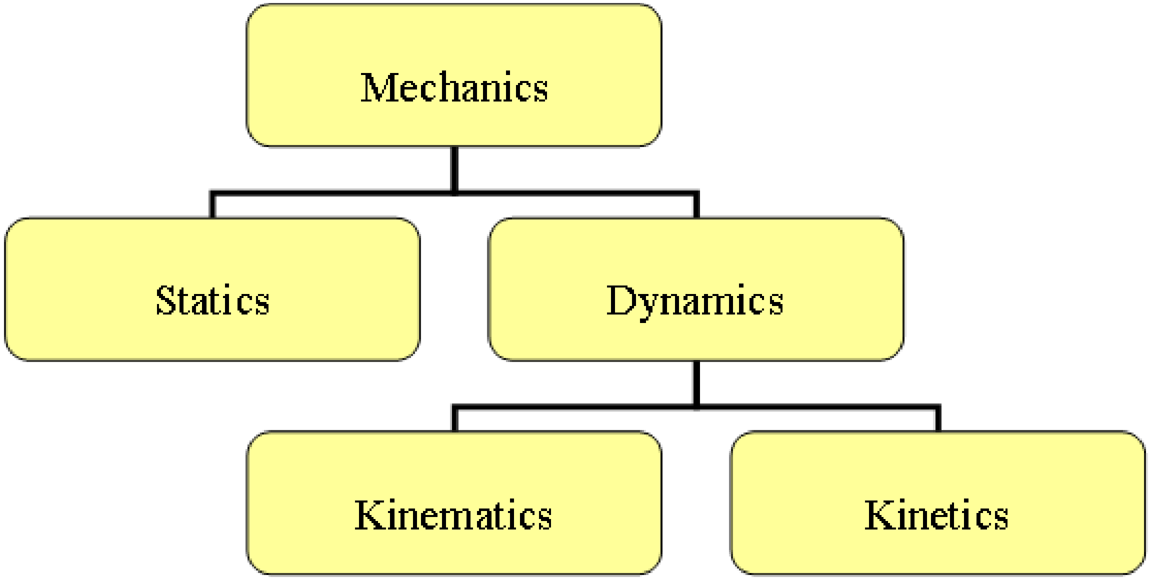
Let us now determine whether the chain, as shown in Fig. 5.12 (*a*), is a kinematic chain or not. We know that $l = 11$ and $j = 15$. We know that,

$$j = \frac{3}{2}\,l - 2, \quad \text{or} \quad 15 = \frac{3}{2} \times 11 - 2 = 14.5, \text{ \textit{i.e.}, L.H.S.} > \text{R.H.S.}$$

Since the left hand side is greater than right hand side, therefore the chain, as shown in Fig. 5.12 (*a*) , is not a kinematic chain. We have discussed in Art 5.9 , that such a type of chain is called locked chain and forms a rigid frame or structure.

Mechanics

Mechanics is a branch of science which deals with **motions, time and forces**.



Statics: It deals with the **analysis of systems at rest**.

Dynamics: It is the study of **systems that change with time**.

Kinematics: It is the study of **position, displacement, rotation, speed, velocity and acceleration**.

Kinetics: It deals with the **inertia force which arises due to mass and motion of the body**.

Mechanism

- When **one of the links of a kinematic chain is fixed**, the chain is known as ***mechanism.***
- *It may* be used for **transmitting or transforming motion**
- A mechanism with **four links** is known as ***simple mechanism****, and the mechanism with* ***more*** **than four links** is known as ***compound mechanism.***
- *When a mechanism is required to* ***transmit power*** or to do some particular type of work, it then becomes a ***machine.***
- *In such cases, the various links or* elements have to be designed to withstand the forces (both static and kinetic) safely.
- A little consideration will show that a mechanism may be regarded as a machine in which each part is reduced to the simplest form to transmit the required motion.

Mechanisms and Simple Machines

- ***Mechanism**: the fundamental physical or chemical processes involved in or responsible for an action, reaction or other natural phenomenon.*
- ***Machine**: an assemblage of parts that transmit forces, motion and energy in a predetermined manner.*
- The term ***mechanism*** is applied to the combination of geometrical bodies which constitute a machine or part of a machine.
- A ***mechanism*** may therefore be defined as a combination of rigid or resistant bodies, formed and connected so that they move with definite relative motions with respect to one another .
- The ***similarity*** between *machines* and *mechanisms* is that
  - they are both combinations of rigid bodies
  - the relative motion among the rigid bodies are definite.
- The *difference* between *machine* and *mechanism* is that machines transform energy to do work, while mechanisms do not necessarily perform this function.
- All machines are mechanisms. But all mechanisms are not machines.

- ✓ **Planar mechanisms:** When **all the links** of a mechanism have **plane motion**, it is called as a **planar mechanism**. All the links in a planar mechanism **move in planes parallel to the reference plane**.
- ✓ **Degrees of freedom/mobility of a mechanism:** It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.
- ✓ **Kutzbach's criterion: Number of degrees of freedom** of a mechanism is given by. (with considering higher pair)

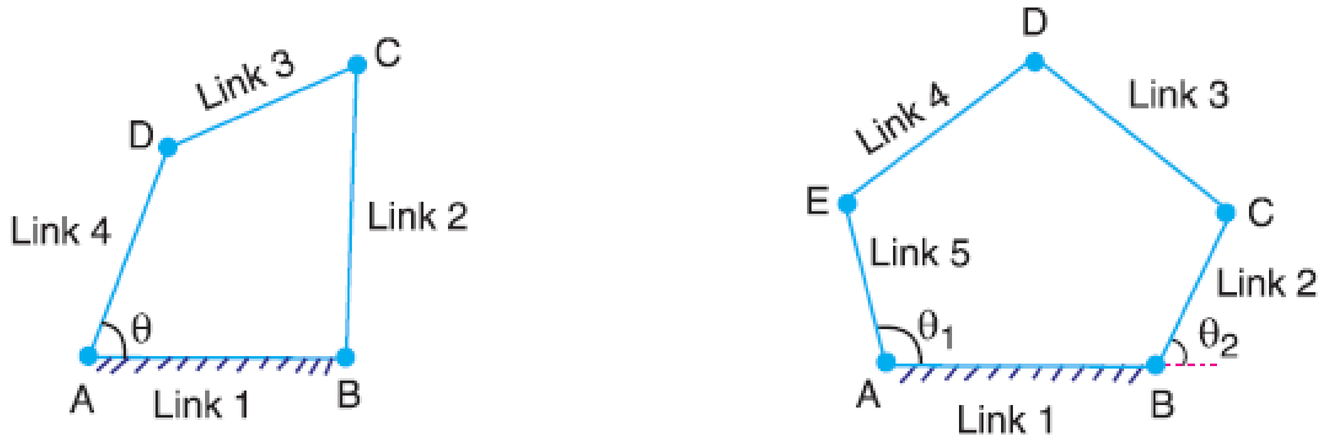
$$n = 3\,(l - 1) - 2\,j - h$$

- ✓ **Grubler's equation: Number of degrees of freedom** of a mechanism is given by (not considering higher pair, h = 0)

$$n = 3\,(l - 1) - 2\,j$$

5.12. Number of Degrees of Freedom for Plane Mechanisms

In the design or analysis of a mechanism, one of the most important concern is the number of degrees of freedom (also called movability) of the mechanism. It is defined as the number of input parameters (usually pair variables) which must be independently controlled in order to bring the mechanism into a useful engineering purpose. It is possible to determine the number of degrees of freedom of a mechanism directly from the number of links and the number and types of joints which it includes.



(*a*) Four bar chain. (*b*) Five bar chain.

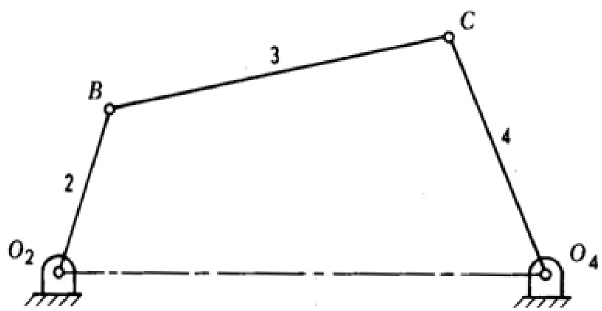
Fig. 5.13

Consider a four bar chain, as shown in Fig. 5.13 (*a*). A little consideration will show that only one variable such as θ is needed to define the relative positions of all the links. In other words, we say that the number of degrees of freedom of a four bar chain is one. Now, let us consider a five bar chain, as shown in Fig. 5.13 (*b*). In this case two variables such as θ_1 and θ_2 are needed to define completely the relative positions of all the links. Thus, we say that the number of degrees of freedom is * two.

Examples of determination of degrees of freedom of planar mechanisms:

(i)

$$n = 3\,(l - 1) - 2\,j - h$$

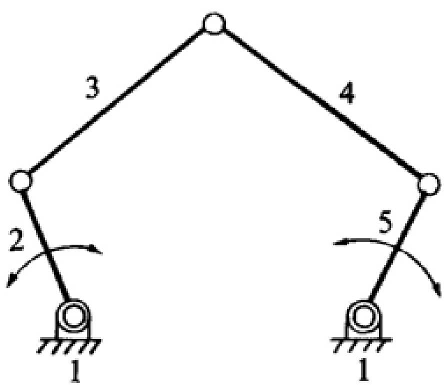


F = 3(n-1)-2l-h

Here, n_2 = 4, n = 4, l = 4 and h = 0.

F = 3(4-1)-2(4) = 1

I.e., one input to any one link will result in definite motion of all the links.

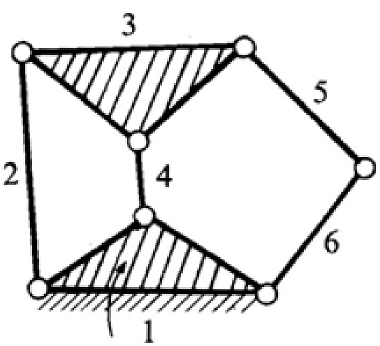


F = 3(n-1)-2l-h

Here, n_2 = 5, n = 5, l = 5 and h = 0.

F = 3(5-1)-2(5) = 2

I.e., two inputs to any two links are required to yield definite motions in all the links.

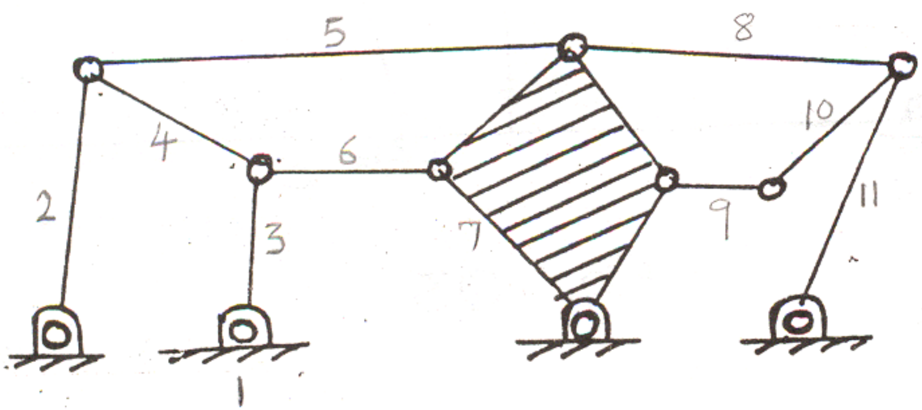


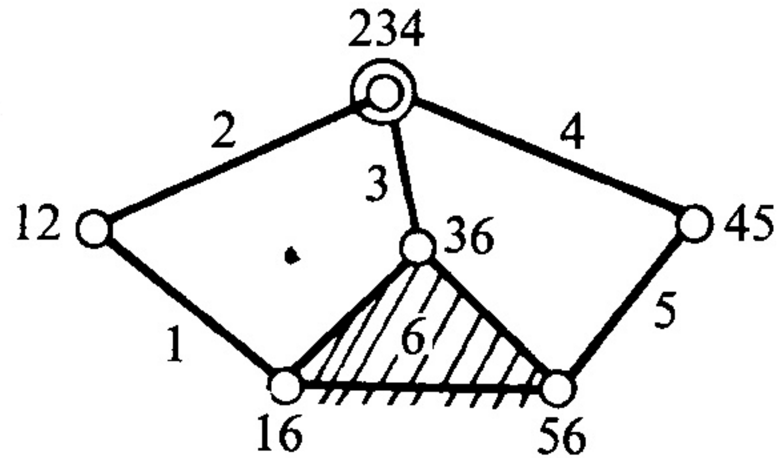
F = 3(n-1)-2l-h

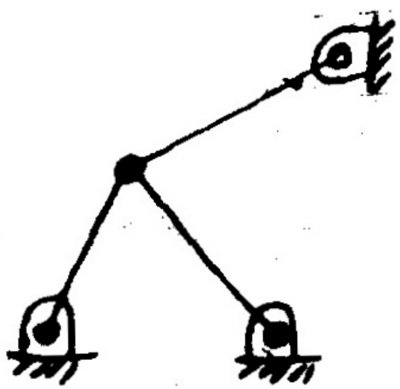
Here, n_2 = 4, n_3 =2, n = 6, l = 7 and h = 0.

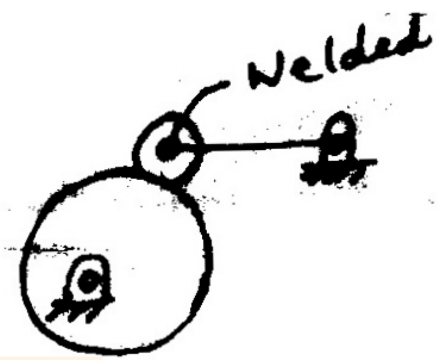
F = 3(6-1)-2(7) = 1

I.e., one input to any one link will result in definite motion of all the links.









$$n = 3\,(l-1) - 2\,j - h$$

(a)

F = 3(n-1)-2l-h
Here, n = 4, l = 5 and h = 0.
F = 3(4-1)-2(5) = -1
I.e., it is a structure

(b)

F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

(c)

F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

Types of Kinematic Chains

- ✓ The most important kinematic chains are those which **consist of four lower pairs**, each pair being a **sliding pair or a turning pair**.
- ✓ The following three types of kinematic chains with four lower pairs are important from the subject point of view :

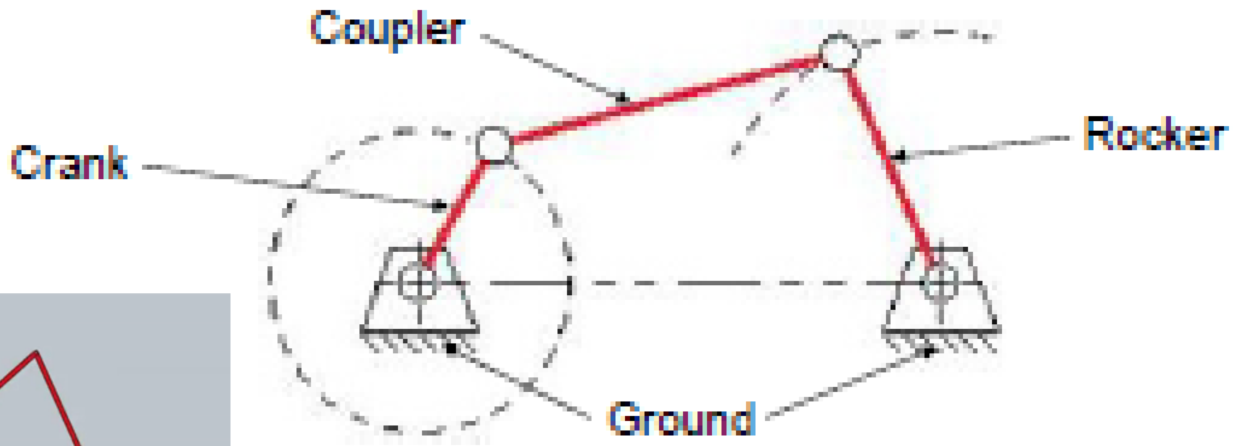
  1. Four bar chain or quadric cyclic chain,

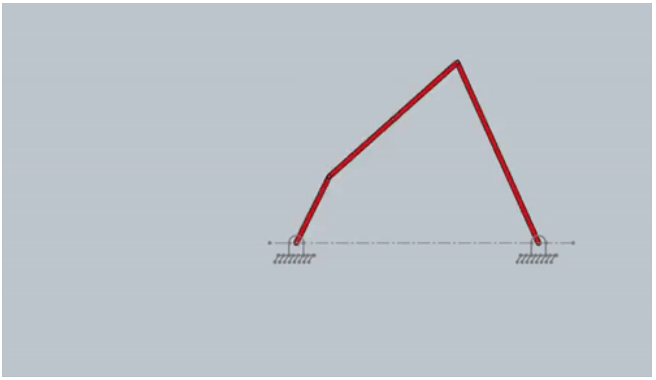
  2. Single slider crank chain, and

  3. Double slider crank chain.

The Grashof Condition

- Fourbar link naming conventions:
  - Ground: Link fixed to the ground
  - Crank: Link which makes a complete revolution
  - Rocker: Link which has oscillatory motion (rocks back and forth)
  - Coupler: Link connecting input and output links





The Grashof Condition

- Grashof condition: Condition used to predict rotational behavior/motion of fourbar linkage based on link lengths
- Linkage is Grashof if $S + L \leq P + Q$ (whiteboard)

  where *S*: length of shortest link

  L: length of longest link

  P: length of intermediate link

  Q: length of another intermediate link
- Else linkage is non-Grashof
- Grashof linkage: At least one link will be able to make a full revolution
- Non-Grashof linkage: No link will be able to make a full revolution

- Class 1: $S + L < P + Q$
  - Either link attached to the shortest is grounded → **Crank-Rocker**
  - Shortest link is grounded → **Double-Crank**
  - Link opposite the shortest is grounded → **Double Rocker**
- Class 2: $S + L > P + Q$ (non-Grashof)
  - **Tripler Rocker**
- Class 3: $S + L = P + Q$ (special case)
  - Will have 'change points' when all links become collinear
  - Output behavior at these points indeterminate

Grashoff's law for four-bar linkage

Identify longest, shortest, intermediate 2 links: L, S, P, Q

I: If L + S < P + Q, then we call this a Grashof Mechanism

G.1 = crank-rocker if S is the crank and either of the adjacent link is the fixed link

G.2 = double-crank if S is the fixed link

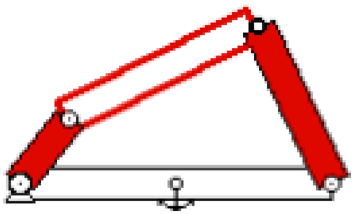
G.3 = double-rocker if the link opposite S is the fixed link

II: If L + S > P + Q, then we call it non-Grashof mechanism
only double-rocker: no link is capable of making a complete revolution

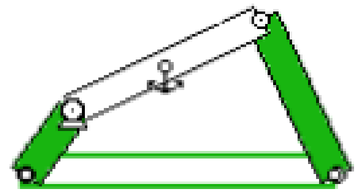
III: If L + S = P + Q, it can have G.1~G.3 and parallelogram form (colinear)

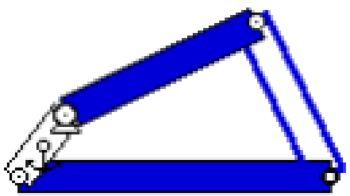
Grashof Mechanisms

All inversions of the Grashof fourbar linkage

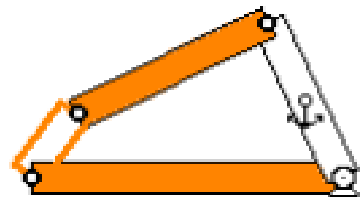


Two non-distinct
crank-rocker inversions





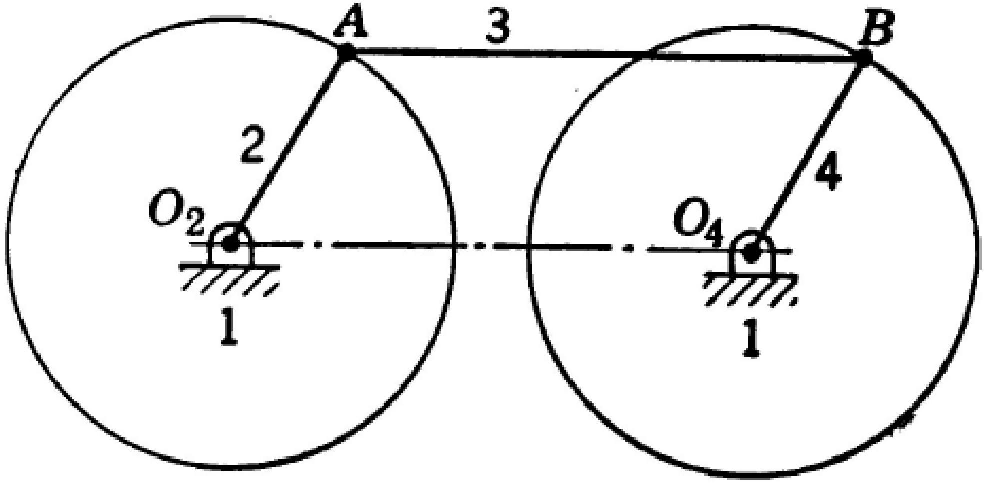
Double-crank inversion
(drag link)



Double-rocker inversion
(coupler rotates)

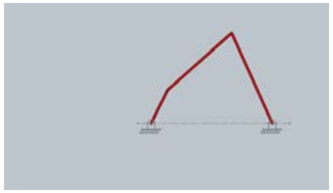
Grashof – Special Case

III: If L + S = P + Q, it can have G.1~G.3 and parallelogram form (colinear)



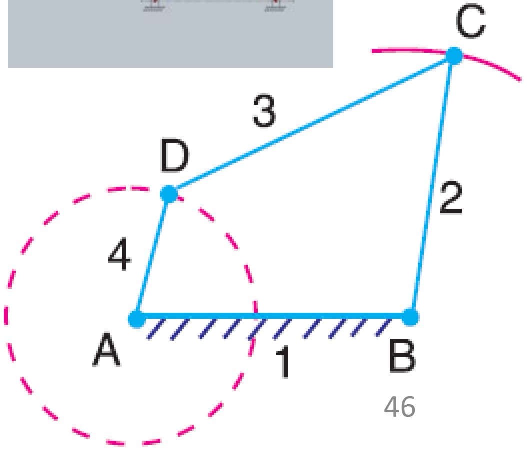
Four Bar Chain or Quadric Cycle Chain

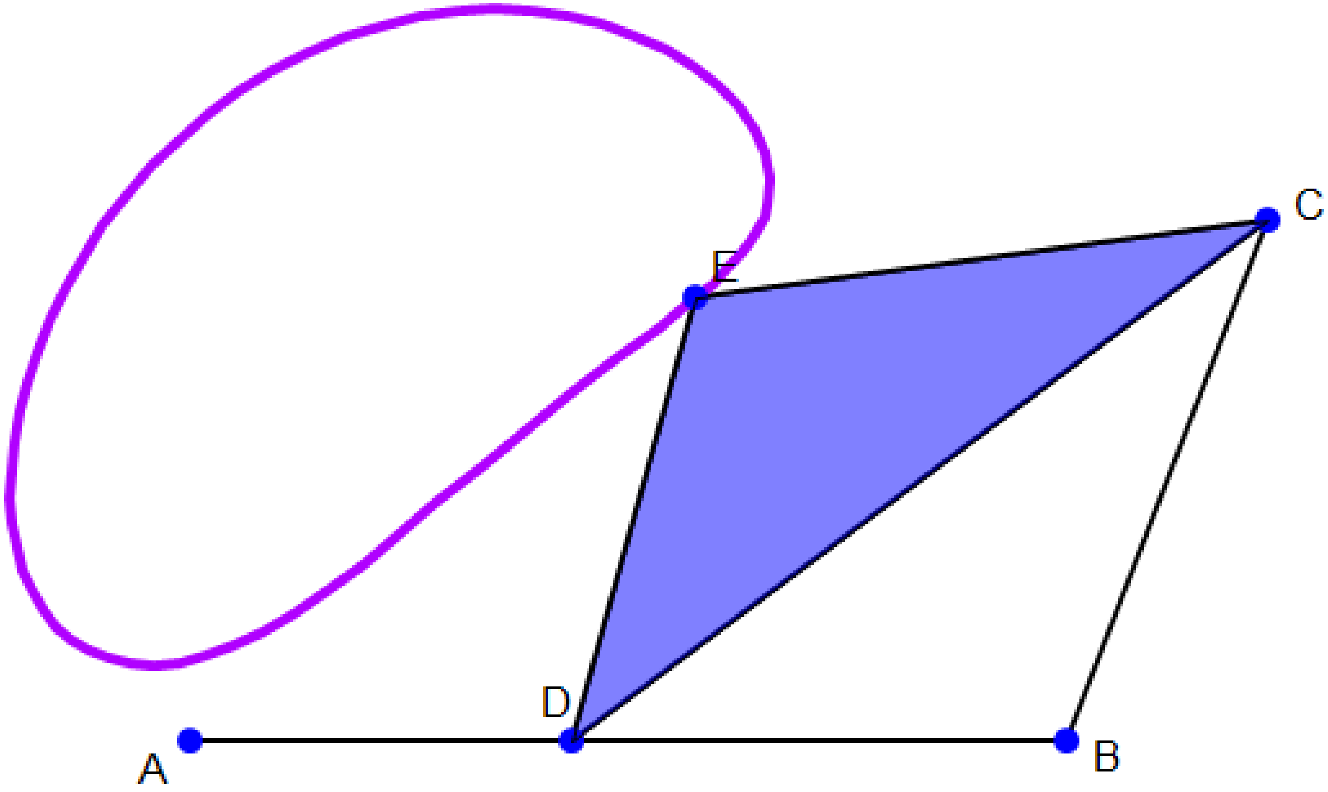
- We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained.
- The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig.
- It consists of four links, each of them forms a turning pair at *A, B, C and D. The four links may be of* different lengths.
- A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful.
- In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links. Such a link is known as ***crank or driver.***



In Fig. *AD (link 4) is a crank. The link BC (link 2) which makes a partial rotation or oscillates is known* as ***lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism.***

When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

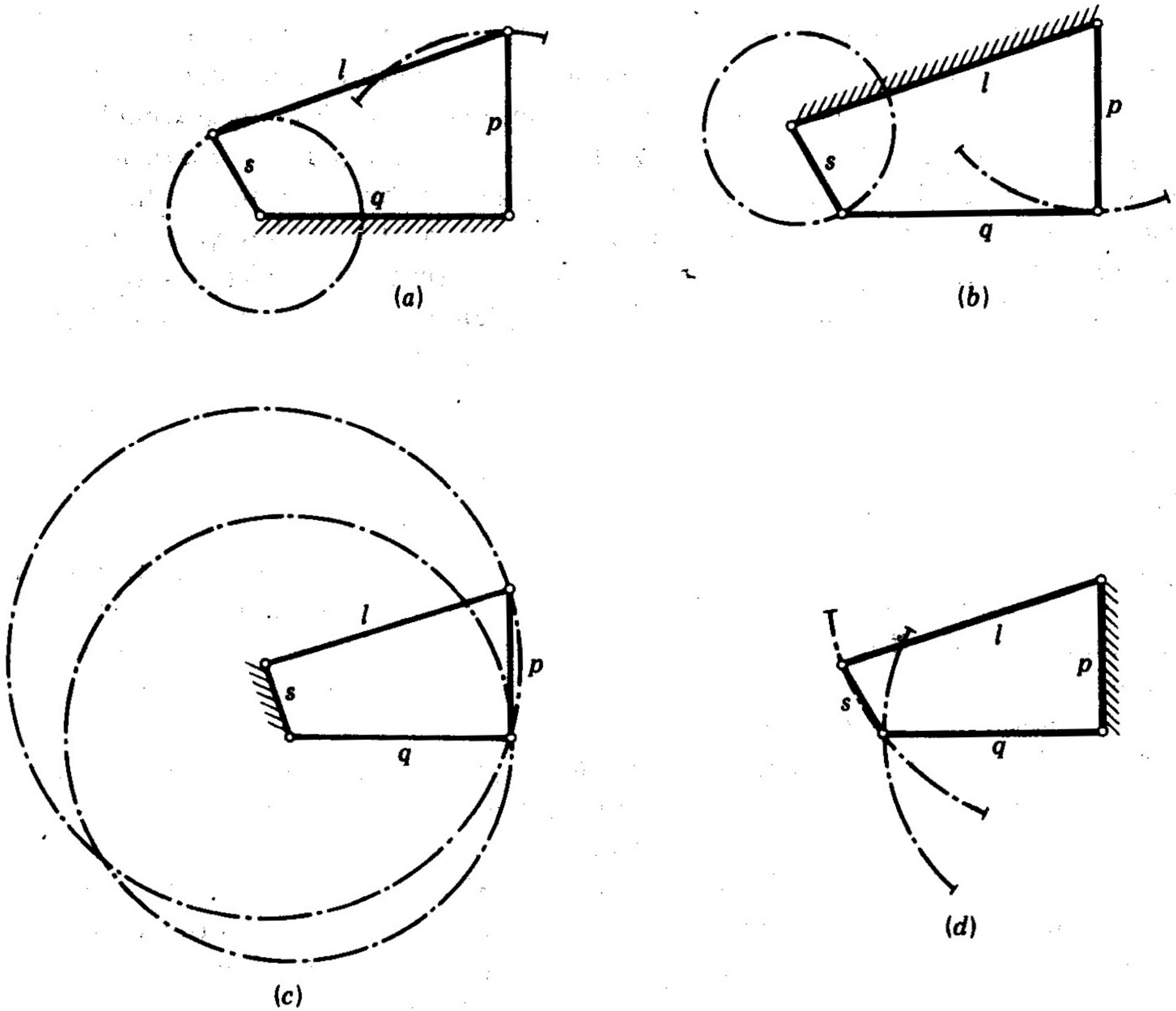



C
E
D
A
B

Inversions of mechanism:

- ✓ A **mechanism** is one in which **one of the links** of a kinematic chain is **fixed**.
- ✓ **Different mechanisms** can be **obtained** by **fixing different links** of the same kinematic chain.
- ✓ These are called as **inversions of the mechanism**.
- ✓ By changing the fixed link, the number of mechanisms which can be obtained is equal to the number of links.
- ✓ Except the original mechanism, all other mechanisms will be known as inversions of original mechanism.
- ✓ The inversion of a mechanism does not change the motion of its links relative to each other.

Inversions of four bar chain:



Inversions of Four Bar Chain

1. *Beam engine (crank and lever mechanism).*

- A part of the mechanism of a beam engine (also known as **crank and lever mechanism**) which consists of four links, is shown in Fig.
- In this mechanism, when the crank rotates about the fixed center *A, the lever oscillates about* a fixed center *D.*
- *The end E of the lever CDE is* connected to a piston rod which reciprocates due to the rotation of the crank.
- In other words, the purpose of this mechanism is to **convert rotary motion into reciprocating motion**.

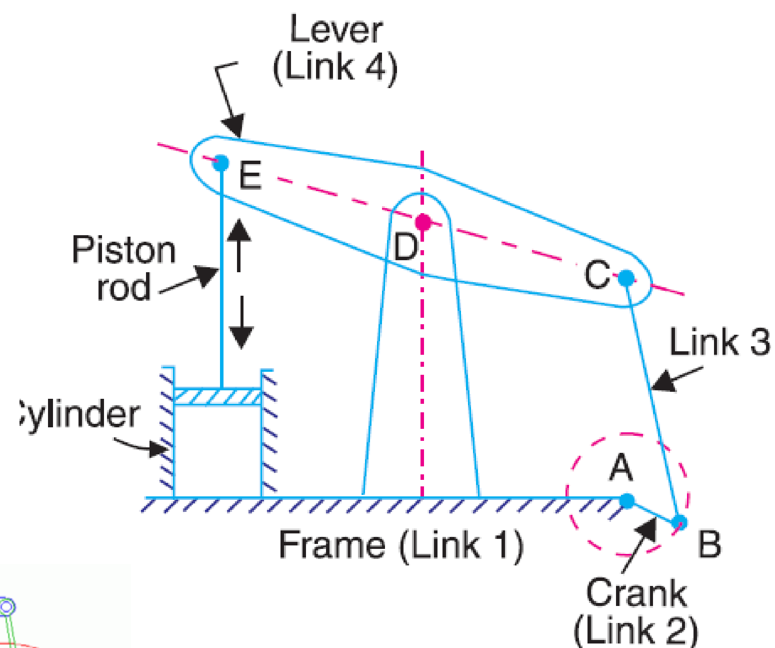
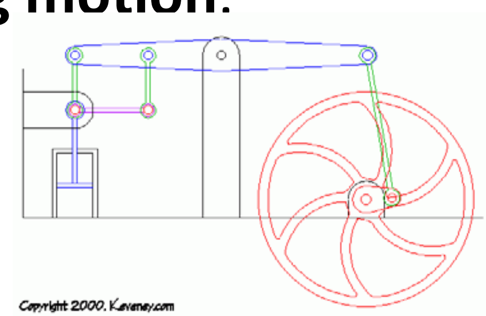
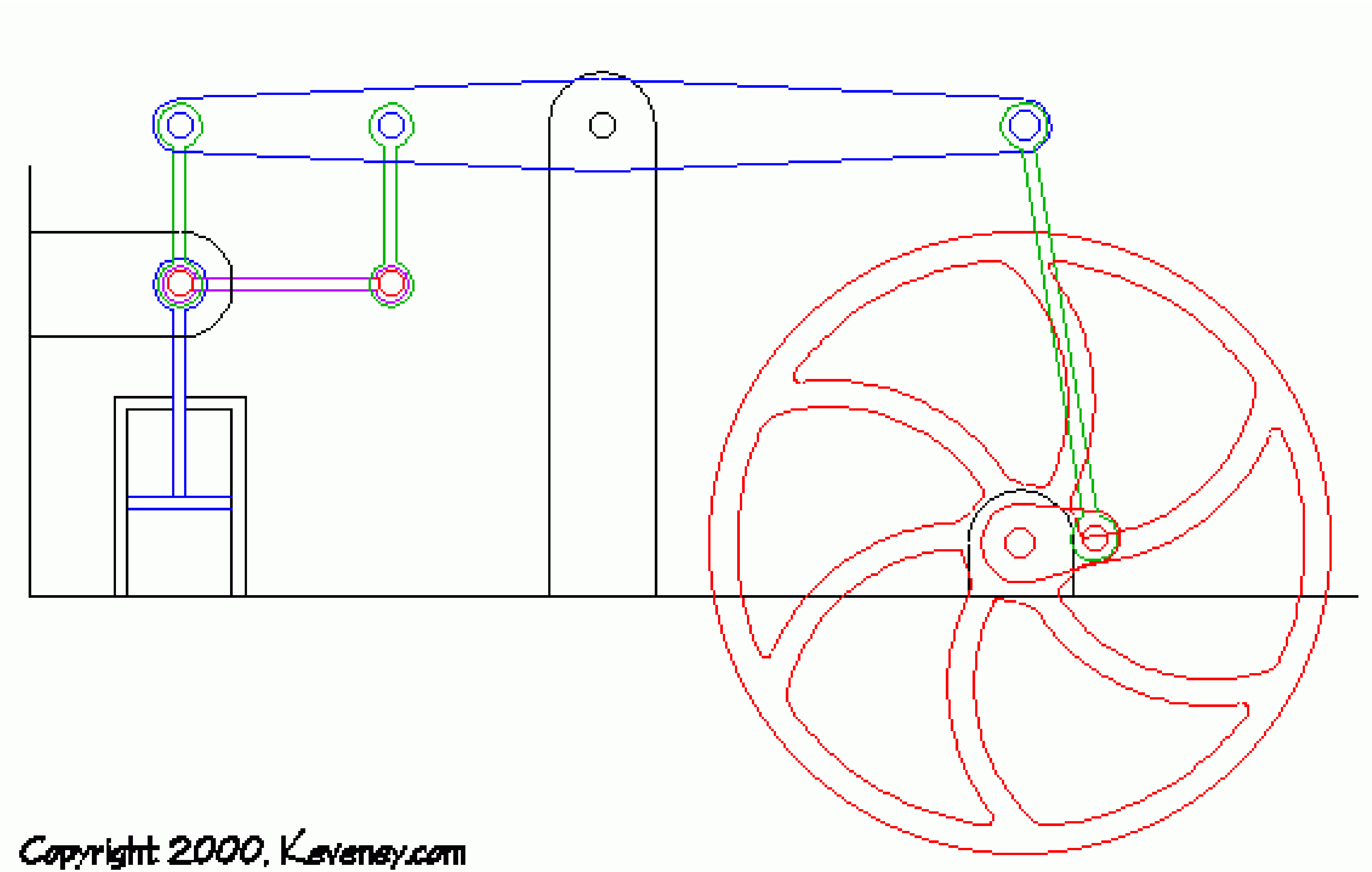


Fig. 5.19. Beam engine.

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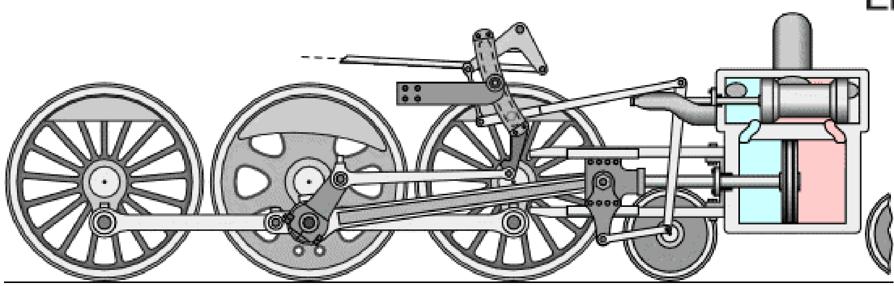
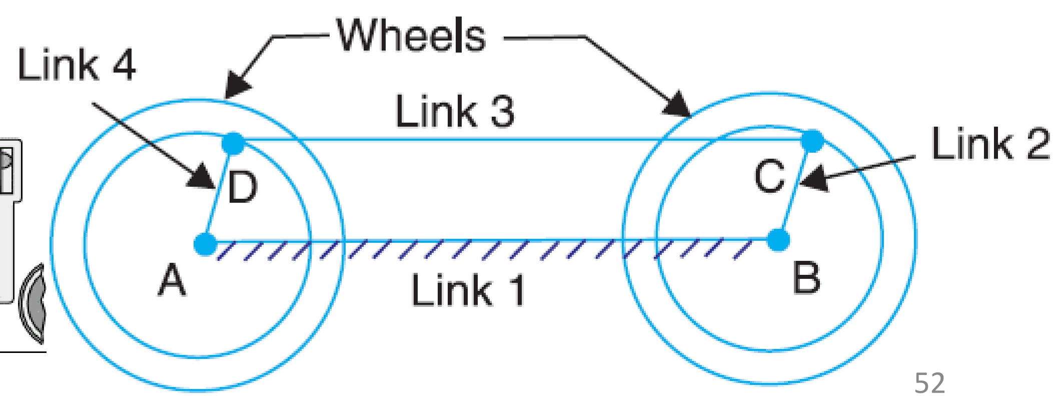
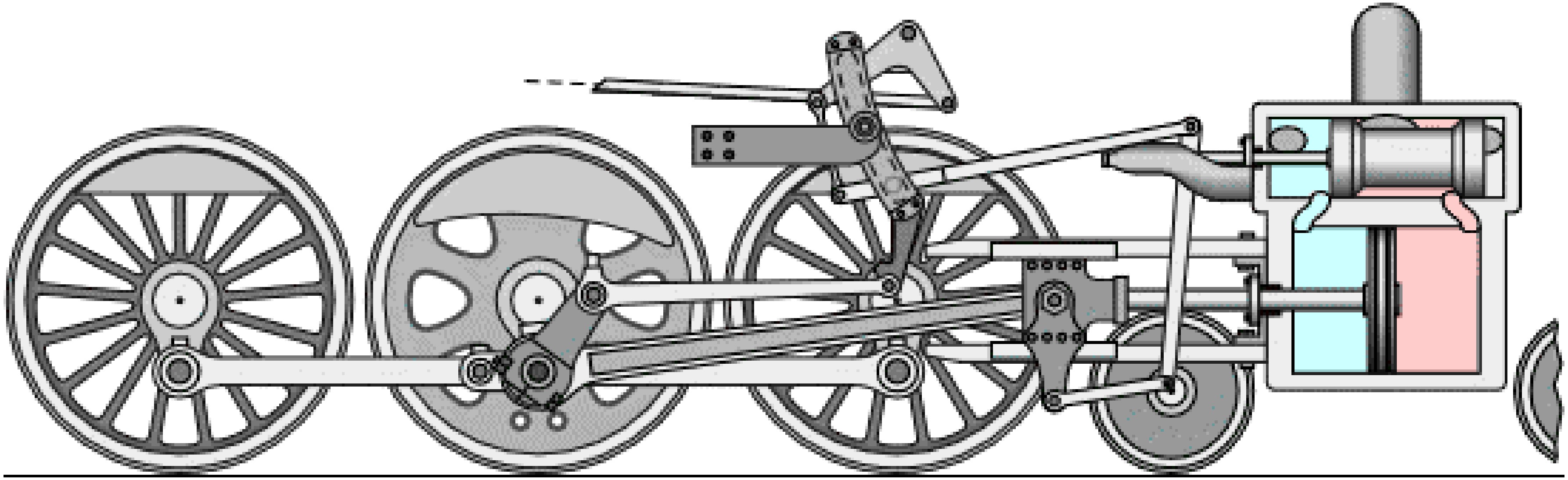
2. *Coupling rod of a locomotive (Double crank mechanism).*

- *The mechanism of a coupling* rod of a locomotive (also known as **double crank mechanism**) which consists of four links, is shown in Fig.
- In this mechanism, the **links *AD and BC*** *(having **equal length**) act as cranks and are connected* to the respective wheels.
- The link *CD acts as a coupling rod and the link AB is fixed in order* to maintain a constant centre to centre distance between them.
- This mechanism is meant for transmitting **rotary motion from one wheel to the other wheel**.

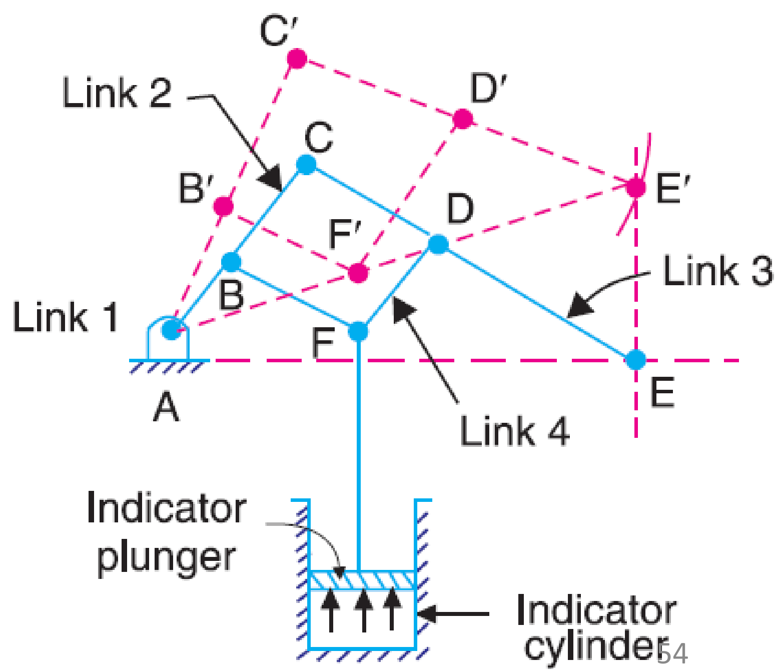






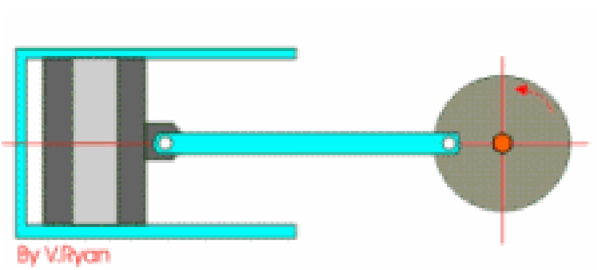
3. *Watt's indicator mechanism (Double lever mechanism).*

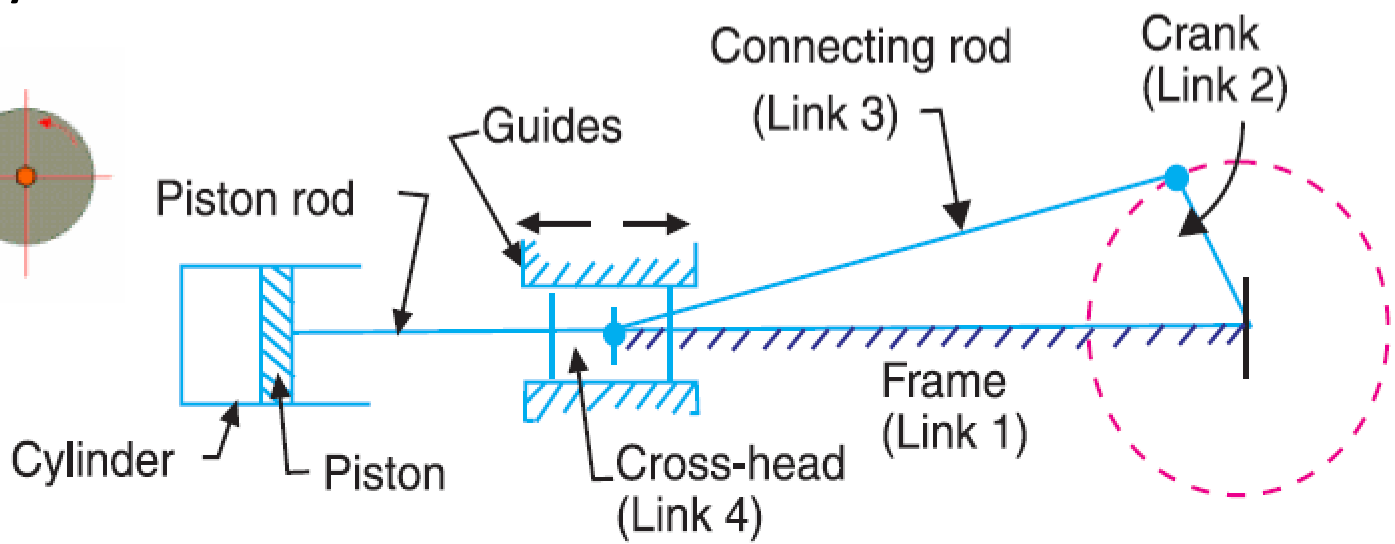
- *A Watt's indicator mechanism* (also known as **Watt's straight line mechanism or double lever mechanism**) which consists of four links, is shown in Fig.
- The four links are : fixed link at *A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no* relative motion between them. The links *CE and BFD act* as levers.
- The displacement of the link *BFD is directly* proportional to the pressure of gas or steam which acts on the indicator plunger.
- On any small displacement of the mechanism, the tracing point *E at the end of the link CE* traces out approximately a straight line.
- The initial position of the mechanism is shown in Fig. by full lines whereas the **dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger**.

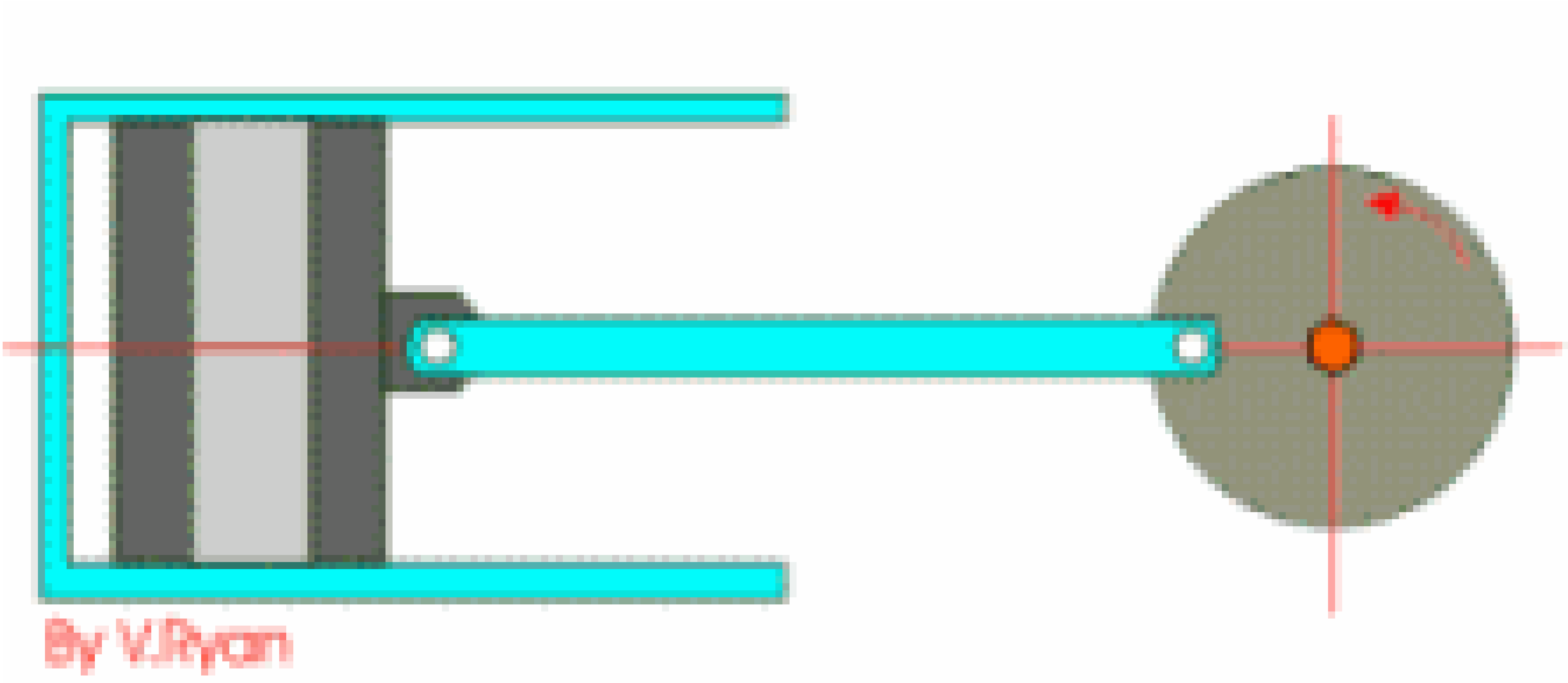


Single Slider Crank Chain

- A single slider crank chain is a modification of the basic four bar chain.
- It consist of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism.
- This type of mechanism converts rotary motion into reciprocating motion and vice versa.
- In a single slider crank chain, as shown in Fig., the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.
- The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank ; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head.
- **As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.**



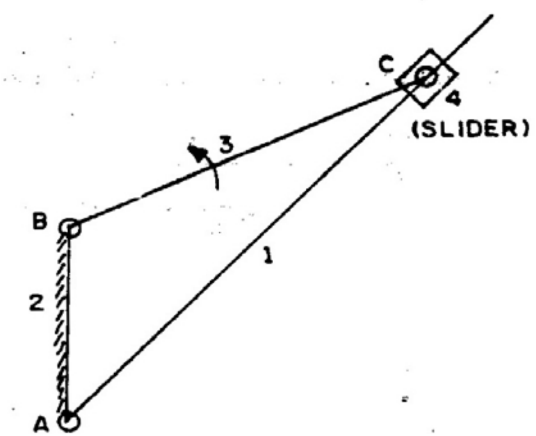



By V.Ryan

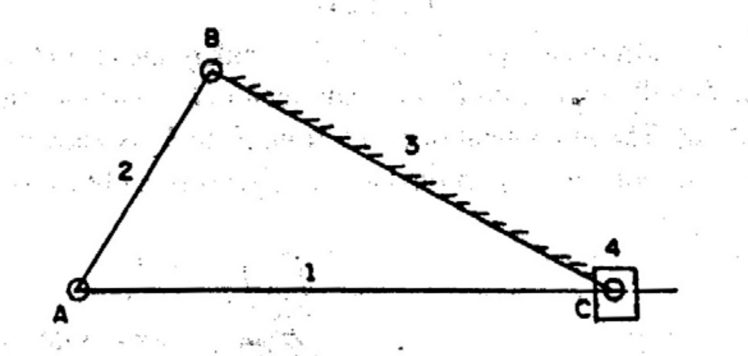
Inversions of Single Slider Crank Chain

- Four inversions of a single slider crank chain are possible.
- These inversions are found in the following mechanisms.

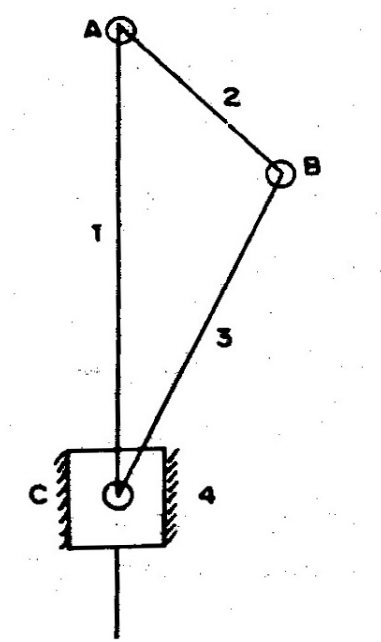
1. ***Pendulum pump or Bull engine.***
2. ***Oscillating cylinder engine***
3. ***Rotary internal combustion engine***
4. ***Crank and slotted lever quick return motion mech***
5. ***Whitworth quick return motion mechanism.***



(a) crank fixed



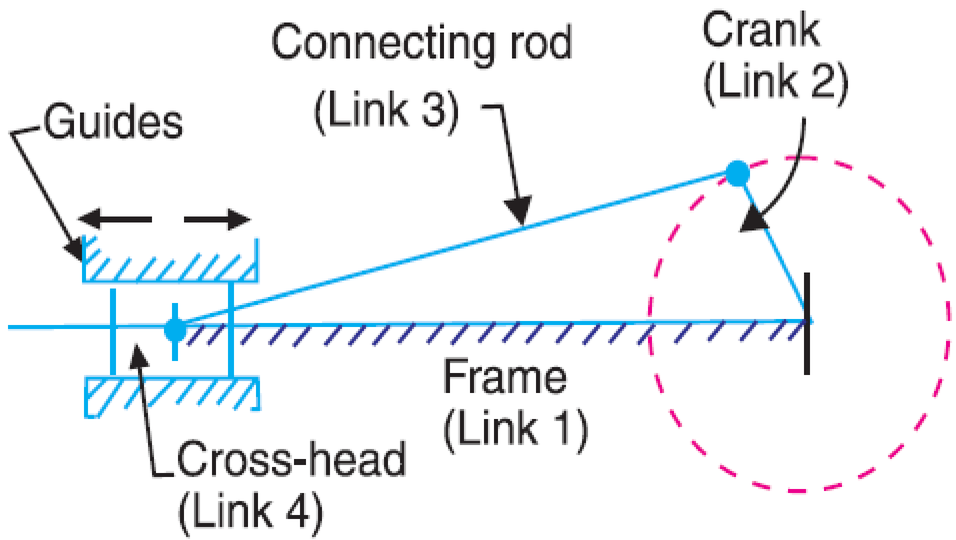
(b) connecting rod fixed

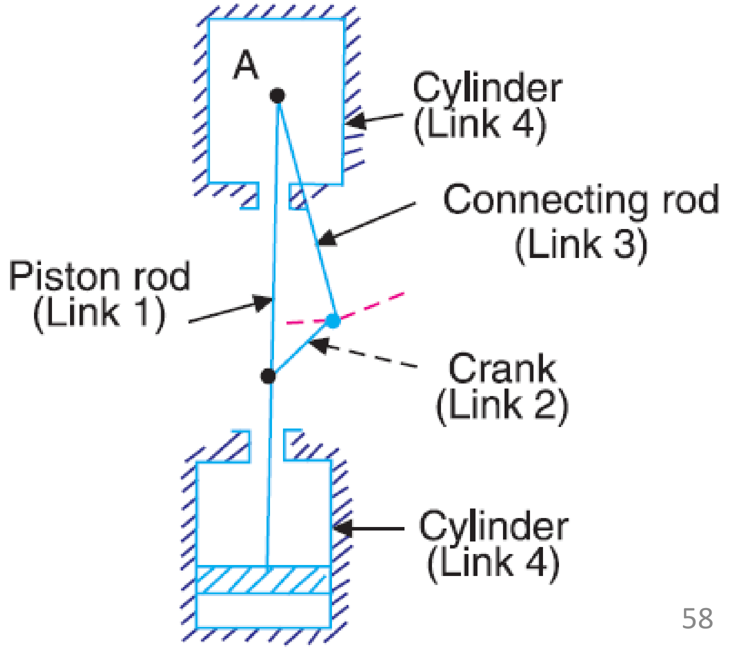


(c) slider fixed

1. Pendulum pump or Bull engine

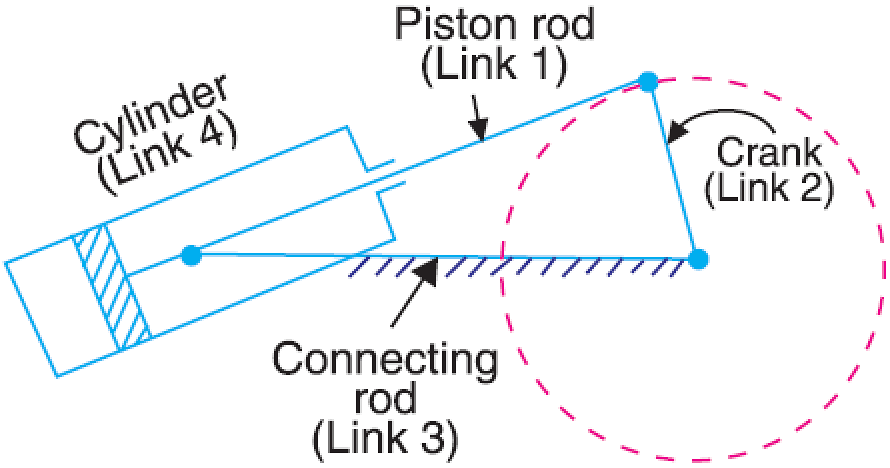
- *In this mechanism, the inversion is obtained by fixing the* cylinder or link 4 (*i.e. sliding pair), as shown in Fig.*
- *In this case, when the crank (link 2) rotates,* the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at *A and the piston* attached to the piston rod (link 1) reciprocates.
- The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.





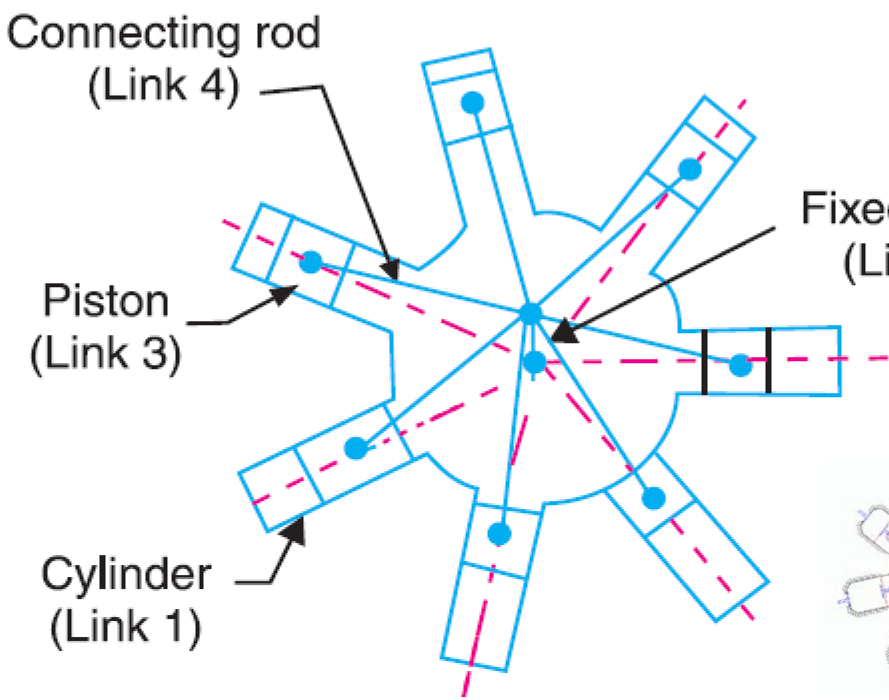
2. Oscillating cylinder engine

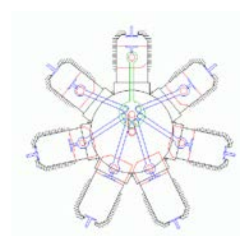
- *The arrangement* of oscillating cylinder engine mechanism, as shown in Fig., is used to convert reciprocating motion into rotary motion.
- In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism.
- When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at *A*.



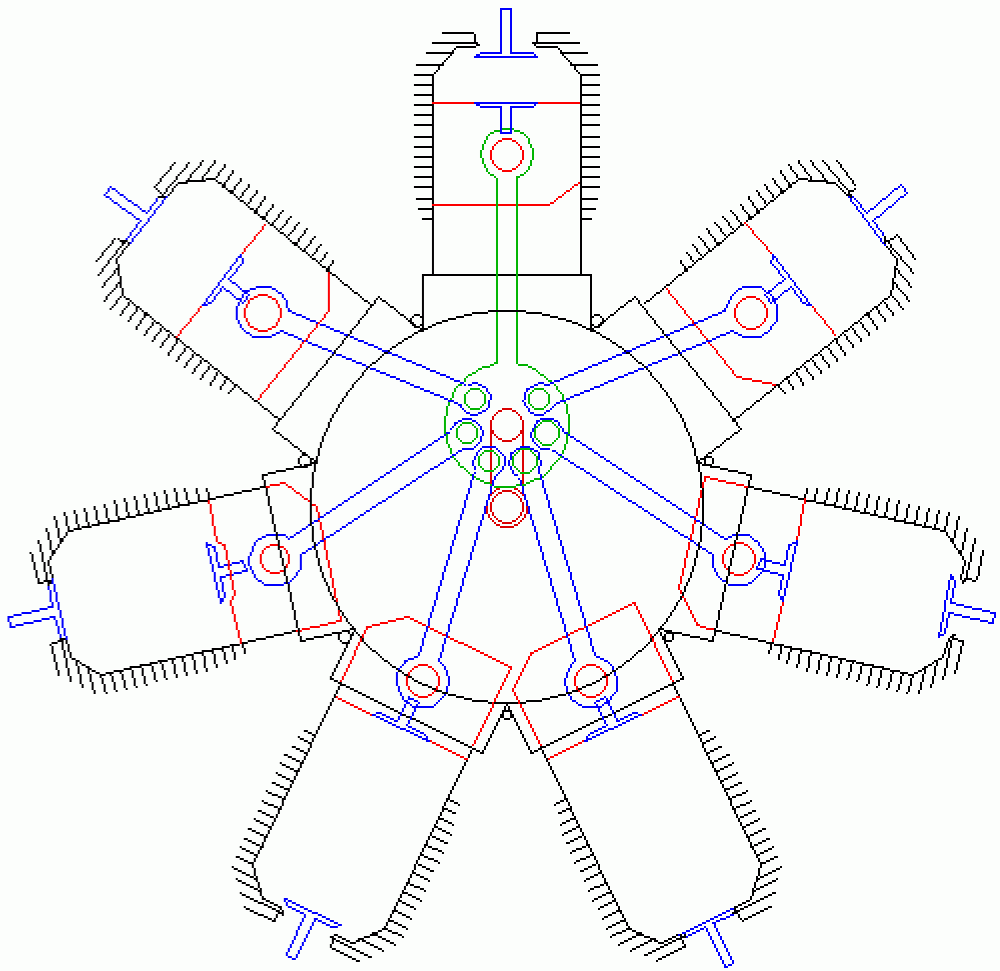
3. Rotary internal combustion engine

- *Sometimes back, rotary internal* combustion engines were used in aviation. But now-a-days gas turbines are used in its place.
- It consists of **seven cylinders** in one plane and all revolves about fixed centre *D, as shown in* Fig., while the crank (link 2) is fixed.
- In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

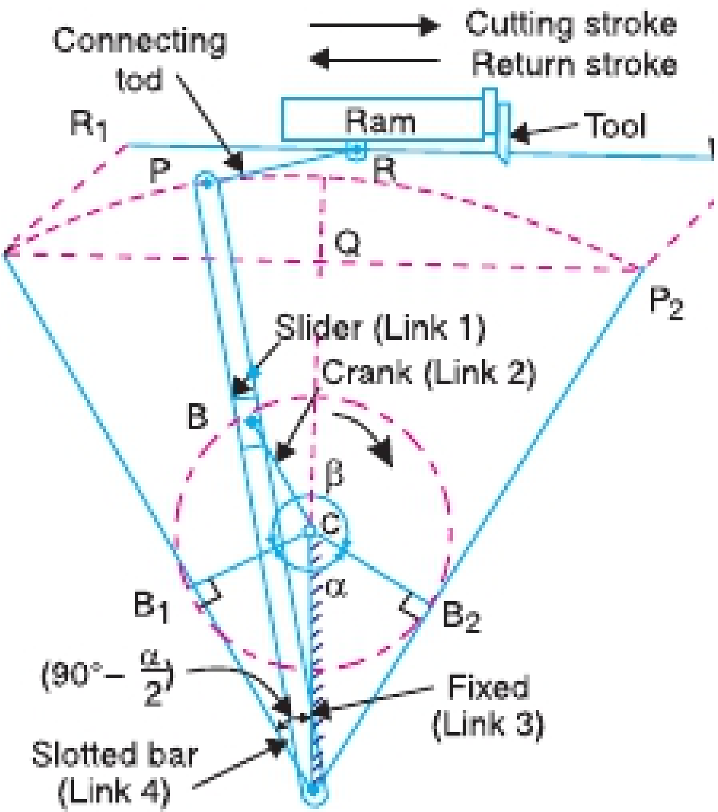


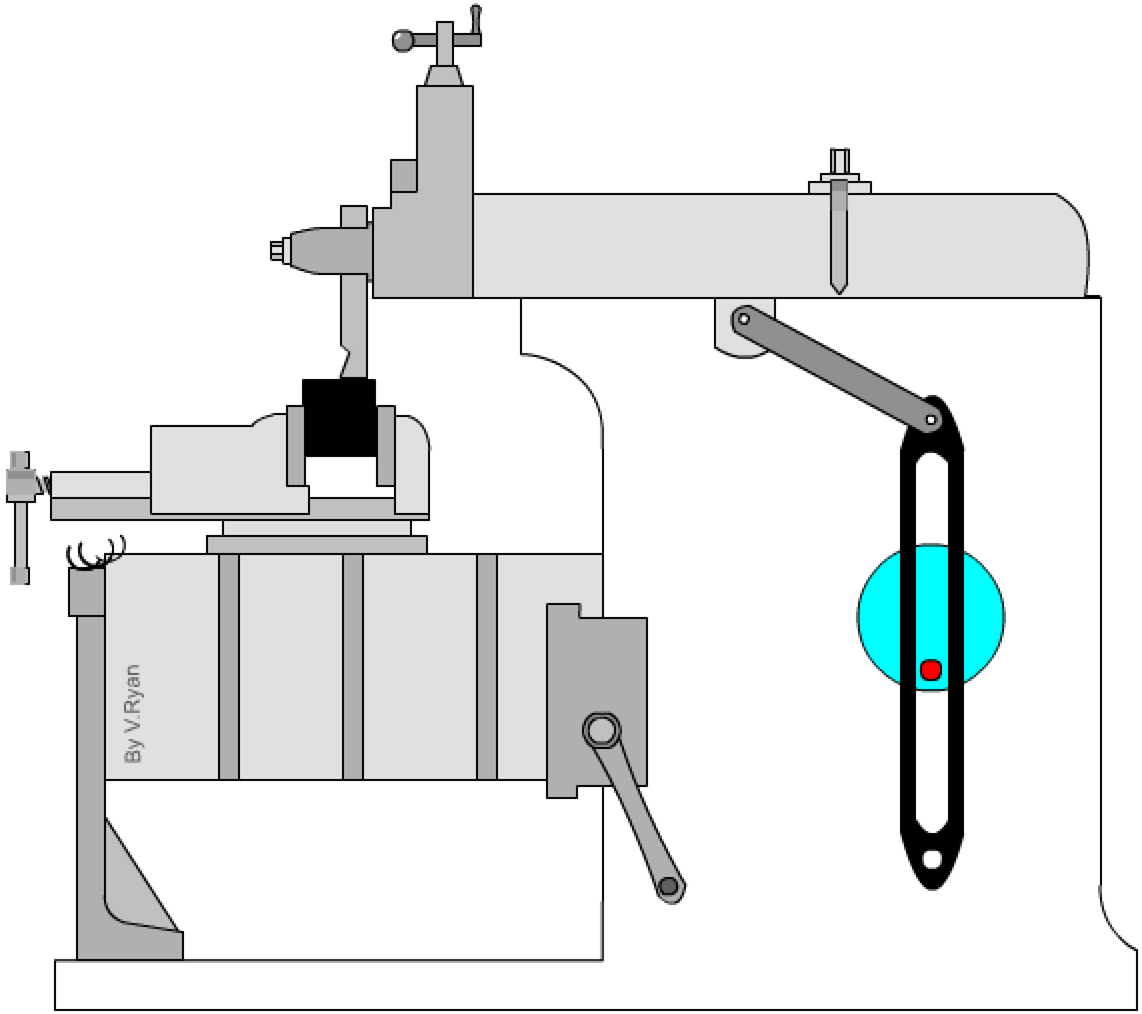


Rotary engine



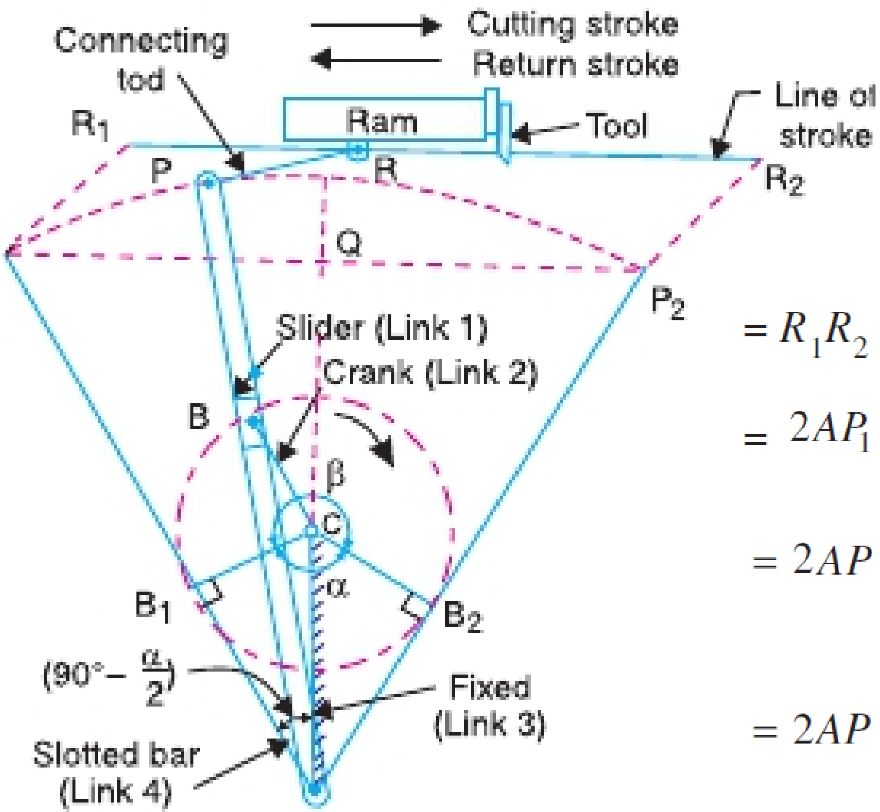
4. Crank and slotted lever quick return motion mechanism





4. Crank and slotted lever quick return motion mechanism

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \text{ or } \frac{360° - \alpha}{\alpha}$$



Travel of tool or length of stroke

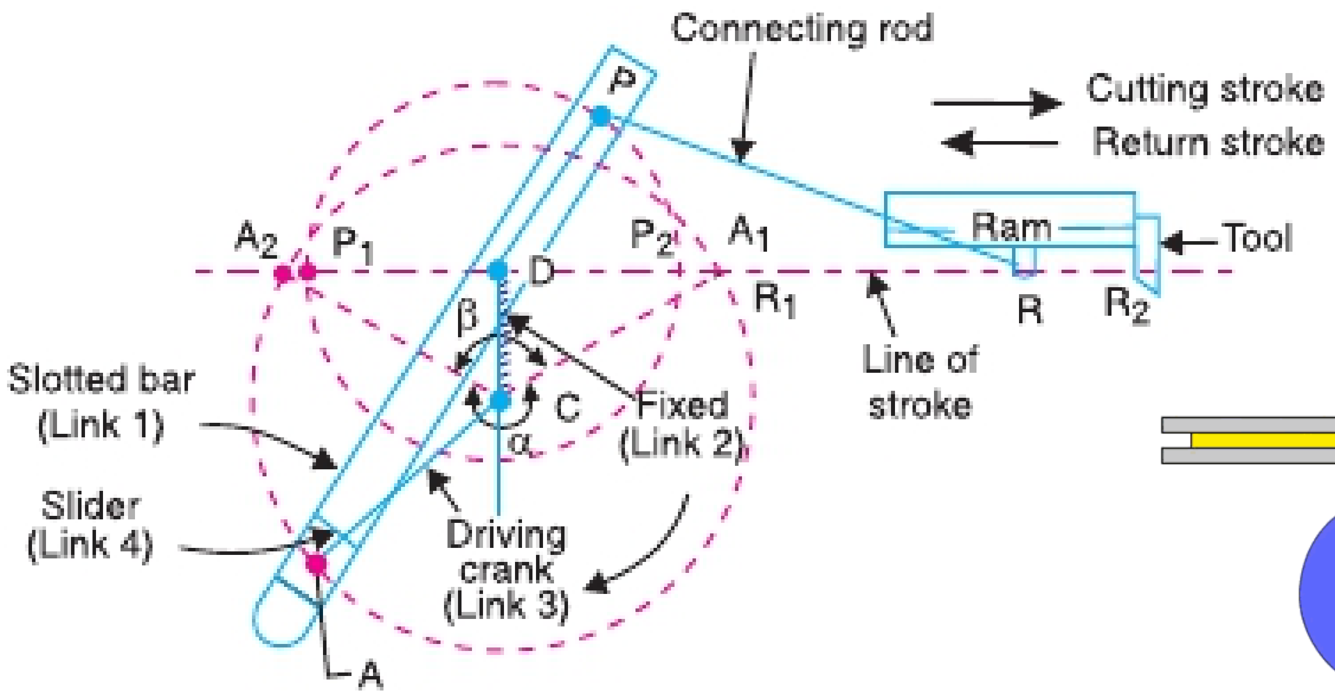
$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

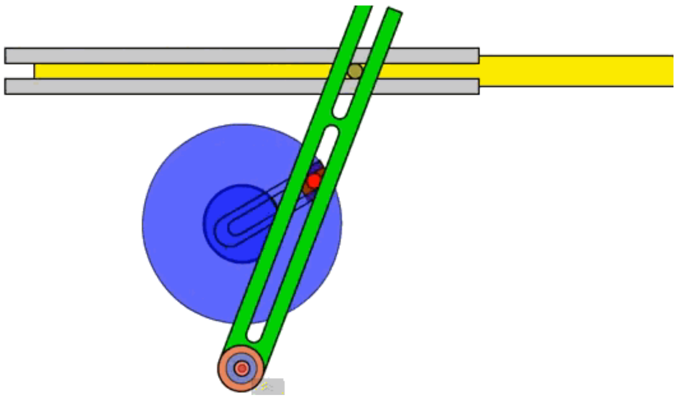
$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2} \qquad \text{..... } (\because AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \qquad \text{... } \left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

$$= 2AP \times \frac{CB}{AC} \qquad \text{... } (\because CB_1 = CB)$$

5. Whitworth quick return motion mechanism





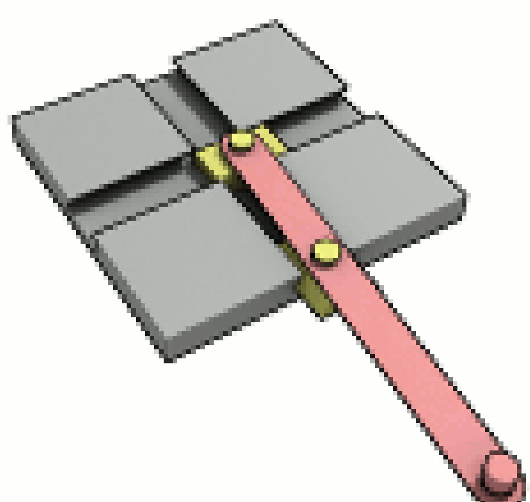
$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \quad \text{or} \quad \frac{360° - \beta}{\beta}$$

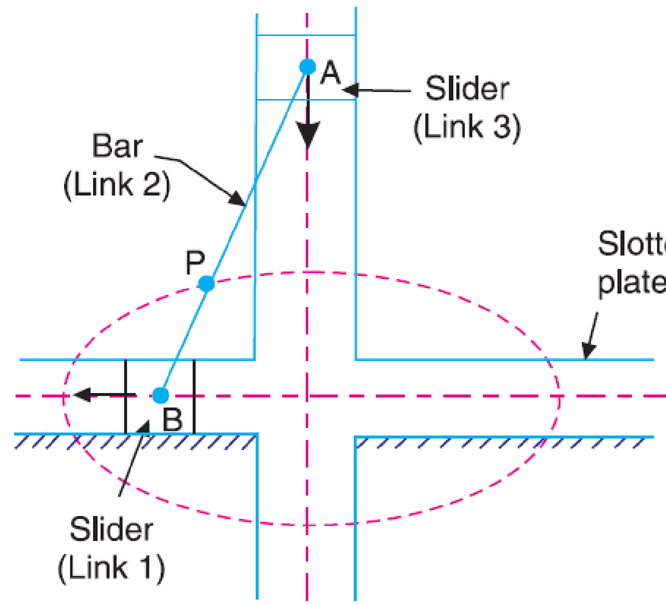
Inversions of double slider crank mechanism

1. *Elliptical trammels*
2. *Scotch yoke mechanism*
3. *Oldham's coupling*

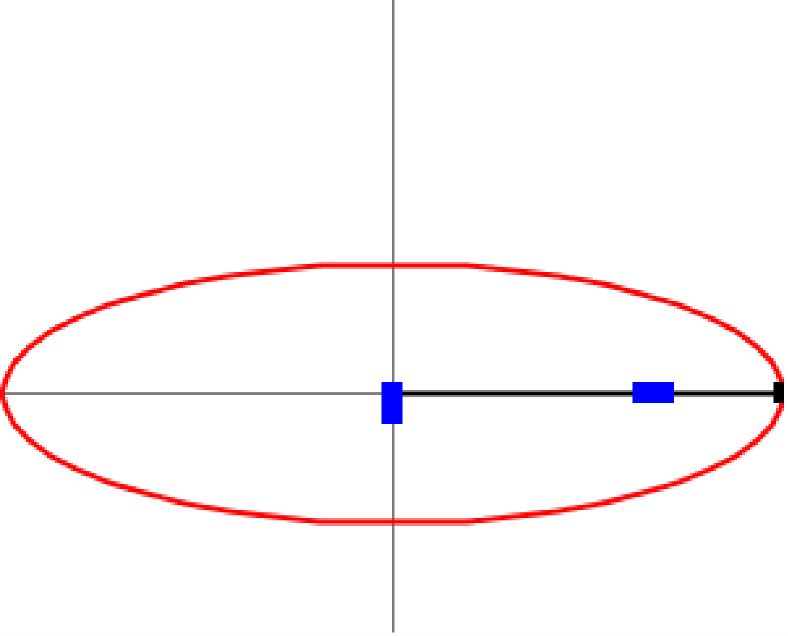
1. *Elliptical trammels*

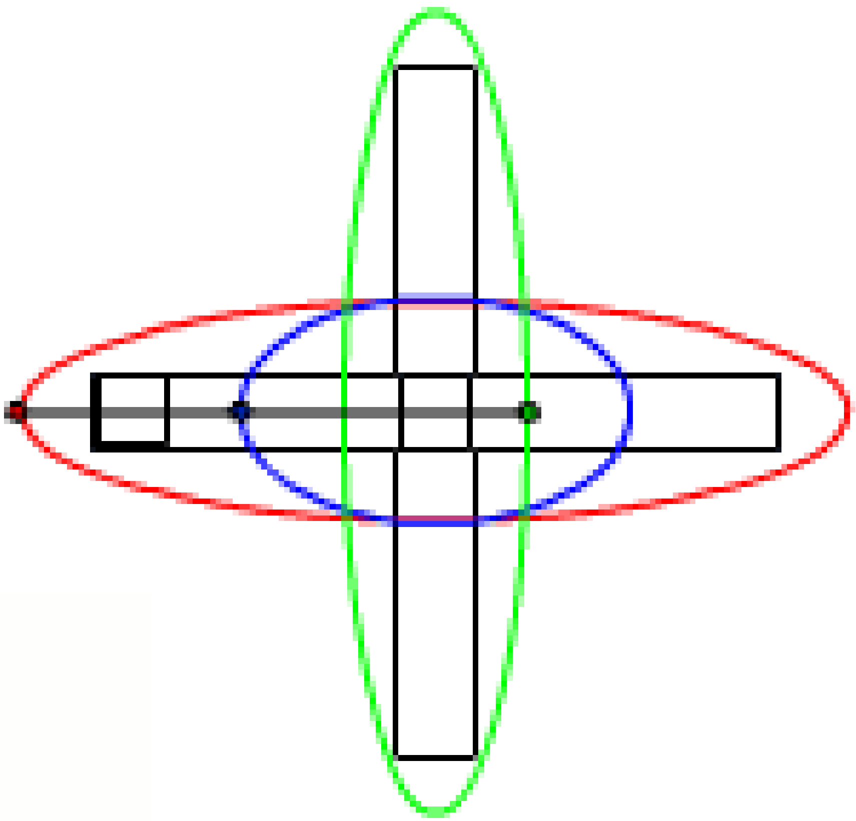
- *It is an instrument used for drawing ellipses. This inversion is obtained* by fixing the slotted plate (link 4), as shown in Fig.
- The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other.
- The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link *AB (link 2) is a bar which forms turning pair with links 1 and 3.*
- When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as *P traces out an ellipse on the surface of link 4, as shown in Fig. A little consideration will* show that *AP and BP are the semi-major axis and semi-minor axis of the ellipse respectively.*

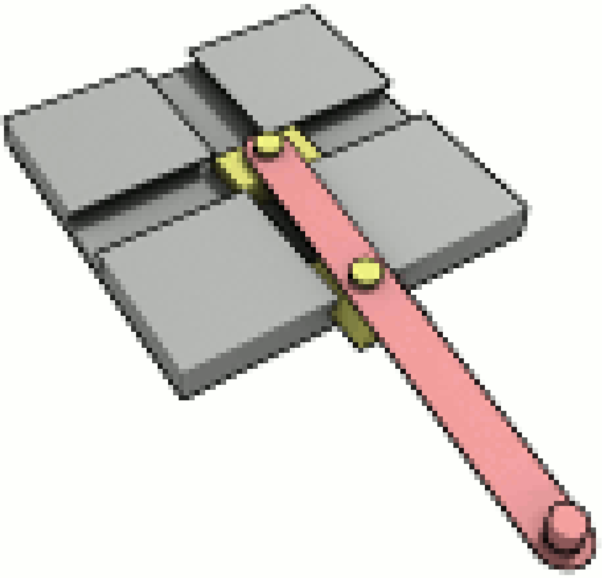




(*a*)

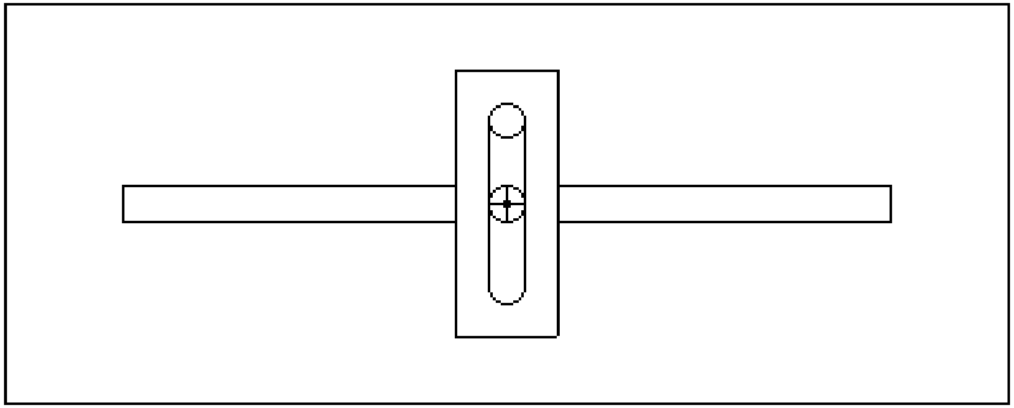


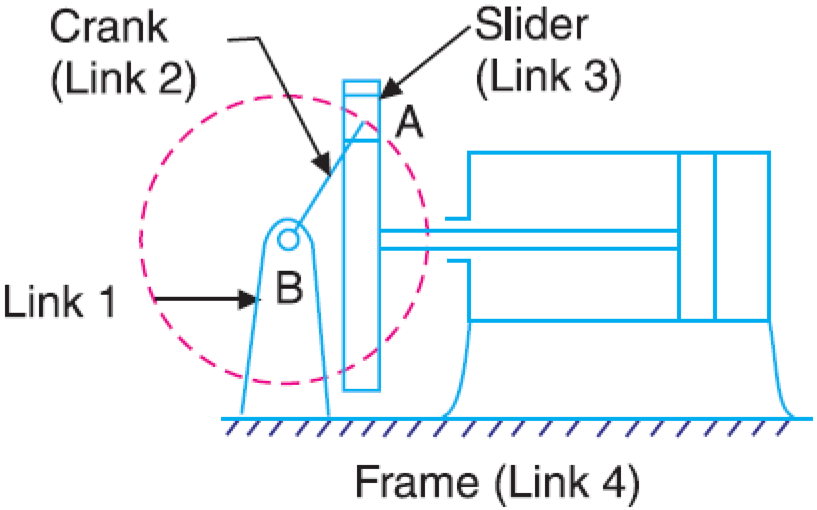




2. *Scotch yoke mechanism*

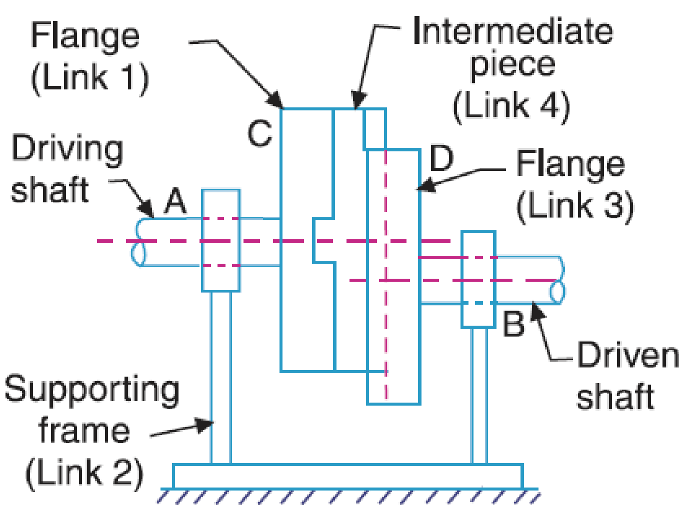
- The **Scotch yoke** is a mechanism for converting the linear motion of a slider into rotational motion or vice-versa. The piston or other reciprocating part is directly coupled to a sliding yoke with a slot that engages a pin on the rotating part.
- *This mechanism is used for converting rotary motion into a* reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig. link 1 is fixed.
- In this mechanism, when the link 2 (which corresponds to crank) rotates about *B as centre, the link* 4 (which corresponds to a frame) reciprocates.
- The fixed link 1 guides the frame.



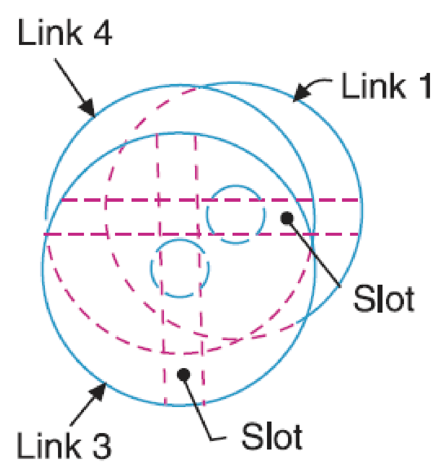


3. *Oldham's coupling*

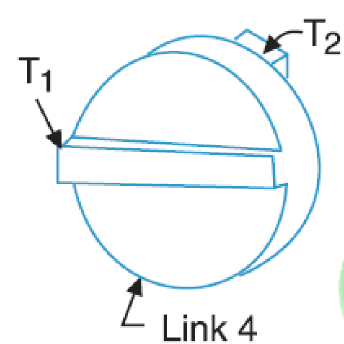
- ***An oldham's coupling is*** used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed.
- This inversion is obtained by fixing the link 2, as shown in Fig (a).
- *The shafts to be* connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.
- The link 1 and link 3 form turning pairs with link 2.
- These flanges have diametrical slots cut in their inner faces, as shown in Fig.(*b).*
- *The intermediate piece (link 4) which is a circular disc,* have two tongues (*i.e. diametrical projections) T*1 and *T*2 on each face at right angles to each other, as shown in Fig. 5.36 (*c).*
- *The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and* link 3). The link 4 can slide or reciprocate in the slots in the flanges.



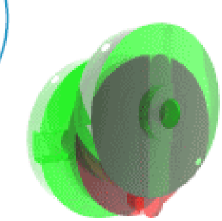
(*a*)

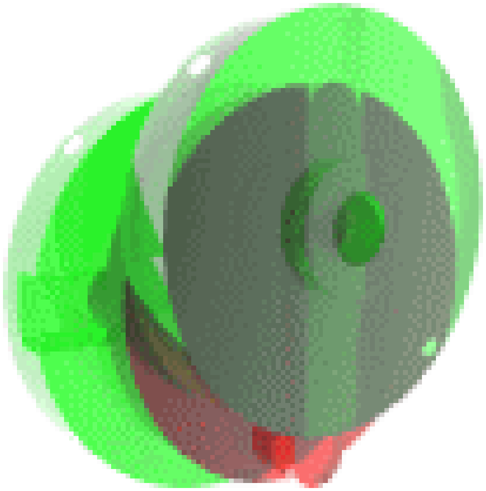


(*b*)



(*c*)



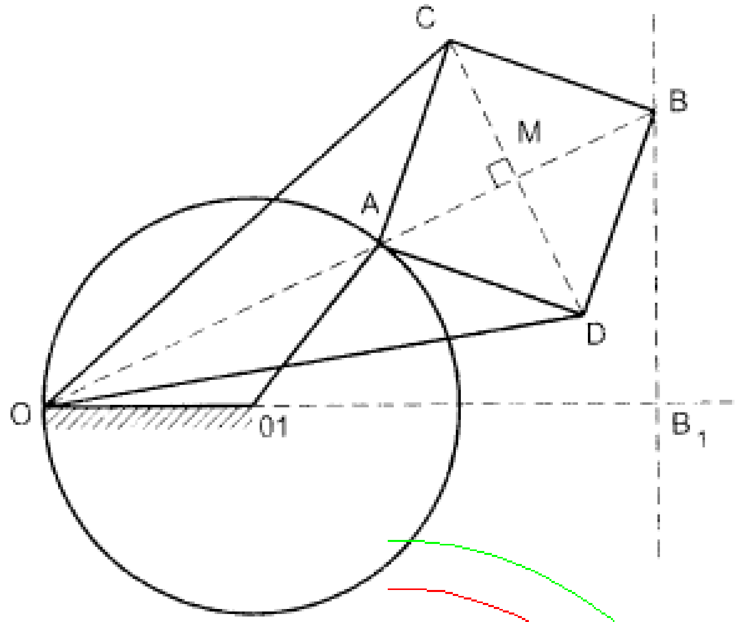


Straight Line Motion Mechanism

- Accurate Straight line Mechanism
  - Peaucellier Mechanism
  - Hart’s Mechanism
  - Scott Russel Mechanism
- Approximate Straight line Mechanism
  - Watt’s Mechanism
  - Grasshoper Mechanism
  - Robert's Mechanism

Accurate Straight line Mechanism

Peaucellier Mechanism



Here $AD = BD = BC = AC$

and $OC = OD$

and $OO_1 = O_1A$

in tringle OMC $\quad OC^2 = OM^2 + CM^2$

$$\therefore CM^2 = OC^2 - OM^2$$

in tringle AMC $\quad AC^2 = AM^2 + CM^2$

$$\therefore CM^2 = AC^2 - AM^2$$

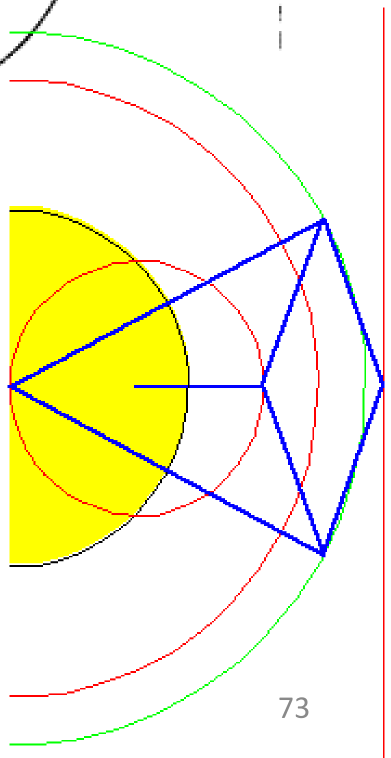
Compare eq. (i) & (ii)

$$OC^2 - OM^2 = AC^2 - AM^2$$

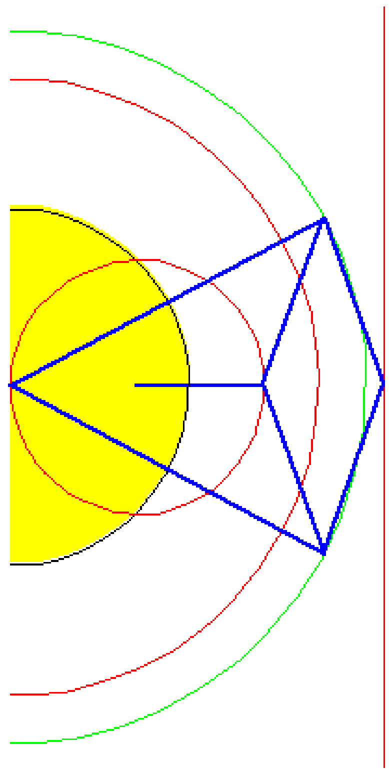
$$OC^2 - AC^2 = OM^2 - AM^2$$

$$= (OM - AM)(OM + AM)$$

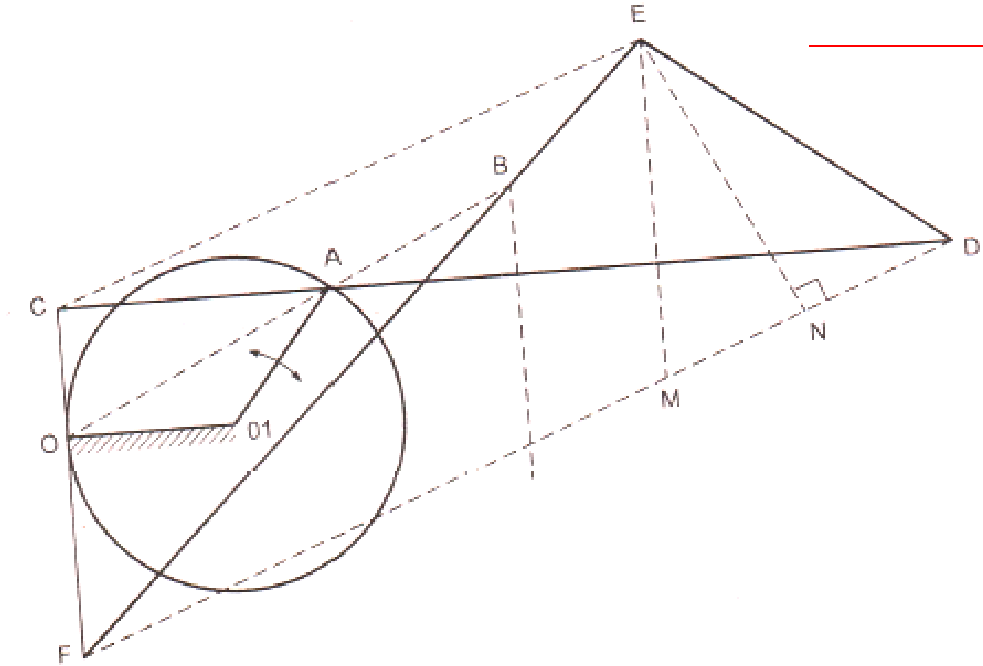
$$= OA \cdot OB \, [\text{as } AM = BM]$$

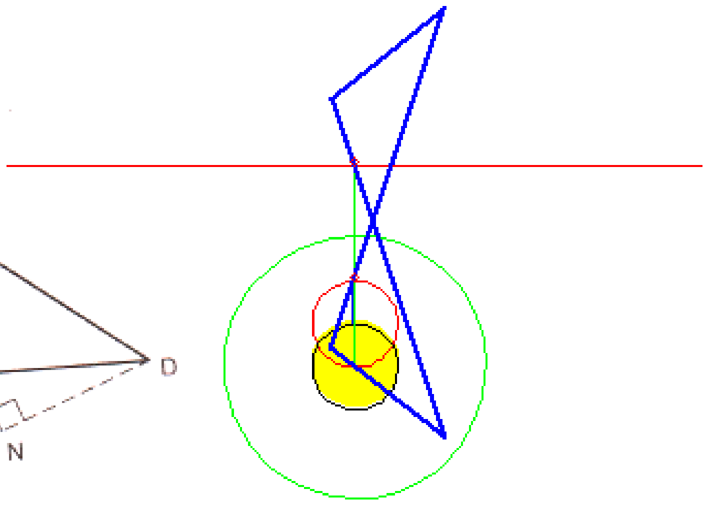


Peaucellier Mechanism



Hart’s Mechanism





In ΔFCE, point O and B divide the link CF and FE in same ratio so

$$\frac{CO}{CF} = \frac{EB}{EF}$$

First consider triangler CFE and OFB

in which $\frac{CE}{FC} = \frac{OB}{OF} \Rightarrow OB = \frac{CE \times OF}{FC}$ ....(i)

Hart's Mechanism

Now consider triangle FCD and OCA

in which $\frac{FD}{FC} = \frac{OA}{OC} \Rightarrow OA = \frac{FD \times OC}{FC}$ ....(ii)

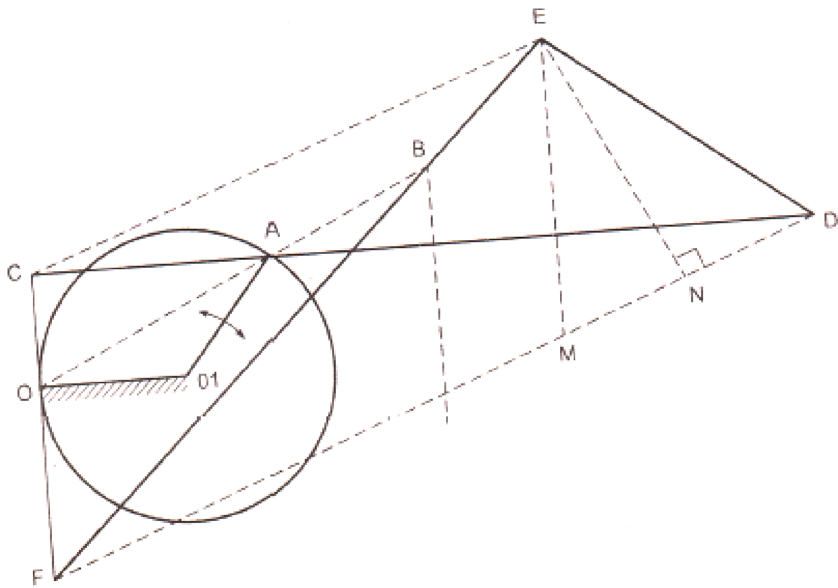
From eq. (i) and (ii)

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

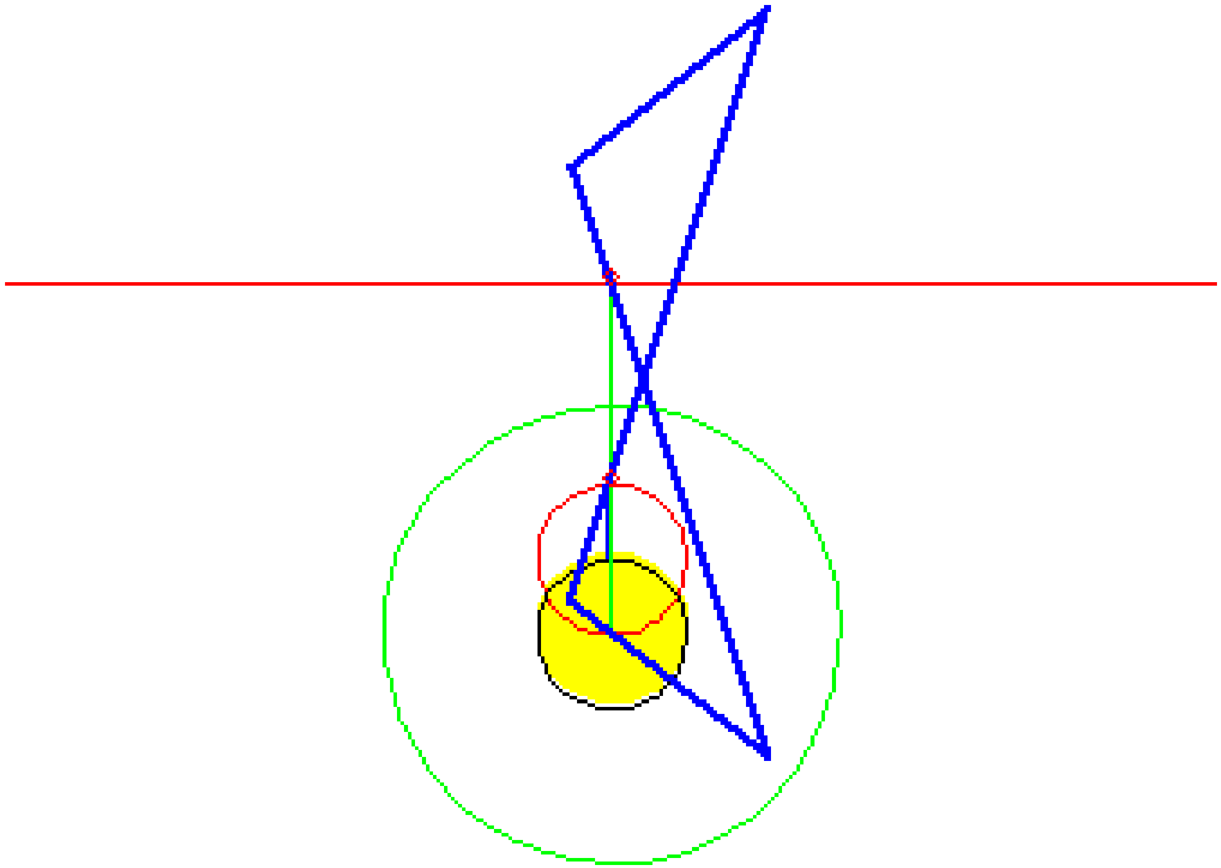
$$OA \times OB = FD \times CE \times \frac{OC \times OF}{FC^2}$$ [As OC, OF and FC have constant length....]

$$= FD \times CE \times \text{constant}$$

$$FD \times CE = FD \times FM$$
$$= (FN + ND)(FN - MN)$$
$$= (FN + ND)(FN - ND) \quad \text{as } MN = ND$$
$$= FN^2 - ND^2$$
$$= (FE^2 - EN^2) - (ED^2 - EN^2)$$
$$= FE^2 - ED^2$$
$$= \text{Constant}$$

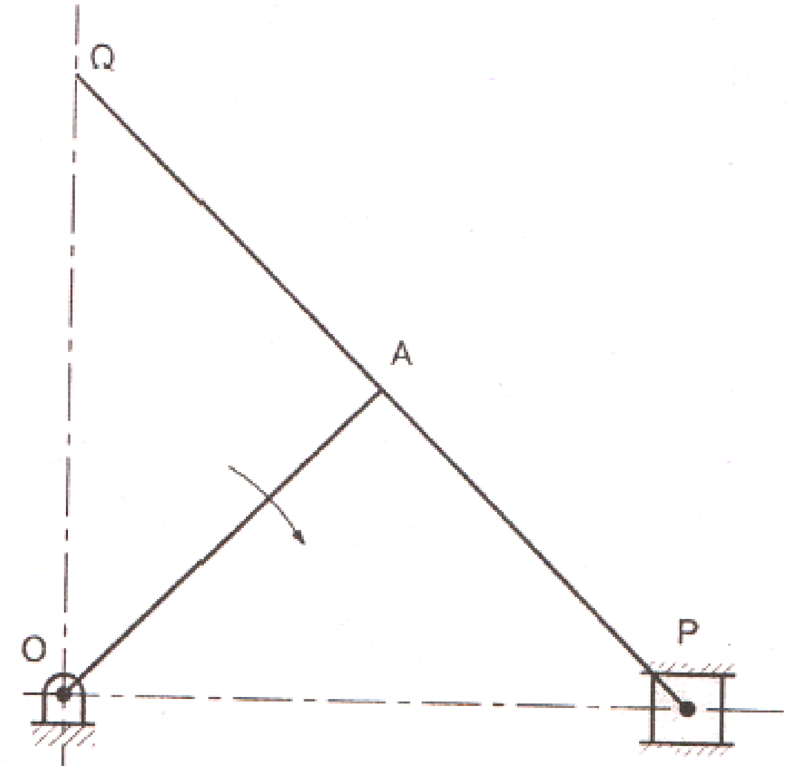


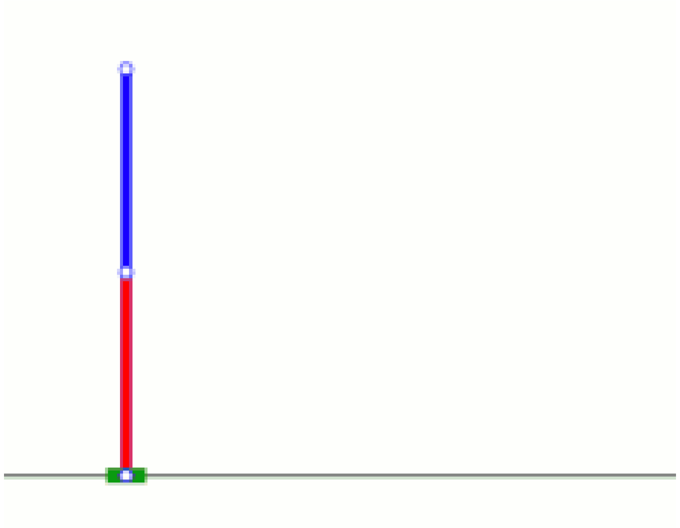
Hart's Mechanism



Scott Russel Mechanism

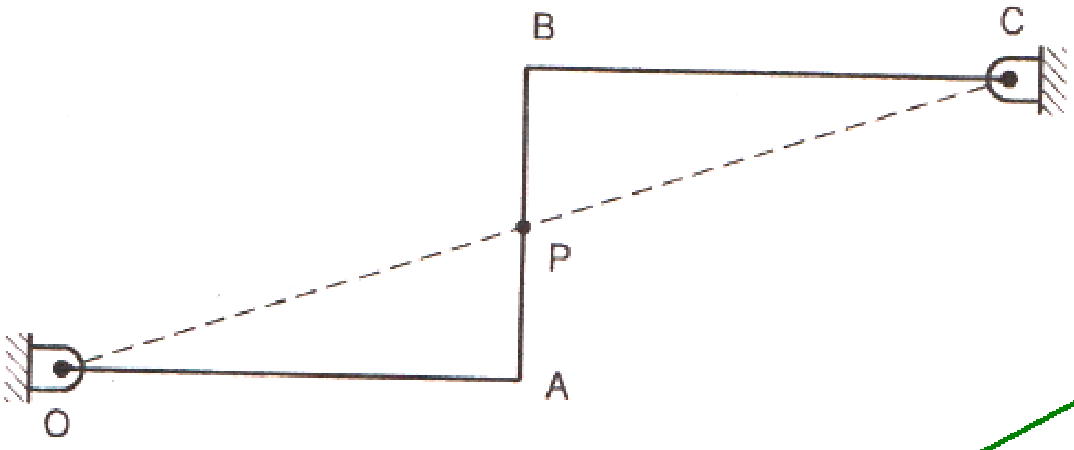
- 2 links
- AQ = AP
- OQ = 4 x OA

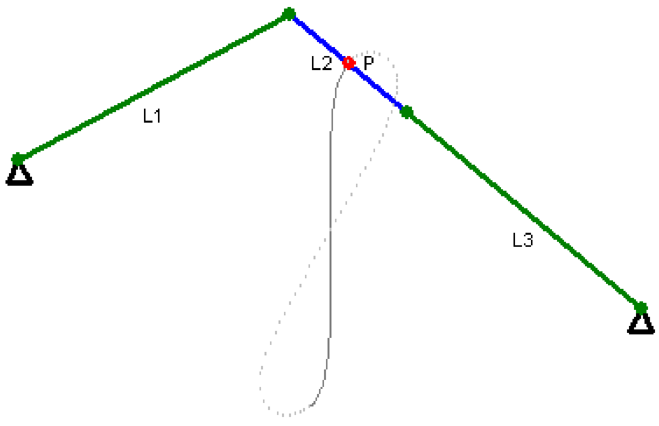




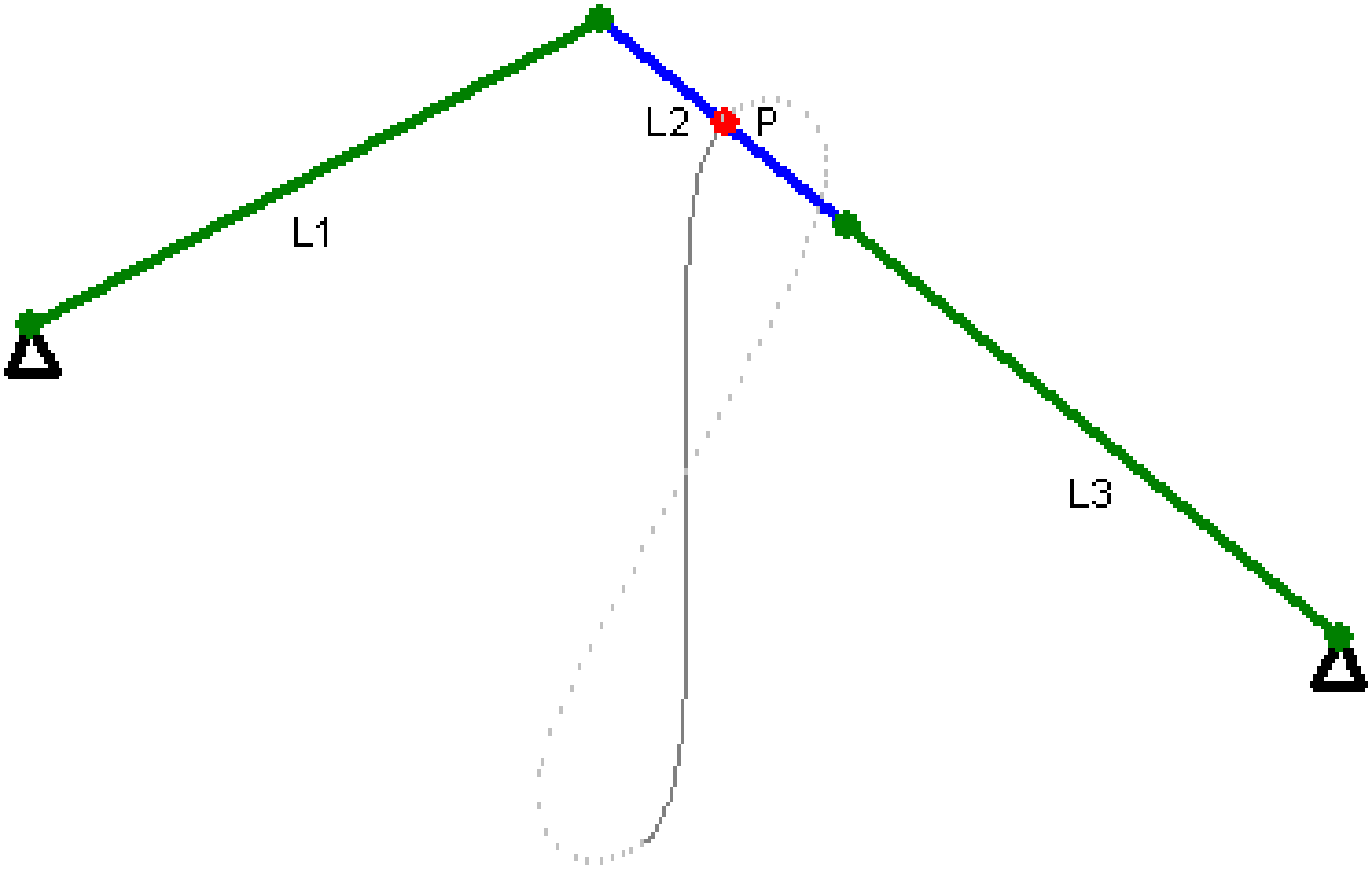
Approximate Straight line Mechanism

Watt’s Mechanism

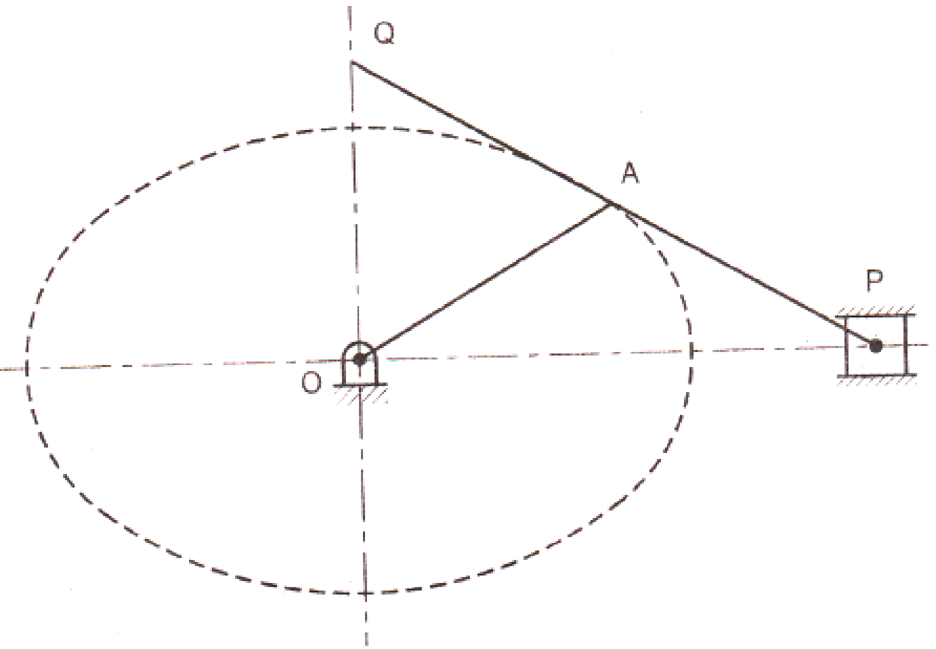




Watt's Mechanism

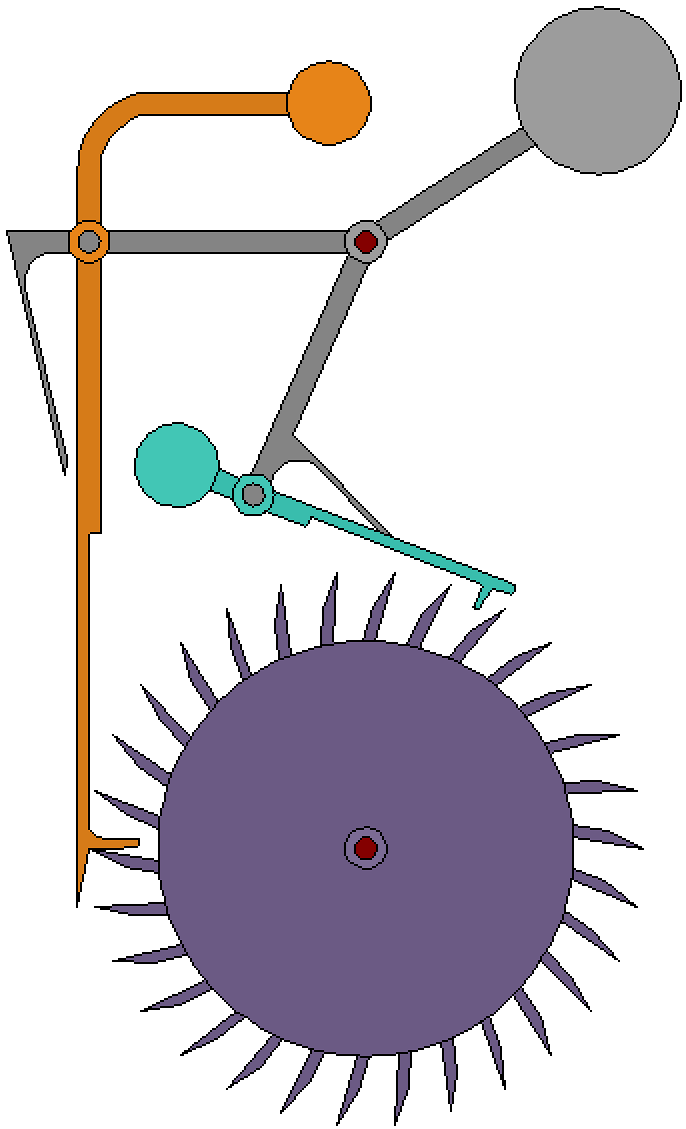


Modified Scott Russel Mechanism

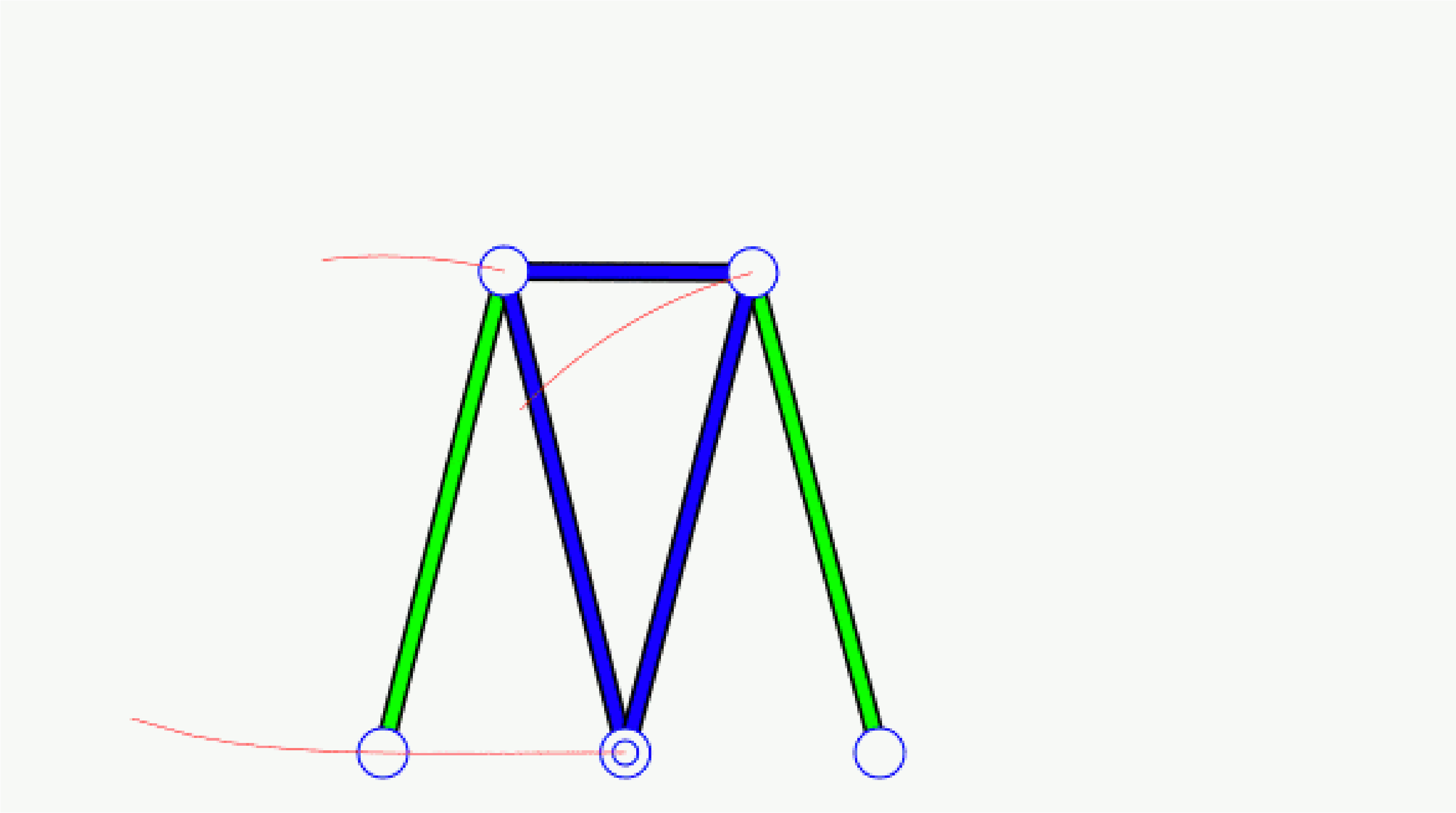


- 2 links
- AQ is not equal to AP
- AQ = ½ x major axis of ellipse
- AP = ½ x minor axis of ellipse

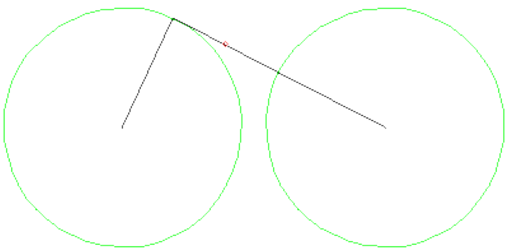
Grasshoper Mechanism

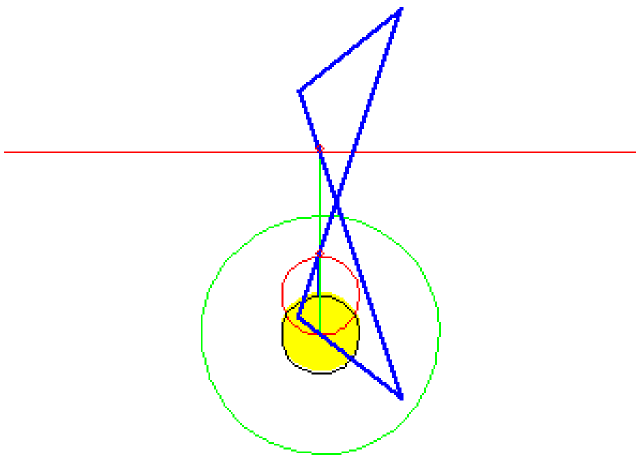


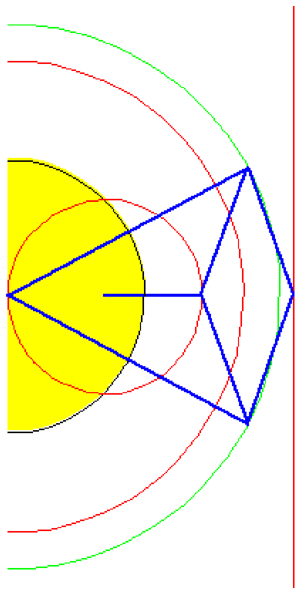
Robert's Mechanism

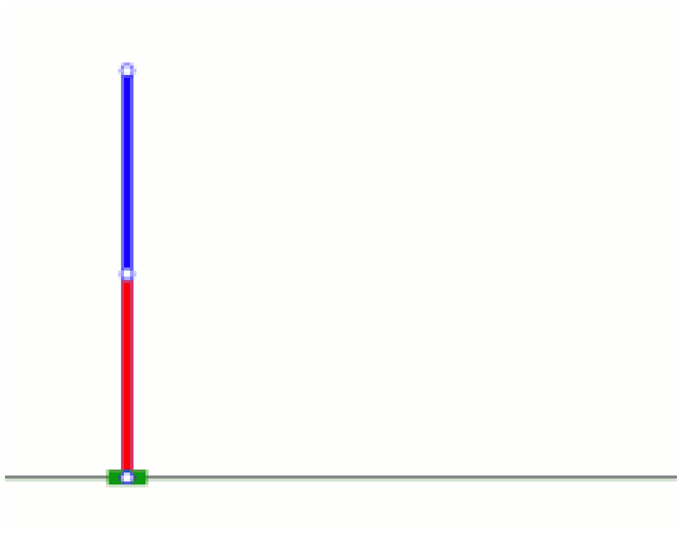


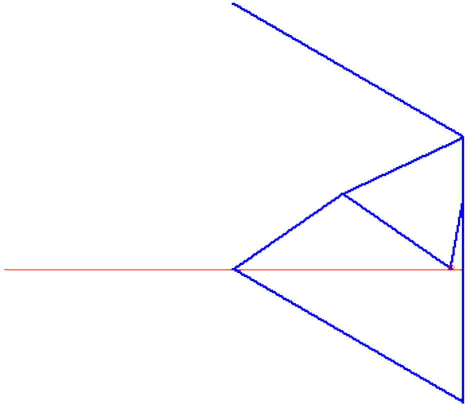
Enjoy Mechanism





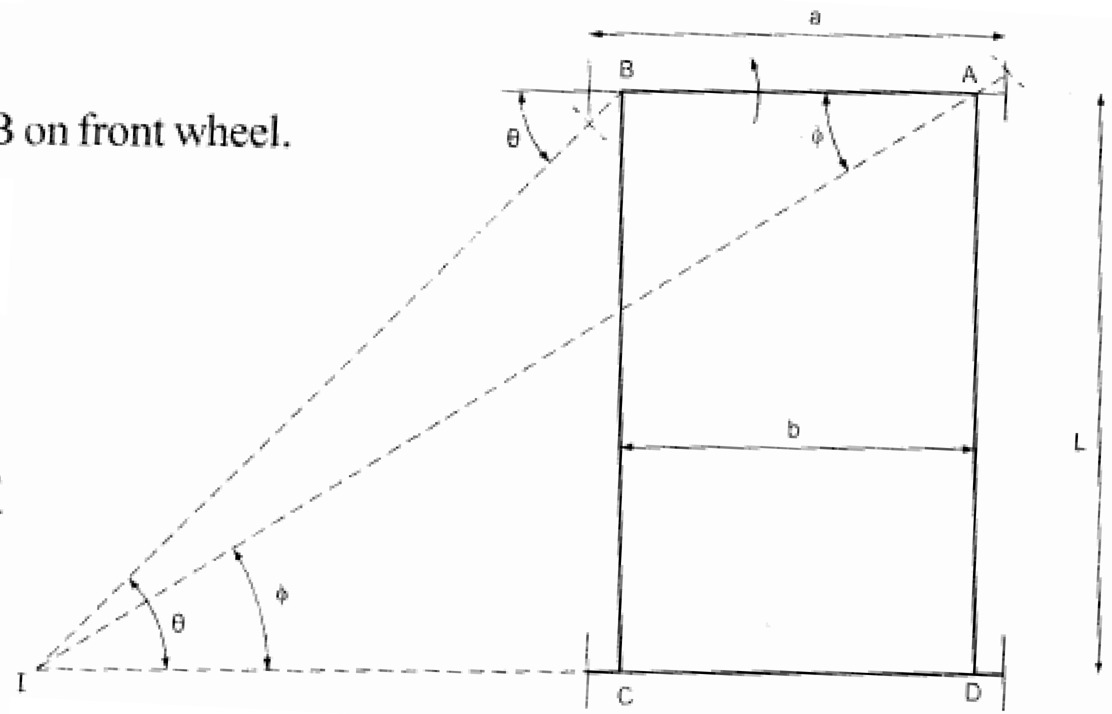






Automobile steering Gear Mechanism

L = Wheel base
a = Distance between two wheels.
b = Distance between two pivots A & B on front wheel.



For inner wheel; ΔIBC : $\cot\theta = \frac{IC}{BC} = \frac{IC}{L}$

For outer wheel ΔIAD : $\cot\phi = \frac{ID}{AD} = \frac{ID}{L}$

$$\cot\phi - \cot\theta = \frac{ID}{L} - \frac{IC}{L} = \frac{1}{L}(ID - IC) = \frac{b}{L}$$

$$\cot\phi - \cot\theta = \frac{b}{L}$$

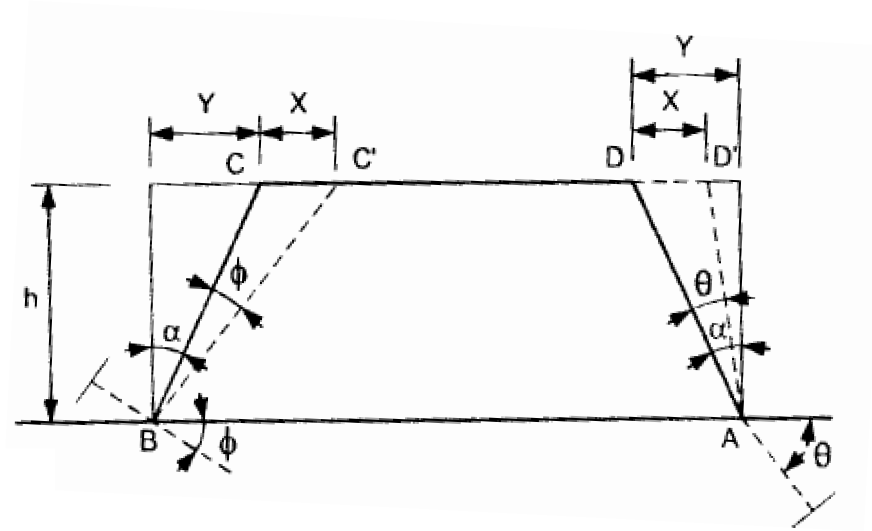
Automobile steering Gear Mechanism

1. Devis Steering Gear Mechanism
2. Ackermann Steering Gear Mechanism

Devis Steering Gear Mechanism

$$\tan \propto = \frac{y}{h}; \quad \tan(\propto + \phi) = \frac{y+x}{h}$$

$$\tan(\propto + \phi) = \frac{\tan \propto + \tan \phi}{1 - \tan \propto . \tan \phi}$$



$$\frac{y+x}{h} = \frac{\frac{y}{h} + \tan \phi}{1 - \frac{y}{h} \tan \phi} \qquad \left[\text{as } \tan \propto = \frac{y}{h}\right]$$

$$(y+x)(h - y \tan \phi) = h(y + h \tan \phi)$$

$$y.h - y^2 \tan \phi + x.h - x.y \tan \phi = h.y + h^2 \tan \phi$$

$$\tan \phi = \frac{x.h}{h^2 + y^2 + xy} \qquad \therefore \cot \phi = \frac{h^2 + y^2 + xy}{x.h}$$

Devis Steering Gear Mechanism

Now, $\tan(\propto - \theta) = \dfrac{y - x}{h}$

$$\tan(\propto - \theta) = \frac{\tan \propto - \tan\theta}{1 + \tan \propto . \tan\theta}$$

and same way $\cot\theta = \dfrac{h^2 + y^2 - xy}{x.h}$

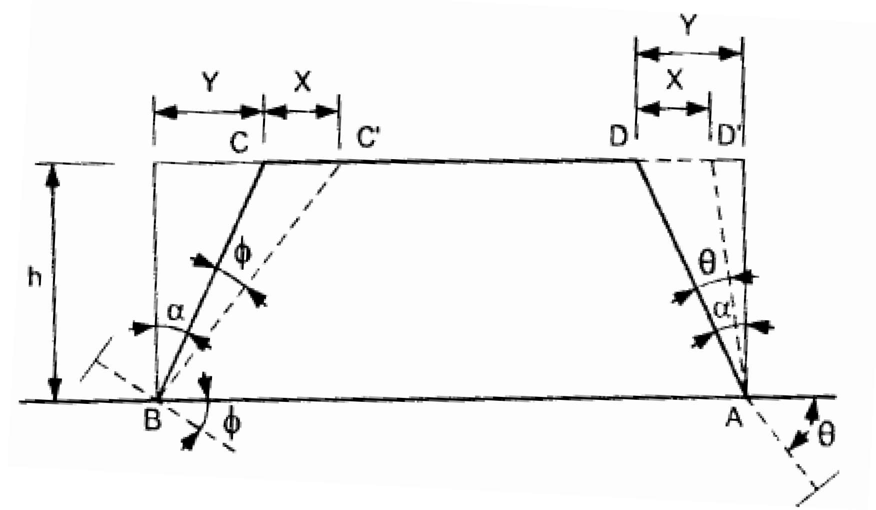
for correct steering

$$\cot\phi - \cot\theta = \frac{b}{L}$$

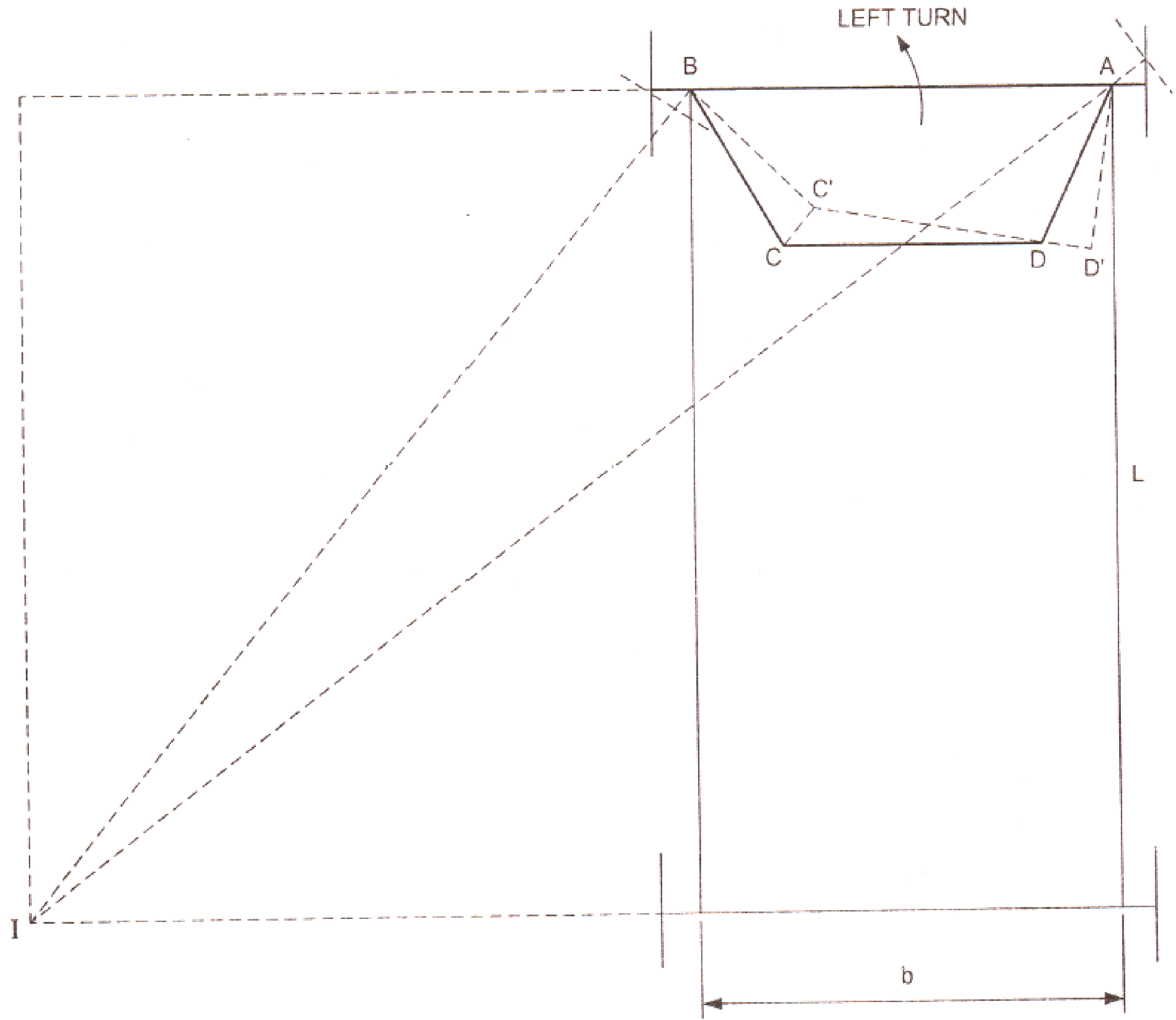
$$\frac{h^2 + y^2 + xy}{x.h} - \frac{h^2 + y^2 - xy}{x.h} = \frac{b}{L}$$

$$\frac{2xy}{xh} = \frac{b}{L} \Rightarrow \frac{2y}{h} = \frac{b}{L}$$

$$\therefore 2\tan \propto = \frac{b}{L} \Rightarrow \tan \propto = \frac{b}{2L}$$



Ackermann Steering Gear Mechanism



Difference Between Two Steering Gear Mechanism

The construction of devis mechanism is on the front side of the front axle where as ackermann mechanism is at back side of the front axle.

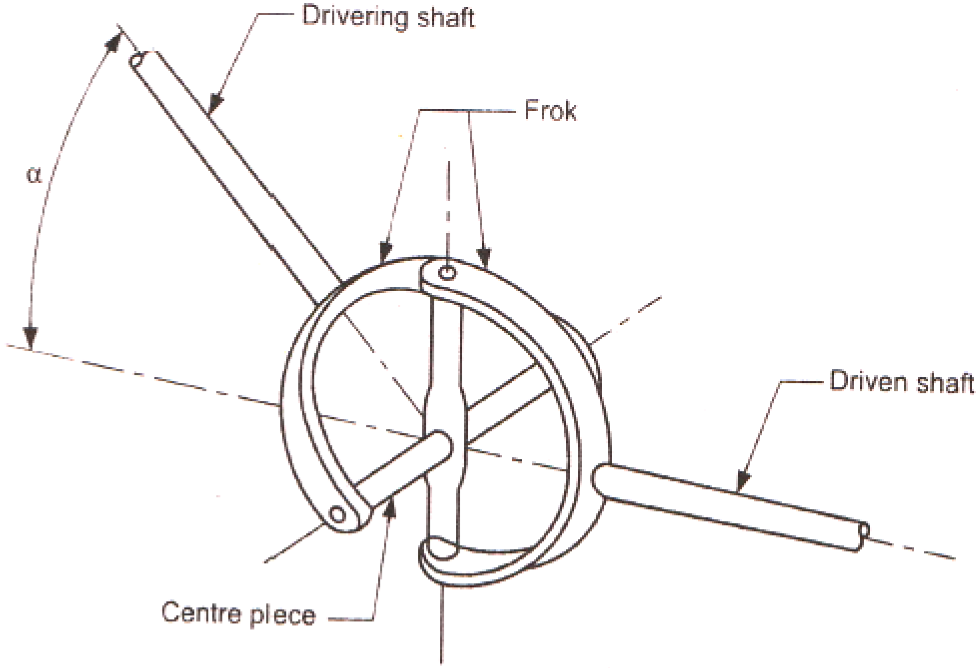
The Devis steering gear mechanism consist four turning pair and two sliding pair where Ackerman steering gear mechanism consists four turning pairs.

In Devis steering mechanism link AB is a fixed link. Link BC and AD are of same length and connected to both the wheels pivoted at B and A.

When the vehicle moves on a straight path the link AB and CD are parallel to each other and link BC and AD are equally inclined to link AB.

Universal Coupling or Hook’s Joint

Universal Coupling or Hook's Joint



Ratio of Velocities

in triangle OC_1M; $\tan\theta = \dfrac{OM}{MC_1}$

In triangle OC_2N $\tan\phi = \dfrac{ON}{NC_2}$

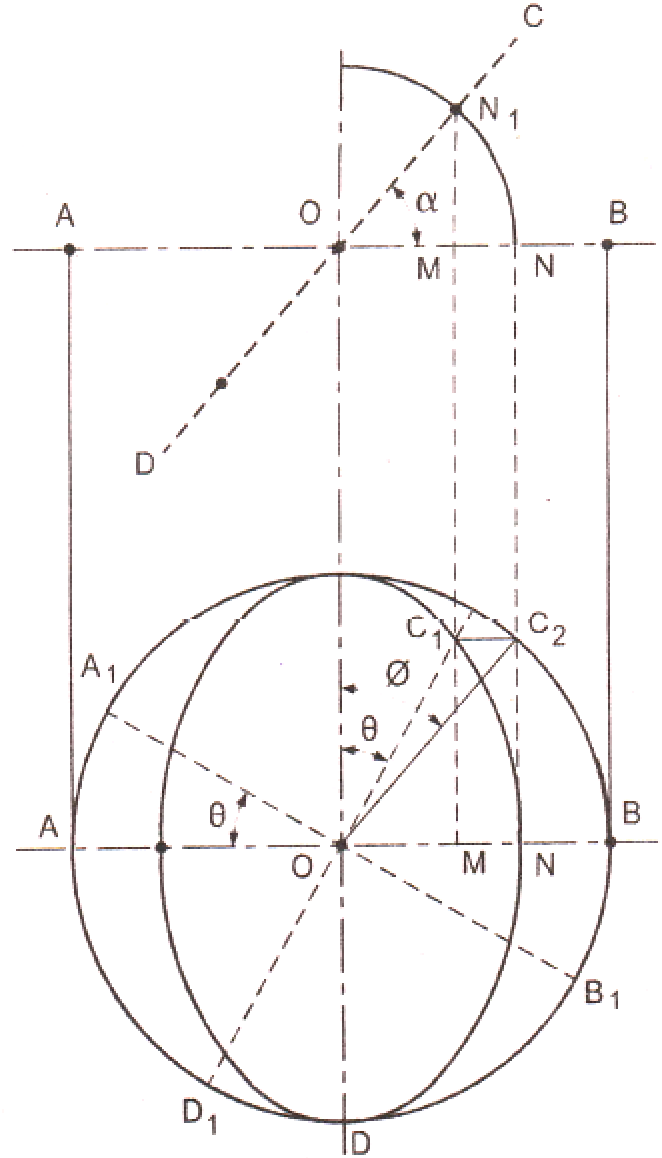
$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{NC_2}{ON} = \frac{OM}{ON}$$

[as $MC_1 = NC_2$]

$$OM = ON_1 COS \propto = ON \, Cos \propto$$

$$\frac{\tan\theta}{\tan\phi} = \frac{ON \, Cos \propto}{ON} = Cos \propto$$

$$\tan\theta = \cos\propto . \tan\phi$$



Ratio of Velocities

θ is the angle turned by driving shaft; then $\frac{d\theta}{dt} = \omega_1$

ϕ is the angle turned by output shaft then $\frac{d\phi}{dt} = \omega_2$

differentiating equaion (iii).

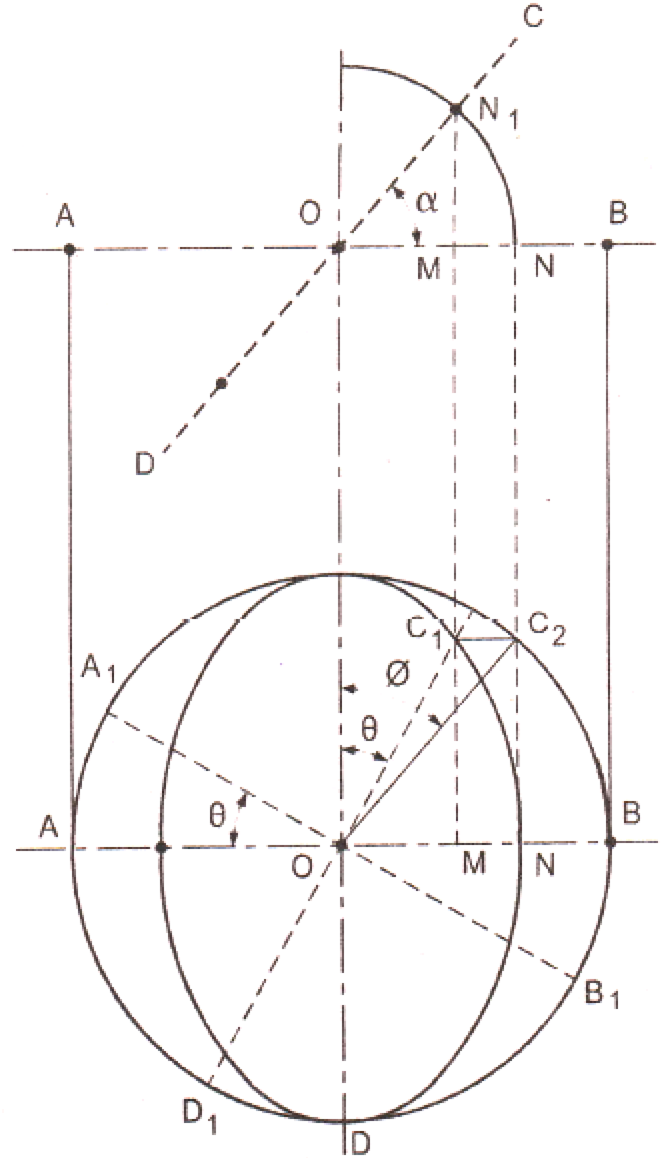
$$\sec^2\theta \frac{d\theta}{dt} = \cos\propto . \sec^2\phi . \frac{d\phi}{dt}$$

$$\omega_1 \sec^2\theta = \omega_2 . \cos\propto . \sec^2\phi$$

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta}{\cos\propto . \sec^2\phi} = \frac{1}{\cos\propto . \cos^2\theta . \sec^2\phi}$$

$= \omega_1 =$ angular velocity of driving shaft.

$\omega_2 =$ angular velocity of driven shaft.



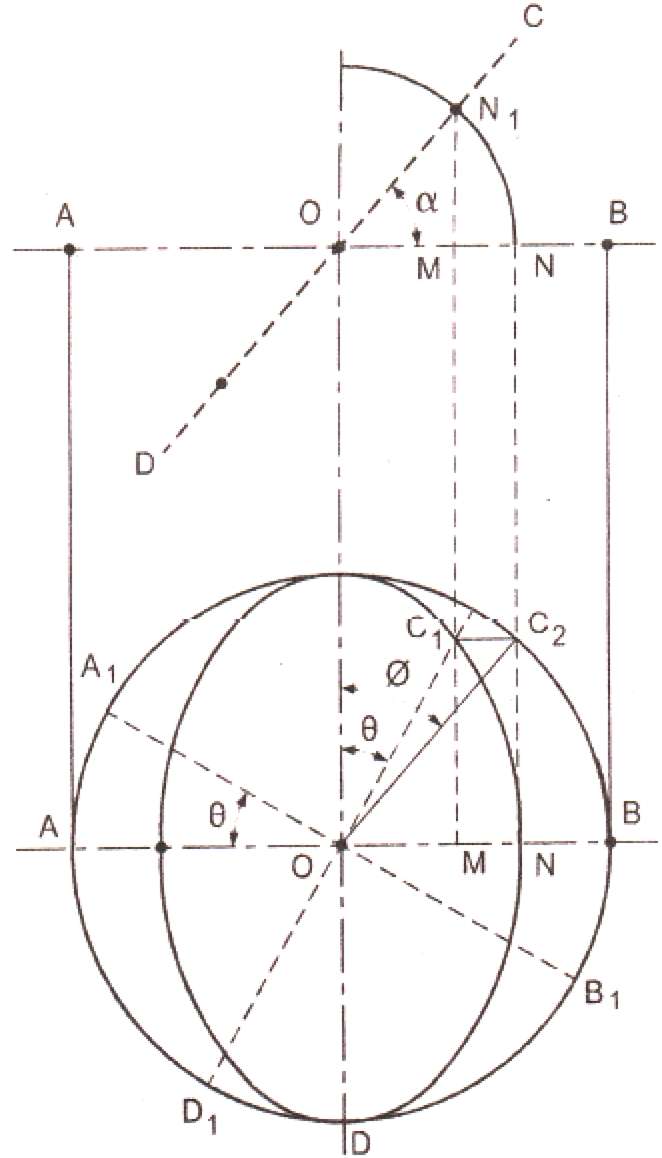
Ratio of Velocities

differentiating equaion (iii).

$$\sec^2\theta \frac{d\theta}{dt} = \cos\propto .\sec^2\phi.\frac{d\phi}{dt}$$

$$\omega_1 \sec^2\theta = \omega_2.\cos\propto .\sec^2\phi$$

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta}{\cos\propto .\sec^2\phi} = \frac{1}{\cos\propto .\cos^2\theta.\sec^2\phi}$$



Ratio of Velocities

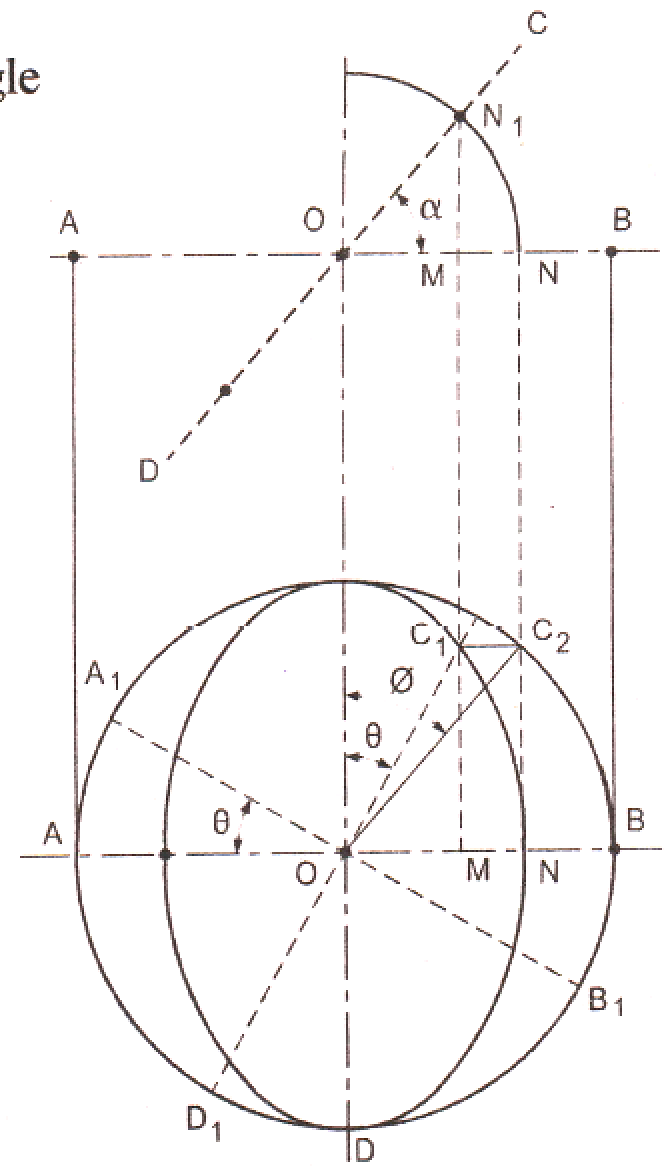
Now convert the driven shaft angle ϕ in form of driving shaft angle

$$\sec^2\phi = 1+\tan^2\phi = 1+\frac{\tan^2\theta}{\cos^2\alpha} \qquad \text{[From eq.(iii)]}$$

$$= 1+\frac{\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta\cos^2\alpha+\sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{\cos^2\theta(1-\sin^2\alpha)+\sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{\cos^2\theta-\cos^2\theta\sin^2\alpha+\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{1-\cos^2\theta\sin^2\alpha}{\cos^2\theta.\cos^2\alpha}$$

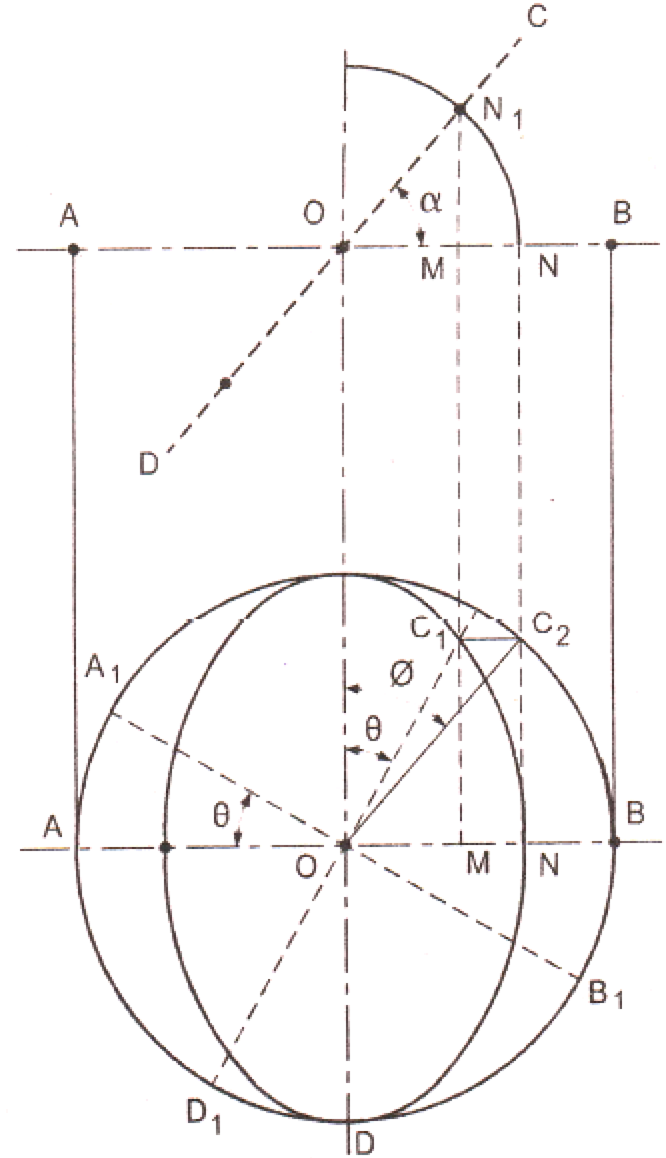


Ratio of Velocities

Put the value of $\sec^2\phi$ in equation

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta \times \cos^2\theta \times \cos^2\alpha}{\cos\alpha\left(1-\cos^2\theta.\sin^2\alpha\right)}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} = \frac{N_2}{N_1}$$



Maximum and Minimum speed of driven shaft

Put the value of $\sec^2\phi$ in equation

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta \times \cos^2\theta \times \cos^2 \propto}{\cos \propto \left(1-\cos^2\theta.\sin^2 \propto\right)}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos \propto}{1-\cos^2\theta.\sin^2 \propto} = \frac{N_2}{N_1}$$

When denominator $\left(1-\cos^2\theta\sin^2 \propto\right)$ is minimum $\frac{\omega_2}{\omega_1}$ is maximum. For that $\cos^2\theta$ must be maximum

When $\theta = 0^o, 180^o, 360^o$ ....$\cos^2\theta$ is max $= 1$; Now put $\theta = 0$ in eq (v)

$$\omega_{2\max} = \frac{\omega_1 \cos \propto}{1-\sin^2 \propto} = \frac{\omega_1 \cos \propto}{\cos^2 \propto} = \frac{\omega_1}{\cos \propto}$$

Maximum and Minimum speed of driven shaft

Put the value of $\sec^2\phi$ in equation

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta \times \cos^2\theta \times \cos^2\propto}{\cos\propto\left(1-\cos^2\theta.\sin^2\propto\right)}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\propto}{1-\cos^2\theta.\sin^2\propto} = \frac{N_2}{N_1}$$

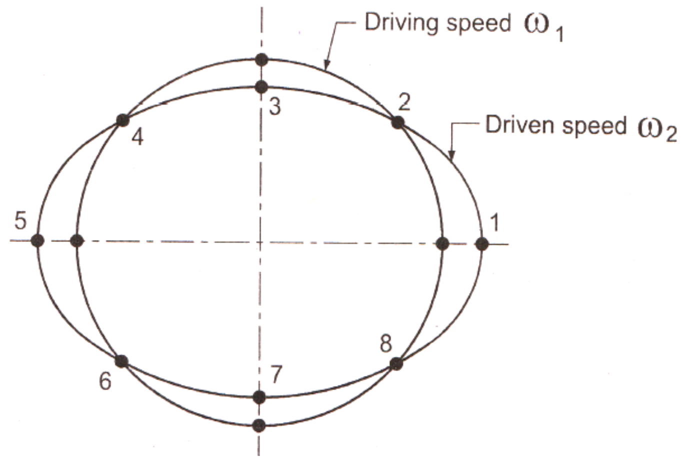
Similary when denominator $\left(1-\cos^2\theta\sin^2\propto\right)$ is maximum $\frac{\omega_2}{\omega_1}$ is minimum. For that $\cos^2\theta$ must be minimum.

When $\theta = 90^\circ, 270^\circ, \ldots\ldots\cos^2\theta$ is min $= 0$; Now put $\theta = 90^\circ$ in eq (v)

$$\omega_{2min} = \frac{\omega_1.\cos\propto}{1}$$

$$\omega_{2max} = \frac{\omega_1}{\cos\propto} \quad \& \quad \omega_{2min} = \omega_1\cos\propto$$

Polar Diagram of Shaft Velocities



In hooke's joint the angular velocity of driving shaft (ω_1) is constant through out the rotation, which shown by a circle in fig.

The velocity of driven shaft (ω_2) is vary during rotation. At 0^0 and 180^0 (Points 1 & 5), it is maximum and more than ω_1. Whereas at 90^0 and 270^0 (Points 3 & 7), it is minimum and less than ω_1. The angular velocity behaviour of output shaft is shown by ellipse in figure

The point 2,4,6,8 in figure shows the location where both the velocities are equal.

Condition for Equal Speed

The ratio of angular velocity of shaft is written as

$$\frac{\omega_2}{\omega_1} = \frac{\cos \propto}{1 - \cos^2\theta \sin^2 \propto}$$

for equal speed of both shaft $\omega_2 = \omega_1$

$$\therefore \cos \propto = 1 - \cos^2\theta \sin^2 \propto$$

$$\cos^2\theta = \frac{1 - \cos \propto}{\sin^2 \propto}$$

$$\text{Now } \sin^2\theta = 1 - \cos^2\theta = 1 - \frac{(1 - \cos \propto)}{\sin^2 \propto}$$

$$= 1 - \frac{(1 - \cos \propto)}{1 - \cos^2 \propto}$$

$$= 1 - \frac{(1 - \cos \propto)}{(1 - \cos \propto)(1 + \cos \propto)}$$

$$\sin^2\theta = \frac{1 + \cos \propto - 1}{1 + \cos \propto} = \frac{\cos \propto}{1 + \cos \propto}$$

Condition for Equal Speed

$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos\alpha}{1+\cos\alpha} \times \frac{\sin^2\alpha}{(1-\cos\alpha)}$$

$$\tan^2\theta = \frac{\cos\alpha.\sin^2\alpha}{1-\cos^2\alpha} = \frac{\cos\alpha.\sin^2\alpha}{\sin^2\alpha} = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha}$$

There are two positive and two negative values of θ where the angular velocities of driving shaft (ω_1) and driven shaft (ω_2) are same.

Angular Acceleration of the driven shaft

$$\text{Angular Acceleration } \propto = \frac{d\omega}{dt}$$

$$\text{Angular Acceleration of driven shaft } \propto = \frac{d\omega_2}{dt}$$

$$\propto = \frac{d}{dt}\left[\frac{\omega_1 . \cos \propto}{1 - \cos^2 \theta \sin^2 \propto}\right]$$

$$\propto = \frac{d}{dt}\left[\omega_1 . \cos \propto \left(1 - \cos^2 \theta \sin^2 \propto\right)^{-1}\right]$$

$$= \omega_1 \cos \propto \left[-1\left(1 - \cos^2 \theta \sin^2 \propto\right)^{-2} \times \left(2 \cos \theta \sin \theta \sin^2 \propto\right)\right] \frac{d\theta}{dt}$$

$$\propto = -\frac{\omega_1^2 \cos \propto . \sin 2\theta . \sin^2 \propto}{\left(1 - \cos^2 \theta \sin^2 \propto\right)^2} \qquad \left[\text{As } \frac{d\theta}{dt} = \omega_1\right]$$

For getting maximum acceleration differentiate $\propto$ with respect to θ and equate to zero. The condition for max acceleration is

$$\cos 2\theta = \frac{2 \sin^2 \propto}{2 - \sin^2 \propto}$$

Maximum Fluctuation

Fluctuation means the difference between maximum and minimum speed. It is expressed by 'q'.

$$q = (\omega_2)_{max} - (\omega_2)_{min}$$

$$= \frac{\omega_1}{\cos \propto} - \omega_1 \cos \propto$$

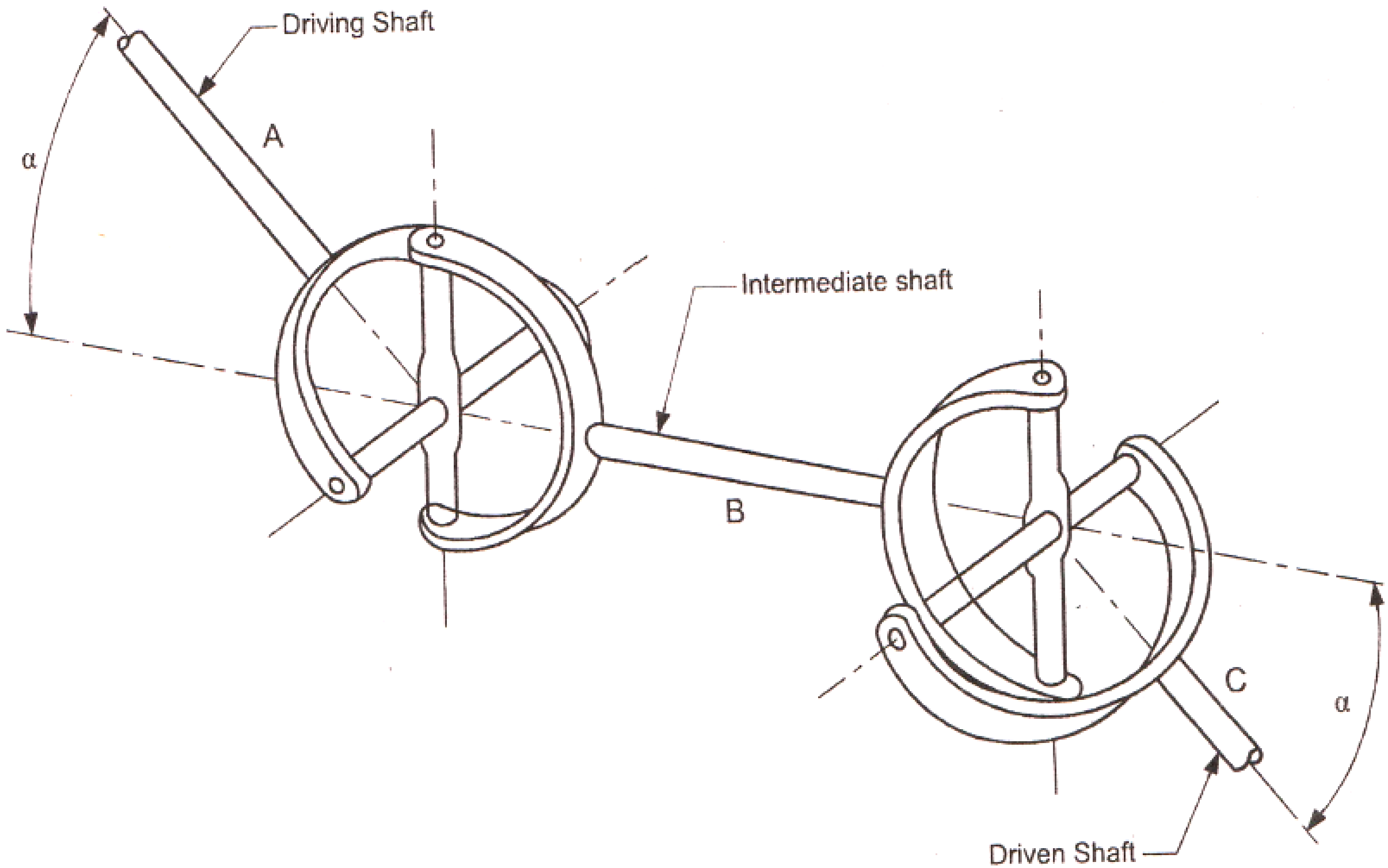
$$= \omega_1 \left(\frac{1}{\cos \propto} - \cos \propto \right)$$

$$= \omega_1 \left(\frac{1 - \cos^2 \propto}{\cos \propto} \right) = \frac{\omega_1 \sin^2 \propto}{\cos \propto}$$

$q = \omega_1 \tan \propto . \sin \propto$ (When $\propto$ is small $\cos \propto \cong 1$ and $\sin \propto = \propto$ radian)

so $q = \omega_1 . \propto^2$

Double Hook's Joint



Double Hook's Joint

In a single hooke's joint the angular velocity of driven shaft is not uniform throughout the rotation, which creates a vibration in system. To get uniform output speed a double hooke joint is preferred.

For getting uniform output speed the following conditions must be maintained. Axis of driving and driven shaft are in same plane and both shaft makes an equal angle with intermediate shaft.

For shaft A and B

$\tan\theta = \tan\phi . \cos\propto$

For shaft B and C

$\tan\gamma = \tan\phi , \cos\propto$

here if $\theta = \gamma$ then $\omega_1 = \omega_2$ i.e. the angular velocity of driven and driving shaft are same.

References

1. Shigley, J.E and Uicker, J.J, Theory of Machines and Mechanisms, Oxford
2. Robert L. Norton, Design of Machinery, The McGraw-Hill Companies.
3. Rattan S.S., Theory of Machines Tata McGraw-Hill Publishing Co. Ltd.
4. R. S. Khurmi, Theory of Machines, S. Chand.
5. http://www.technologystudent.com

Thanks