Mathematical representation of solids

Chapter 3

Dr M B Vaghela

Department of Mechanical Engineering

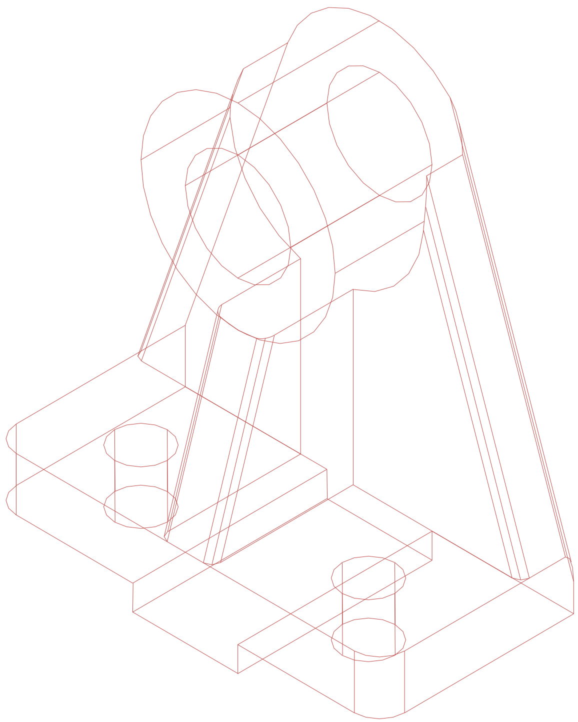
L E College, Morbi

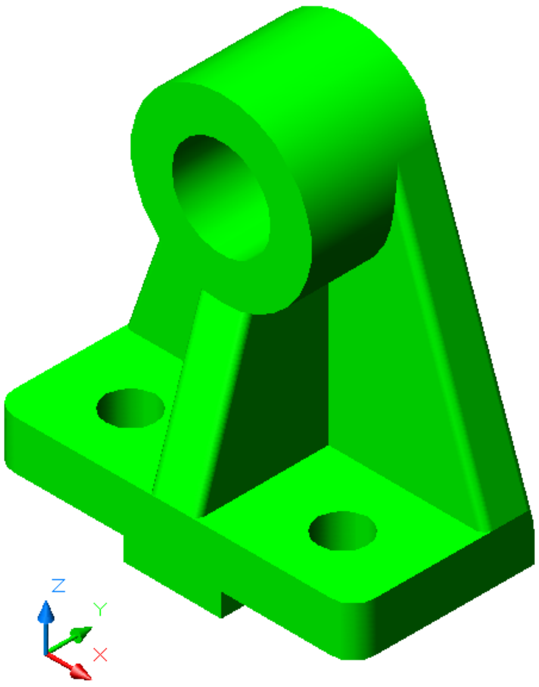
mbvaghela@lecollege.ac.in

Solid Modeling

Solid Model

Solid models give designers a complete descriptions of constructs, shape, surface, volume, and density.





Solid model

- Solid modeling is based on *complete, valid and unambiguous* geometric representation of physical object.
  - Complete → points in space can be classified.(inside/ outside)
  - Valid →vertices, edges, faces are connected properly.
  - Unambiguous → there can only be one interpretation of object

Solid model

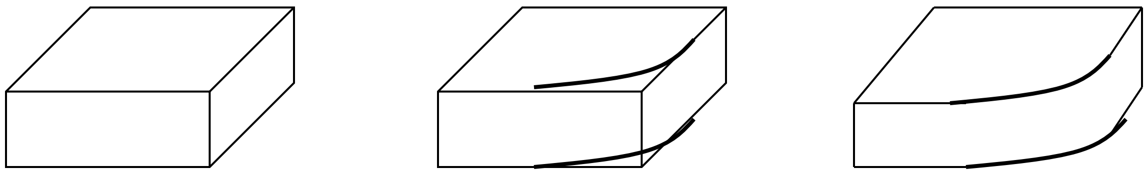
- Analysis automation and integration is possible only with solid models→ has properties such as weight, moment of inertia, mass.
- Solid model consist of geometric and topological data
  - Geometry → shape, size, location of geometric elements
  - Topology →connectivity and associativity of geometric elements →non graphical, relational information

Types of Solid Models

- Constructive Solid Geometry (CSG)
- Boundary Representation (B-Rep)
- Feature Based Modeling (FBM)

Solid object construction method

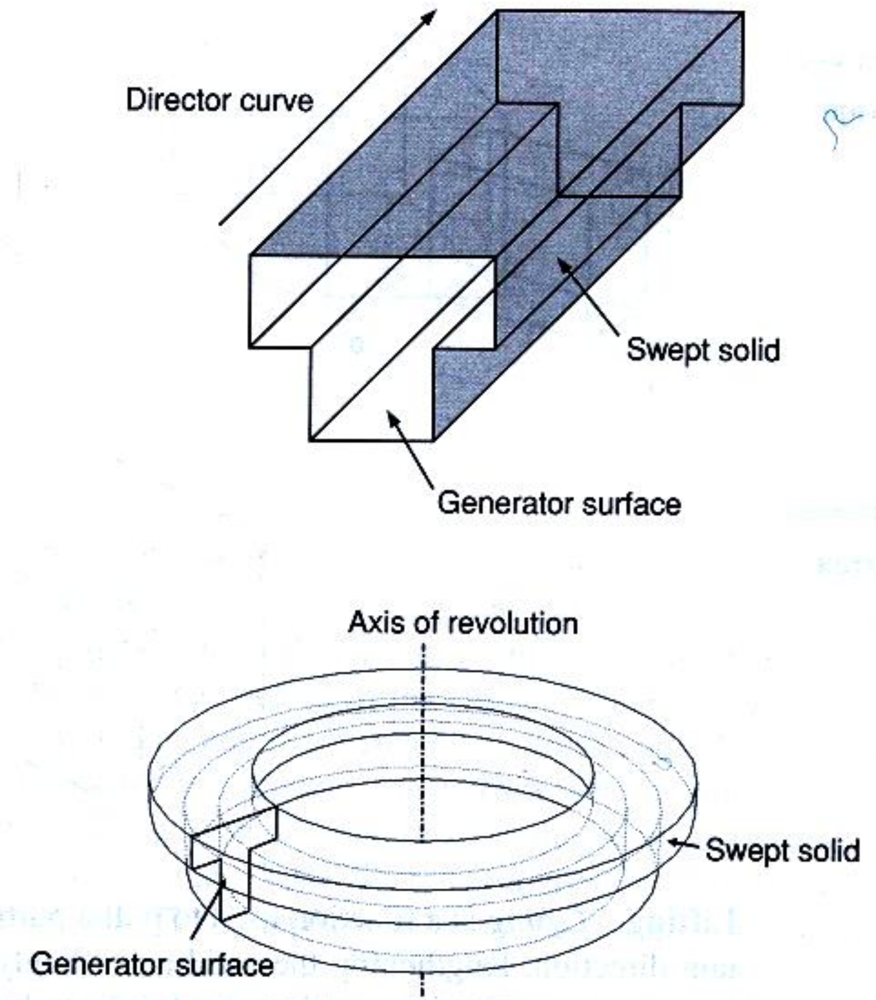
- Sweeping
- Boolean
- Automated filleting and chambering



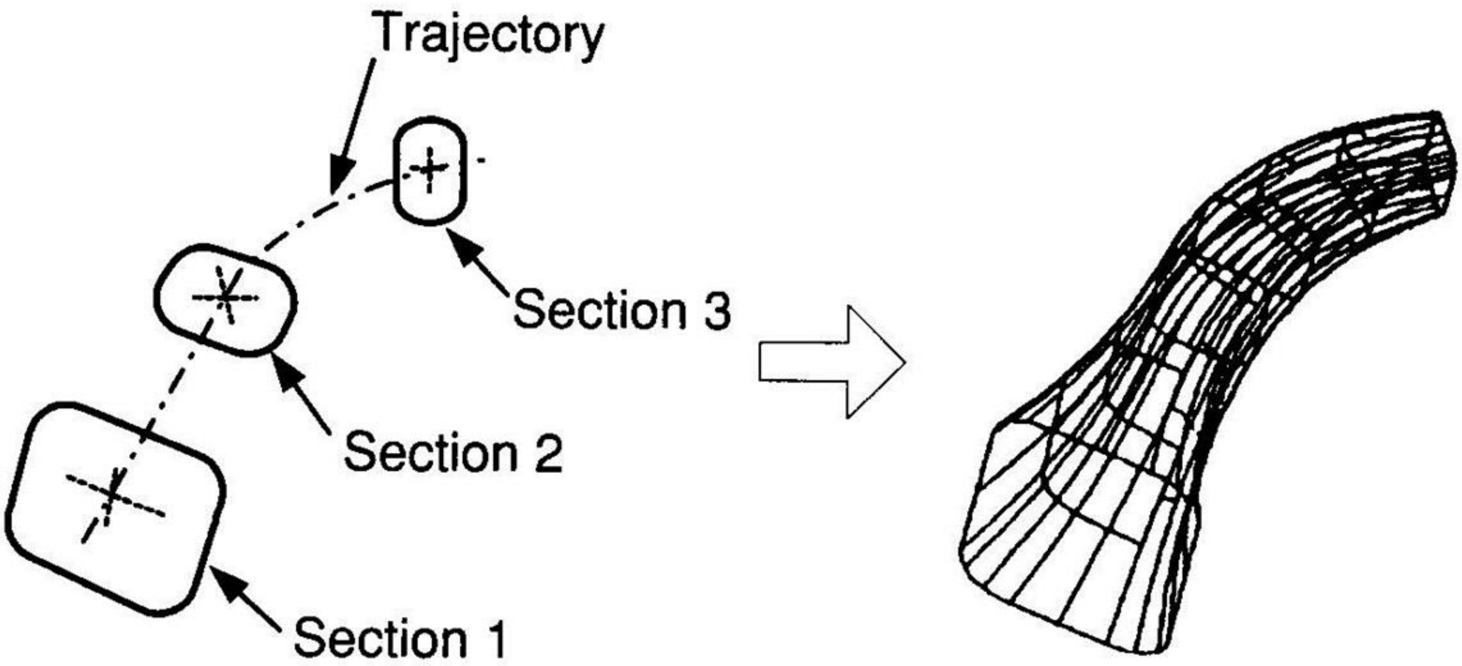
- Tweaking
  - Face of an object is moved in some way

Sweeping

Sweeping *Sweeping* is a modeling function in which a planar closed domain is translated or revolved to form a solid. When the planar domain is translated, the modeling activity is called *translational sweeping;* when the planar region is revolved, it is called *swinging, or rotational sweeping*.

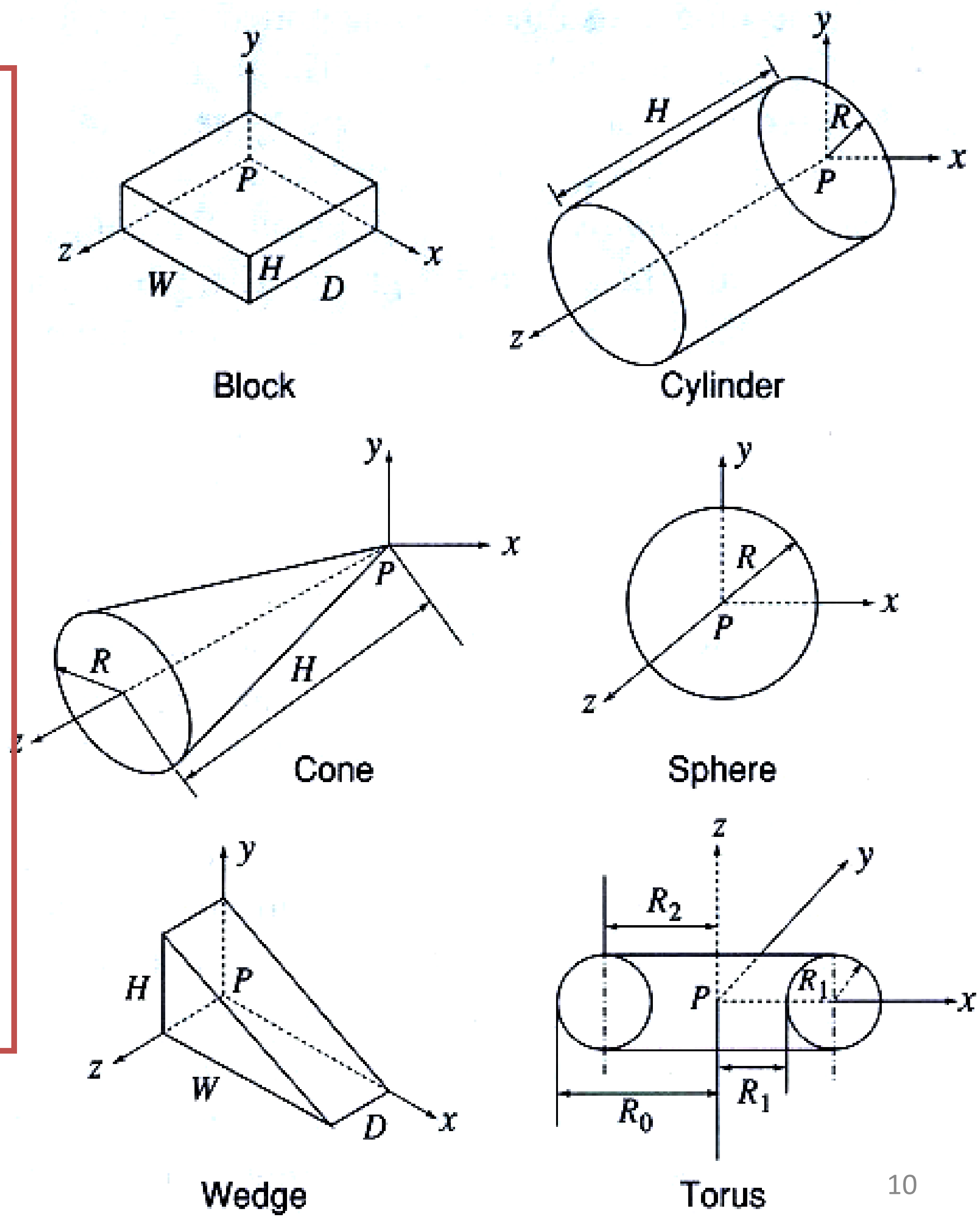


skinning operation

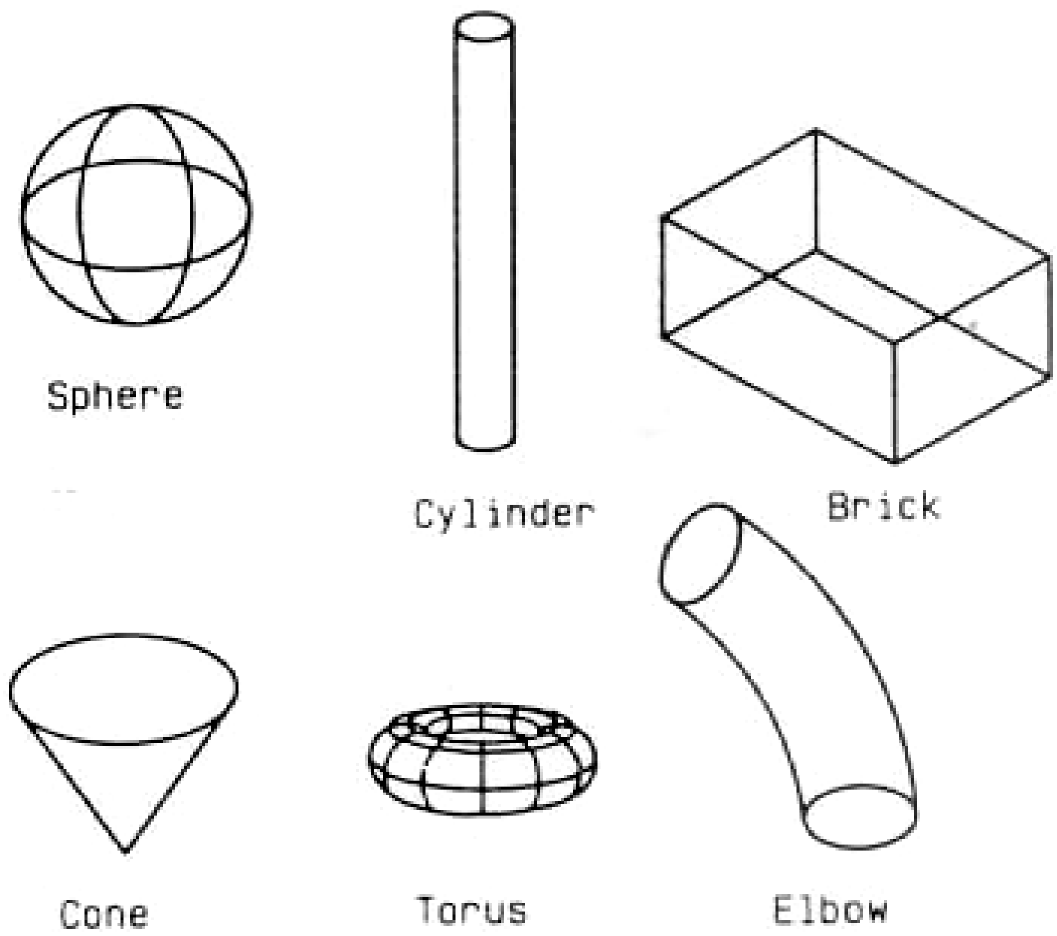


Primitive creation functions:

These functions retrieve a solid of a simple shape from among the primitive solids stored in the program in advance and create a solid of the same shape but of the size specified by the user.



Primitives in CSG



Features of CSG Solid Models

- CSG is the first solid rep. scheme developed at University of Rochester.
- In CSG, a complex object is constructed from simple shapes such as box, cylinder etc. using Boolean operations.
- The CSG model of an object is represented in the form of a binary tree *called CSG tree* whose leaves are the primitives and the branches are the Boolean operators.

Constructive solid geometry (CSG)

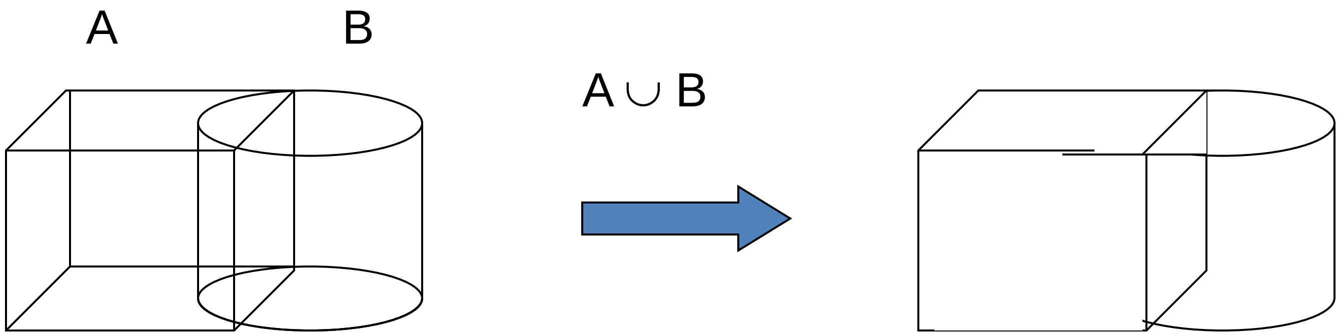
- Objects are represented as a combination of simpler solid objects (*primitives*).
- The primitives are such as cube, cylinder, cone, torus, sphere etc.
- Copies or “instances” of these primitive shapes are created and positioned.
- A complete solid model is constructed by combining these “instances” using set specific, logic operations (Boolean)

Constructive solid geometry (CSG)

- Boolean operation
  - each primitive solid is assumed to be a set of points, a boolean operation is performed on point sets and the result is a solid model.
  - Boolean operation → union, intersection and difference
  - The relative location and orientation of the two primitives have to be defined before the boolean operation can be performed.
  - Boolean operation can be applied to two solids other than the primitives.

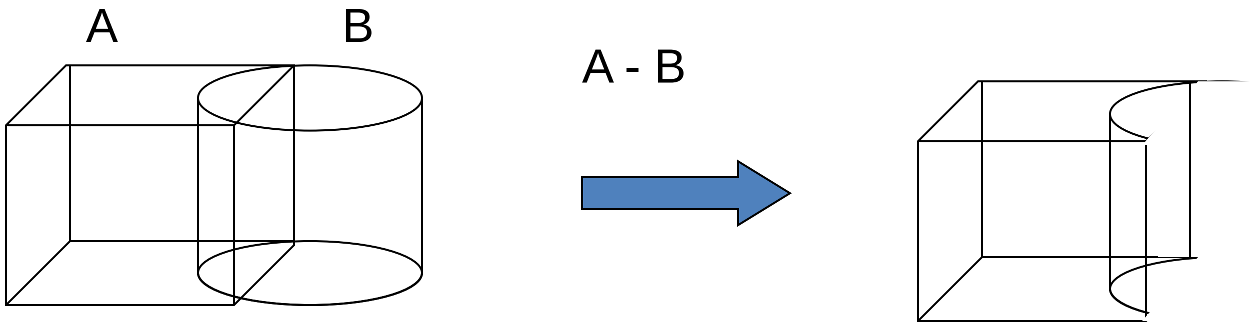
Constructive solid geometry (CSG)- Boolean operation

- Union
  - The sum of all points in each of two defined sets. (logical “OR”)
  - Also referred to as Add, Combine, Join, Merge



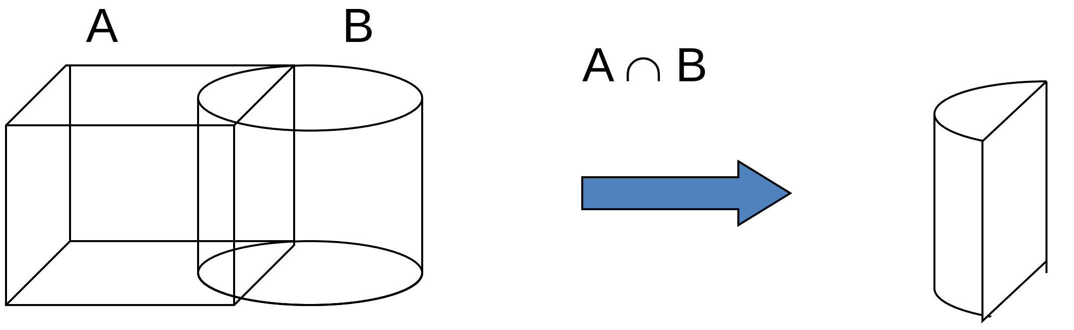
Constructive solid geometry (CSG)- Boolean operation

- Difference
  - The points in a source set minus the points common to a second set. (logical “NOT”)
  - Set must share common volume
  - Also referred to as subtraction, remove, cut



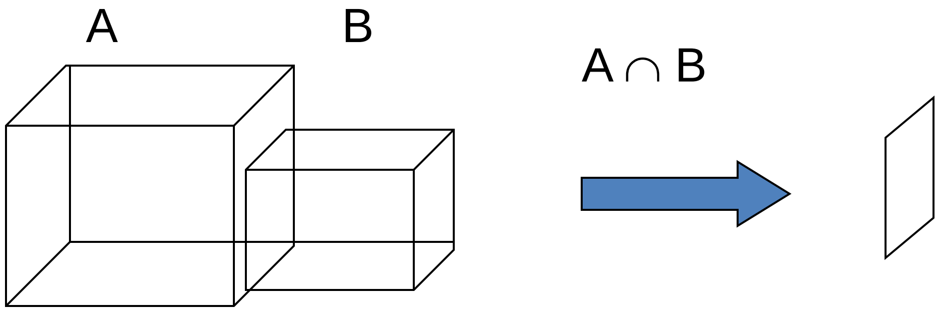
Constructive solid geometry (CSG)- Boolean operation

- intersection
  - Those points common to each of two defined sets (logical “AND”)
  - Set must share common volume
  - Also referred to as common, conjoin



Constructive solid geometry (CSG)- Boolean operation

- When using boolean operation, be careful to avoid situation that do not result in a valid solid



Constructive solid geometry (CSG)- Boolean operation

- Boolean operation
  - Are intuitive to user
  - Are easy to use and understand
  - Provide for the rapid manipulation of large amounts of data.
- Because of this, many non-CSG systems also use Boolean operations

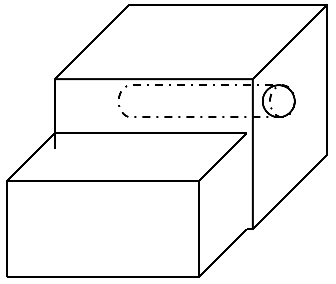
Constructive solid geometry (CSG)-data structure

- Data structure does not define model shape explicitly but rather implies the geometric shape through a procedural description
  - E.g: object is not defined as a set of edges & faces but by the instruction : *union primitive1 with primitive 2*
- This procedural data is stored in a data structure referred to as a CSG tree
- The data structure is simple and stores compact data → easy to manage

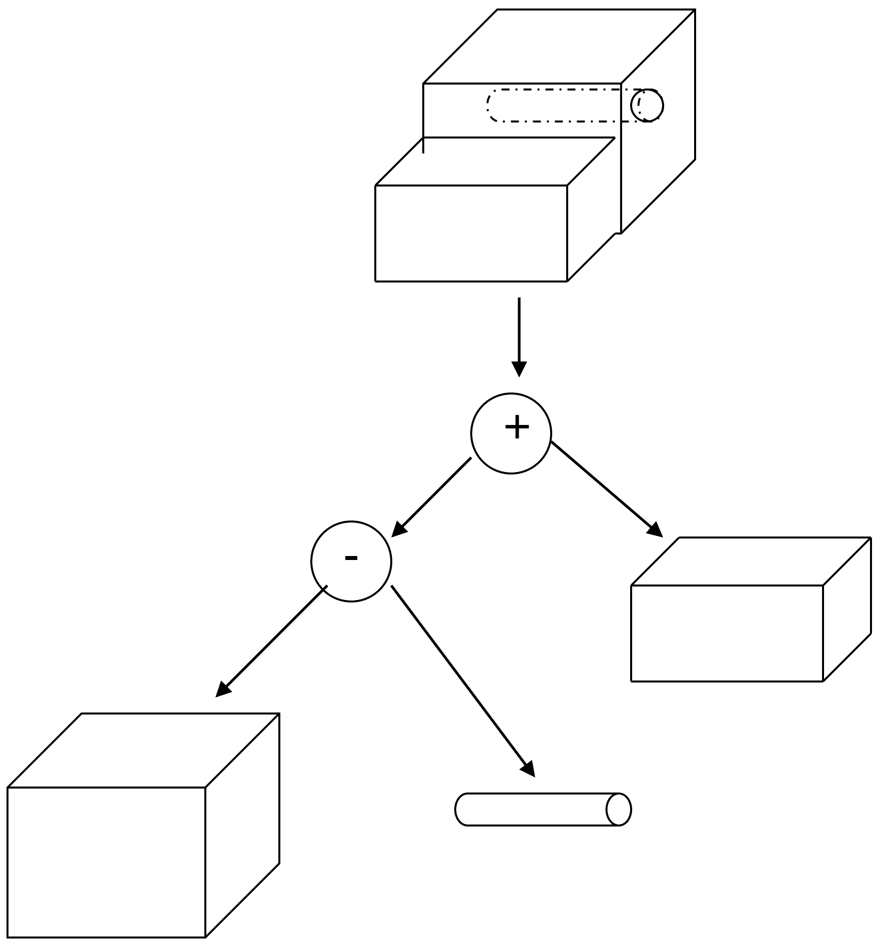
Constructive solid geometry (CSG)- CSG tree

- CSG tree → stores the history of applying boolean operations on the primitives.
  - Stores in a binary tree format
  - The outer leaf nodes of tree represent the primitives
  - The interior nodes represent the boolean operations performed.

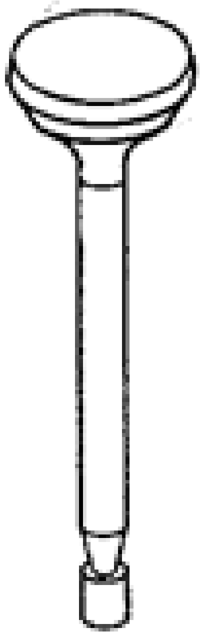
Constructive solid geometry (CSG)-CSG tree



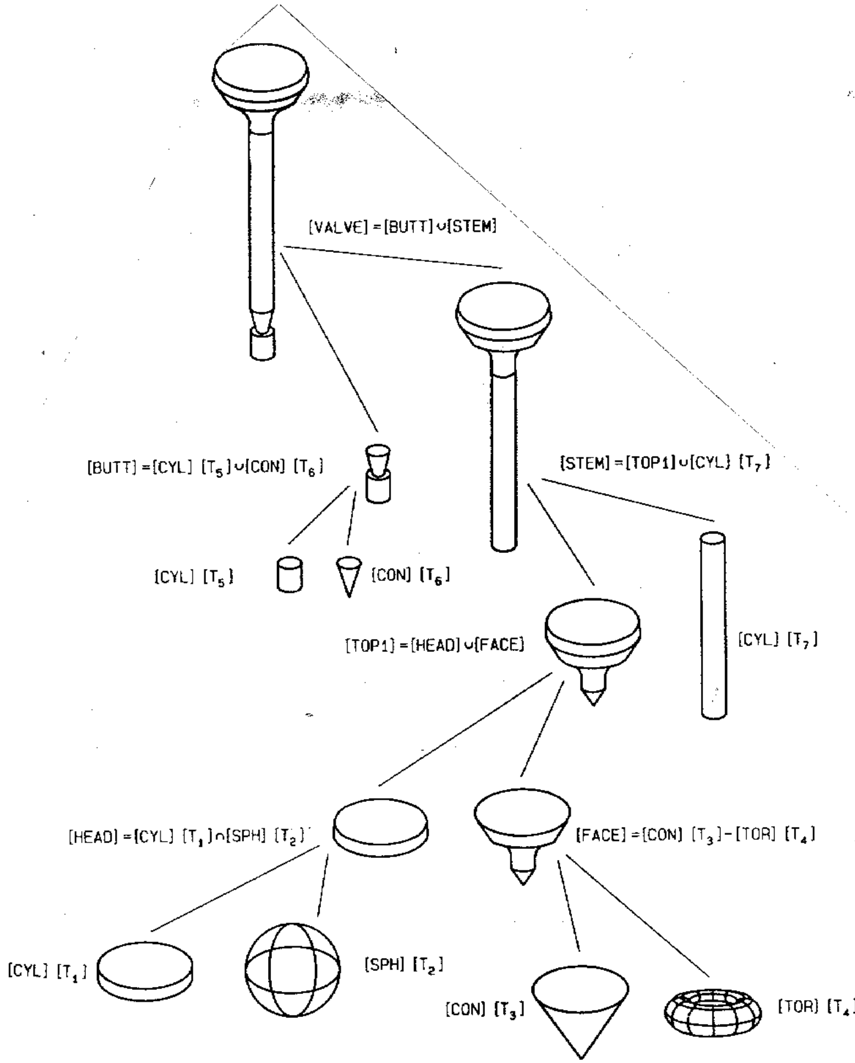
Constructive solid geometry (CSG)- CSG tree



CSG Tree of Tappet Valve

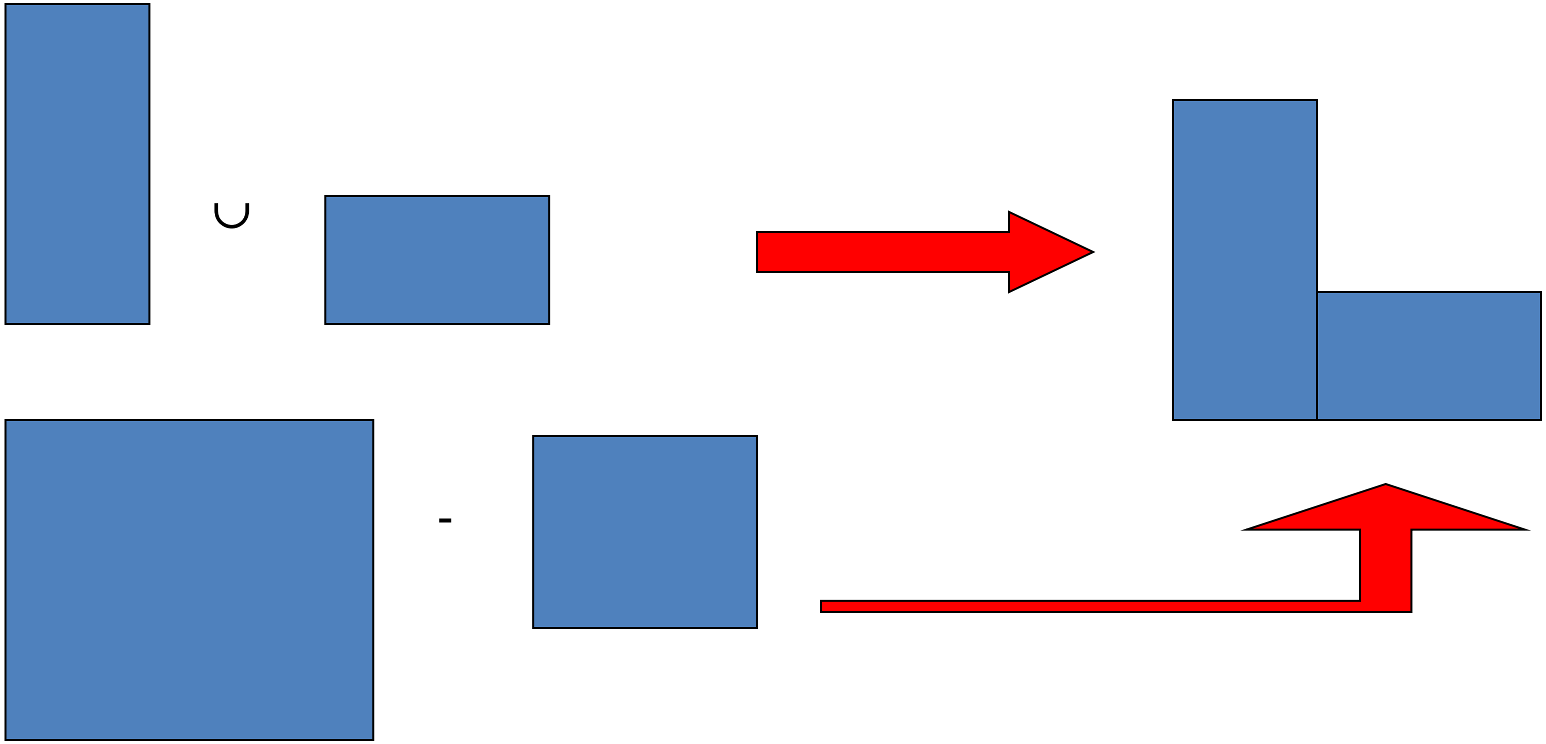


CSG Tree of Tappet Valve



Constructive solid geometry (CSG)- not unique

- More than one procedure (and hence database) can be used to arrive at the same geometry.



Constructive solid geometry (CSG) representation

- CSG representation is unevaluated
  - Faces, edges, vertices not defined in explicit
- CSG model are always valid
  - Since built from solid elements.
- CSG models are complete and unambiguous

CSG - advantage

- Simple to understand and use.
- Low memory requirements.
- CSG is powerful with high level command.
- Easy to construct a solid model – minimum step.
- CSG modeling techniques lead to a concise database→ less storage.
  - Complete history of model is retained and can be altered at any point.
- Can be converted to the corresponding boundary representation.

CSG - disadvantage

- Only boolean operations are allowed in the modeling process → with boolean operation alone, the range of shapes to be modeled is severely restricted → not possible to construct unusual shape.
- Requires a great deal of computation to derive the information on the boundary, faces and edges which is important for the interactive display/ manipulation of solid.

solution

- CSG representation tends to accompany the corresponding boundary representation → *hybrid representation*
- Maintaining consistency between the two representations is very important.

Features of B-Rep Models

- B-Rep. is a very powerful solid rep. scheme using the concept of half spaces.
- In B-Rep., a complex object is constructed out of its constituent surfaces.
- The B-Rep. model of an object is generally represented in the form of winged data structure which has some amount of data redundancy to enhance speed.

Boundary representation (B-Rep)

- Solid model is defined by their enclosing surfaces or boundaries. This technique consists of the geometric information about the faces, edges and vertices of an object with the topological data on how these are connected.

Boundary representation (B-Rep)

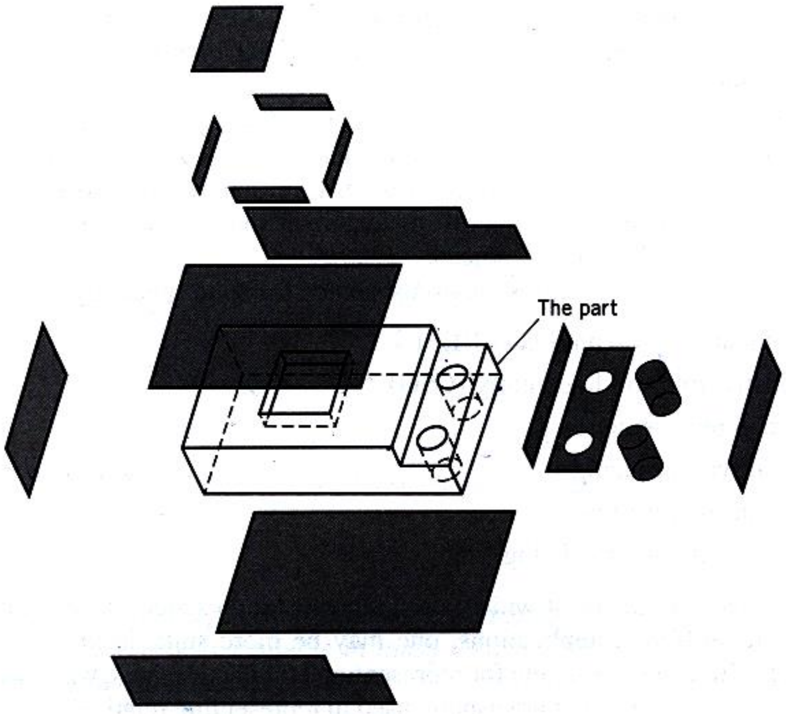
- Why B-Rep includes such topological information?
  - A solid is represented as a closed space in 3D space (surface connect without gaps)
  - The boundary of a solid separates points inside from points outside solid.

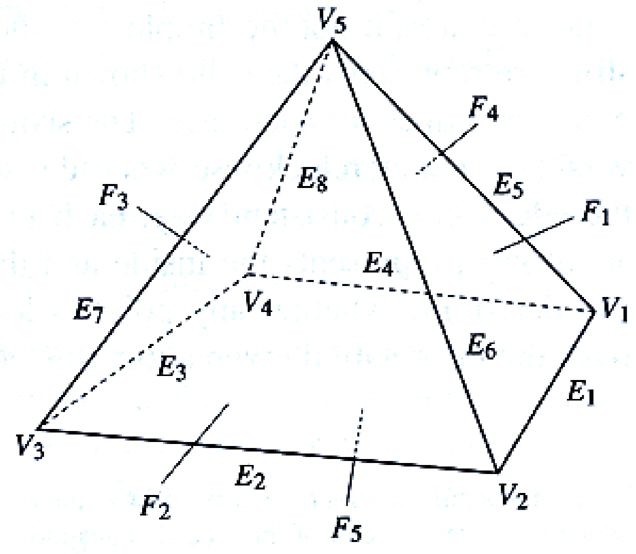
B-Rep vs surface modeling

- Surface model
  - A collection of surface entities which simply enclose a volume lacks the connective data to define a solid (i.e topology).
- B- Rep model
  - Technique guarantees that surfaces definitively divide model space into solid and void, even after model modification commands.

B-Rep data structure

- B-Rep graph store face, edge and vertices as nodes, with pointers, or branches between the nodes to indicate connectivity.





B-Rep data structure

Three tables for storing B-Rep

Face Table

Face	*Edges*
F_1	E_1, E_5, E_6
F_2	E_2, E_6, E_7
F_3	E_3, E_7, E_8
F_4	E_4, E_8, E_5
F_5	E_1, E_2, E_3, E_4

Edge Table

Edge	*Vertices*
E_1	V_1, V_2
E_2	V_2, V_3
E_3	V_3, V_4
E_4	V_4, V_1
E_5	V_1, V_5
E_6	V_2, V_5
E_7	V_3, V_5
E_8	V_4, V_5

Vertex Table

Vertex	*Coordinates*
V_1	x_1, y_1, z_1
V_2	x_2, y_2, z_2
V_3	x_3, y_3, z_3
V_4	x_4, y_4, z_4
V_5	x_5, y_5, z_5
V_6	x_6, y_6, z_6

Boundary representation- validity

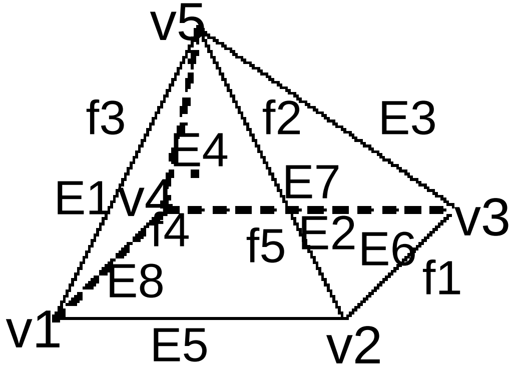
- System must validate topology of created solid.
- B-Rep has to fulfill certain conditions to disallow self-intersecting and open objects
- This condition include
  - Each edge should adjoin exactly two faces and have a vertex at each end.
  - Vertices are geometrically described by point coordinates

Boundary representation- validity

- This condition include (cont)
  - At least three edges must meet at each vertex.
  - Faces are described by surface equations
  - The set of faces forms a complete skin of the solid with no missing parts.
  - Each face is bordered by an ordered set of edges forming a closed loop.
  - Faces must only intersect at common edges or vertices.
  - The boundaries of faces do not intersect themselves

Boundary representation- validity

- Validity also checked through mathematical evaluation
  - Evaluation is based upon Euler's Law (valid for simple polyhedra – no hole)
  - V – E + F = 2 V-vertices E- edges F- face loops



V = 5, E = 8, F = 5

5 – 8 + 5 = 2

Boundary representation-advantages

- Capability to construct unusual shapes that would not be possible with the available CSG→ aircraft fuselages, swing shapes
- Less computational time to reconstruct the image
- Algorithms work very fast due to its presence in evaluated condition and data redundancy.

Boundary representation-disadvantages

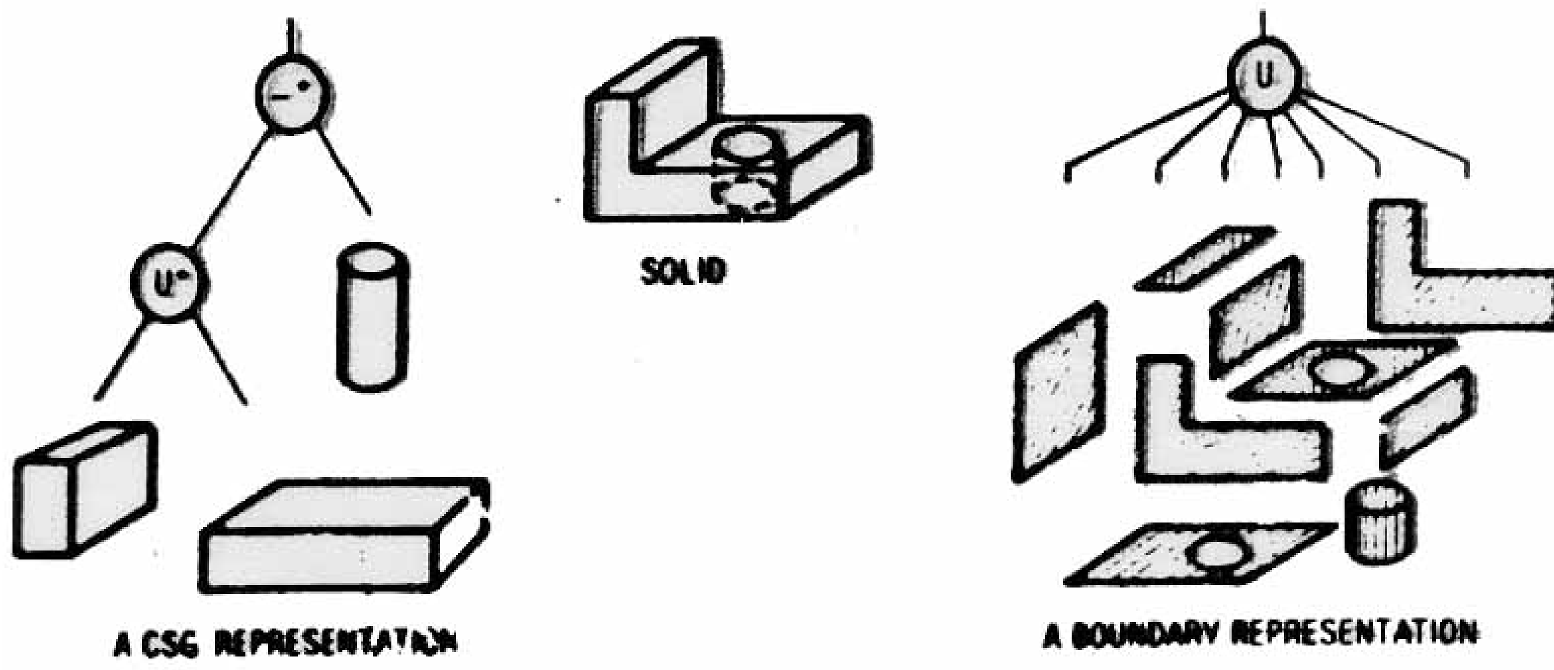
- Requires more storage
- More prone to validity failure than CSG
- Model display limited to planar faces and linear edges
  - complex curve and surfaces only approximated

Limitations of B-Rep Models

Limitations:

- High memory requirements.
- Difficult for a user to create since he has to calculate the intersections of various surfaces, i.e., poor user friendliness.
- Large amount of data redundancy.
- During the manipulative operations, the topology may be disturbed leading to nonsense object if proper check is not made.

Comparison of CSG & B-Rep



Features of FBM

- The shape is constructed out of manufacturable features such as holes, boss, slot etc. instead of a collection of primitives operated by the Booleans. In other words, Boolean operations are implied by the features.
- Each feature has some constraints which it shall always adhere.
- These constraints ensure adherence to topological conditions even when the dimensions are changed.
- FBM is design of a family of parts and not just a part.
- It is driven by constrained parameters.
- Basically 2D sketches are involved.

Features of FBM - Procedure

- Choose a sketch plane.
- Sketch a rough 2D sketch. This gives only topology.
- Just constrain this sketch in three levels:
- Use the rules defined internally
- Add more relational constraints
- Add dimensional parameters for the rest.
- Convert the 2D sketch into 3D features such as extrusion, revolution, sweep, cutout etc.
- The first feature created in this manner is called base feature.
- Use the above steps to create all other features.

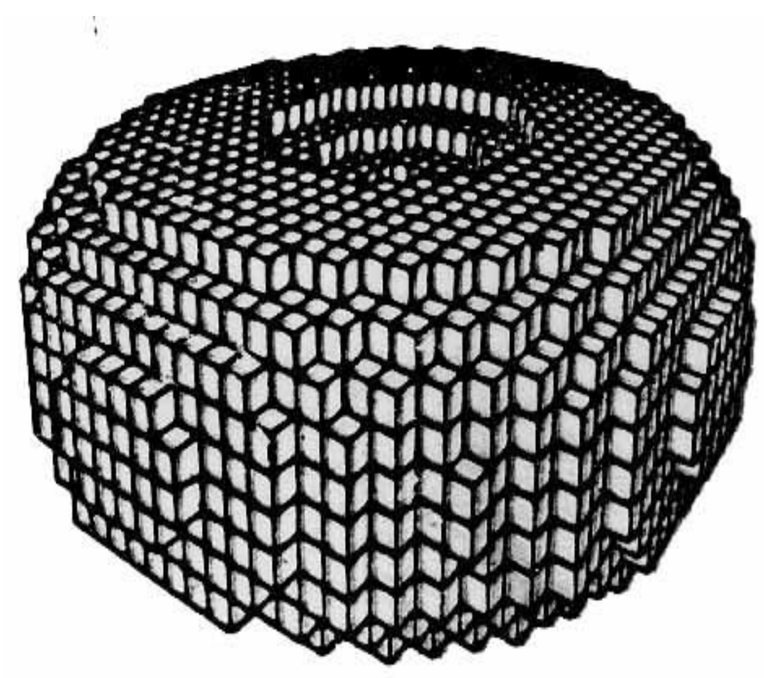
Features of FBM - Advantages

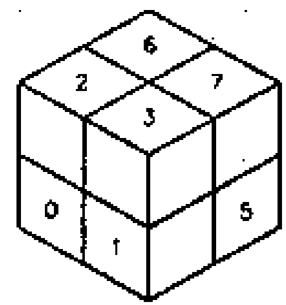
- By changing a few parameters, the object can be changed unambiguously since all the dimensions are related to each other by these parameters. this way, a family of parts can be designed with the same effort required to design a part in CSG.
- Changes propagate throughout the assembly automatically.
- Due to the presence of constraints, even if the dimensions change, the topological relations are preserved. For instance, a through hole remains ‘through’ even if the thickness of the plate is increased.
- Creation of 2D and its conversion to 3D using familiar features makes this approach more elegant and natural.
- Unlike CSG, here, the Boolean operations are not explicit. They depend on the feature characteristics. This minimizes construction effort.
- Automatic and just dimensioning.

Features of FBM - Limitations

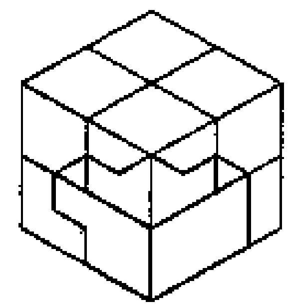
- More intelligence is required for the users. solving constraints requires considerable geometric acumen.
- User should plan well in deciding the parameters to exploit the benefits of this philosophy.
- Not very amenable for freeform modeling (*freeform* and *constraints* do not go together!)

Octree

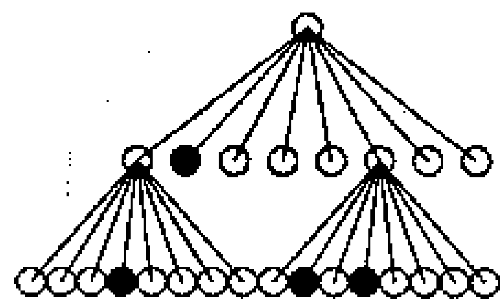




(a)

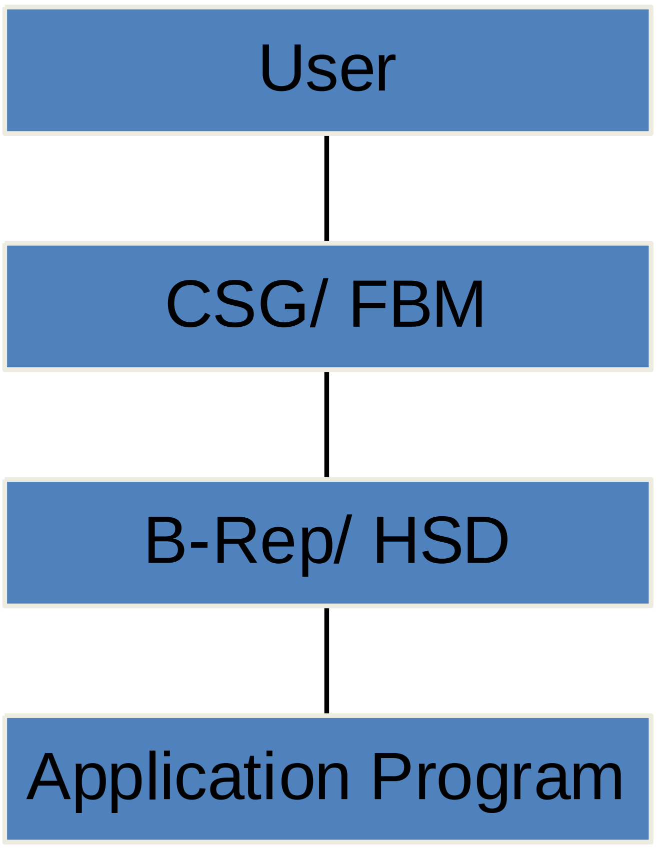


(b)



(c)

Practical Solid Modeling System



No single solid representation scheme is good enough in all aspects. Therefore, any practical solid modeling system invariably makes use of and maintains two or more representations - one being ***user-friendly*** and the other being ***system-friendly***.

Practical Solid Modeling Systems

Software	User-friendly Representation	System-friendly Representation
AutoCAD MDT, I-DEAS, Unigraphics	CSG & FBM	B-Rep
Pro/E, SolidWorks	FBM	B-Rep
GIS, Medicine, NC Simulation …	Images, NC Program (DSG)	HSD (V-Rep or Octree)

Conclusions

- Wire-frame, Surface and Solid are the three possible representations of 3D objects.
- These representations differ in the amount of topological information they store.
- Solid Modeling is the most preferred model as of now.
- There are several solid representations in use, some more user-friendly and others more system-friendly – none have both.
- Any usable solid modeler has at least two representations simultaneously.
- An object which has not only shape and size but many other properties such as tolerance, surface finish, color, smell etc. and is composite is the ultimate aim. This is still a far off goal.

References

- Les Piegl and Wayne Tiller, *The NURBS Book*, Springer (1997)
- G. Farin, J. Hoschek, M.-S. Kim, *Handbook of Computer Aided Geometric Design*, North-Holland (2002)
- Faizack Haidri, CAD CAM & Automation Book
- https://www.google.co.in
- https://en.wikipedia.org

Thanks