# Analog and Digital Electronics

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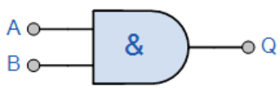
# Logic Gates

| Symbol | Truth Table |
| --- | --- |

AND



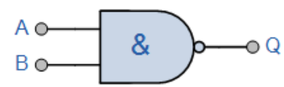

| B | A | Q |
| --- | --- | --- |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Boolean Expression **Q = A.B**

Read as A **AND** B gives Q

| Symbol | Truth Table |
| --- | --- |

NAND



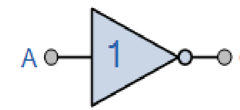

| B | A | Q |
| --- | --- | --- |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Boolean Expression Q = $\overline{A.B}$

Read as A **AND** B gives **NOT** Q

| Symbol | Truth Table |
| --- | --- |

NOT



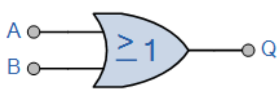

| A | Q |
| --- | --- |
| 0 | 1 |
| 1 | 0 |

Boolean Expression Q = not A or $\overline{A}$

Read as inverse of A gives Q

| Symbol | Truth Table |
| --- | --- |

OR



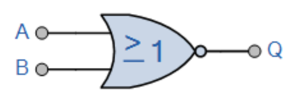

| B | A | Q |
| --- | --- | --- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Boolean Expression Q = A + B

Read as A **OR** B gives Q

| Symbol | Truth Table |
| --- | --- |

NOR



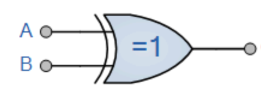

| B | A | Q |
| --- | --- | --- |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

Boolean Expression Q = $\overline{A+B}$

Read as A **OR** B gives **NOT** Q

| Symbol | Truth Table |
| --- | --- |

Ex-OR



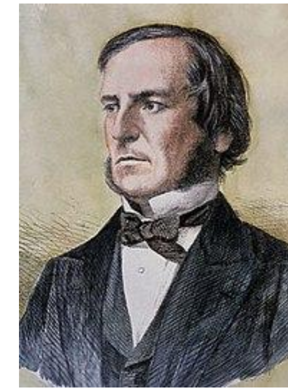

| B | A | Q |
| --- | --- | --- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Boolean Expression Q = A ⊕ B

Read as A **OR** B but not **BOTH** gives Q (odd)

# Boolean Algebra

Boolean algebra was invented by George Boole in 1854



- Boolean Algebra is used to analyze and simplify the digital (logic) circuits.
- It uses only the binary numbers i.e. 0 and 1.
- It is also called as Binary Algebra or logical Algebra.
- This can help lower the cost and raise the speed and efficiency of digital circuits

- Digital circuits are made up of **logic gates**.
- Logic gates are a fundamental digital component that perform simple logical calculations
- Multiple logic gates can be connected with wires, forming a complex digital circuit.
- Outputs of logic gates can be connected to the inputs of other logic gates.
- When multiple logic gates work together, they can perform complex mathematical calculations, as well as remember data.

"Set of rules used to simplify the given logic expression without changing its functionality"

# Boolean Algebra

## Laws of Boolean Algebra:

AND law:

$A.0 = 0$

$A.1 = A$

$A.A = A$

$A.A' = 0$

OR law:

$A + 0 = A$

$A + 1 = 1$

$A + A = A$

$A + A' = 1$

Complement law:

$\left(A'\right)' = A$

Commutative law:

$A.B = B.A$

$A + B = B + A$

Associative law:

$(A.B).C = A.(B.C)$

$(A + B) + C = A + (B + C)$

Distributive law:

$A.(B + C) = A.B + A.C$

$A + B.C = (A + B).(A + C)$

De Morgan's rule:

$\left(A + B\right)' = A' \cdot B'$

$(A \cdot B)' = A' + B'$

Other imp. rules:

$A + AB = A$

$A + A'B = A + B$

$A' + AB = A' + B$

$A' + AB' = A' + B'$

# Boolean Algebra

## Solve

$$AB + AB'$$

$$A \cdot (B + B')$$

$$A \cdot 1$$

$$A$$

$$AB + AB'C + AB'C'$$

$$A(B + B'C + B'C')$$

$$A(B + C + B'C')$$

$$A(B + C + C')$$

$$A(B + 1)$$

$$A(1)$$

$$A$$

$$(A')' = A$$

$$A.0 = 0$$
$$A.1 = A$$
$$A.A = A$$
$$A.A' = 0$$

$$A + 0 = A$$
$$A + 1 = 1$$
$$A + A = A$$
$$A + A' = 1$$

$$A.(B + C) = A.B + A.C$$
$$A + B.C = (A + B).(A + C)$$
$$A + A'B = A + B$$
$$A' + AB = A' + B$$

$$(A + B)' = A' \cdot B'$$
$$(A \cdot B)' = A' + B'$$

# Boolean Algebra

## Solve

$$(A+B+C)(A+B'+C)(A+B+C')$$

$$(X+C)(A+B'+C)(X+C')$$

$$(X+C)(X+C')(A+B'+C)$$

$$(X+CC')(A+B'+C)$$

$$(X+0)(A+B'+C)$$

$$X(A+B'+C)$$

$$(A+B)(A+B'+C)$$

$$A+B(B'+C)$$

$$A+BB'+BC$$

$$A+BC$$

$$(A')' = A$$

$$A.0 = 0$$
$$A.1 = A$$
$$A.A = A$$
$$A.A' = 0$$

$$A+0 = A$$
$$A+1 = 1$$
$$A+A = A$$
$$A+A' = 1$$

$$A.(B+C) = A.B + A.C$$
$$A+B.C = (A+B).(A+C)$$
$$A+A'B = A+B$$
$$A'+AB = A'+B$$

$$(A+B)' = A' \cdot B'$$
$$(A \cdot B)' = A'+B'$$

# Boolean Algebra

## Solve

$$(A+B)(A+B')(A'+B)(A'+B')$$

$$(A+BB')(A'+BB')$$

$$(A+0)(A'+0)$$

$$AA'$$

$$0$$

$$\left(A'\right)' = A$$

$$A.0 = 0$$

$$A.1 = A$$

$$A.A = A$$

$$A.A' = 0$$

$$A+0 = A$$

$$A+1 = 1$$

$$A+A = A$$

$$A+A' = 1$$

$$A.(B+C) = A.B + A.C$$

$$A + B.C = (A+B).(A+C)$$

$$A + A'B = A + B$$

$$A' + AB = A' + B$$

$$\left(A+B\right)' = A' \cdot B'$$

$$(A \cdot B)' = A' + B'$$

# Boolean Algebra

## Redundancy Theorem

- Three variables
- Each variable represented twice
- One variable is complimented

→ Take complemented variable

$$AB + A'C + BC$$

$$AB + A'C$$

$$AB + A'C + BC$$
$$AB + A'C + BC(A + A')$$
$$AB + A'C + ABC + A'BC$$
$$AB(1 + C) + A'C(1 + B)$$
$$AB(1) + A'C(1)$$
$$AB + A'C$$

# Boolean Algebra

## Redundancy Theorem

- Three variables
- Each variable represented twice
- One variable is complimented

→ Take complemented variable

$$AB + BC' + AC$$

$$BC' + AC$$

$$(A+B)(A'+C)(B+C)$$

$$(A+B)(A'+C)$$

$$A'B' + AC' + B'C'$$

$$A'B' + AC'$$

# Sum of Product (SOP)

**SOP → when output is "1"**

| | $A$ | $B$ | $C$ | $Y$ |
|---|---|---|---|---|
| $m_0$ | 0 | 0 | 0 | 0 |
| $m_1$ | 0 | 0 | 1 | 0 |
| $m_2$ | 0 | 1 | 0 | 1 |
| $m_3$ | 0 | 1 | 1 | 0 |
| $m_4$ | 1 | 0 | 0 | 1 |
| $m_5$ | 1 | 0 | 1 | 1 |
| $m_6$ | 1 | 1 | 0 | 1 |
| $m_7$ | 1 | 1 | 1 | 1 |

Standard or canonical SOP form

$$Y = A'BC' + AB'C' + AB'C + ABC' + ABC$$

$$Y = A'BC' + AB'C' + AB'C + ABC' + ABC$$

$$Y = A'BC' + AB'(C'+C) + AB(C'+C)$$

$$Y = A'BC' + AB' + AB$$

$$Y = A'BC' + A(B'+B)$$

$$Y = A'BC' + A$$

$$Y = A + BC'$$

Minimal SOP form

$$F(A,B,C) = m_2 + m_4 + m_5 + m_6 + m_7 = \sum m(2,4,5,6,7)$$

# Sum of Product (SOP)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**For the given Truth Table, Minimize the SOP expression**

$$Y = A' \cdot B + A \cdot B$$

$$Y = B \cdot (A' + A)$$

$$Y = B \cdot 1$$

$$Y = B$$

| | $A$ | $B$ | $Y$ |
|---|---|---|---|
| $m_0$ | 0 | 0 | 1 |
| $m_1$ | 0 | 1 | 0 |
| $m_2$ | 1 | 0 | 1 |
| $m_3$ | 1 | 1 | 1 |

**Simplify the expression for Y(A,B)=Σm(0,2,3)**

$$Y = m_0 + m_2 + m_3$$

$$Y = A'B' + AB' + AB$$

$$Y = B'(A' + A) + AB$$

$$Y = B' + AB$$

$$Y = B' + A$$

# Sum of Product (SOP)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| | $A$ | $B$ | $Y$ |
|---|---|---|---|
| $m_0$ | 0 | 0 | 0 |
| $m_1$ | 0 | 1 | 1 |
| $m_2$ | 1 | 0 | 1 |
| $m_3$ | 1 | 1 | 1 |

**For the given Truth Table, Minimize the SOP expression**

$$Y = A \cdot B + A' \cdot B + A \cdot B$$

$$Y = A \cdot B + B \cdot (A' + A)$$

$$Y = A \cdot B + B$$

$$Y = A \cdot B$$

**Simplify the expression for Y(A,B)=Σm(1,2,3)**

$$Y = m_1 + m_2 + m_3$$

$$Y = A' \cdot B + A \cdot B' + A \cdot B$$

$$Y = A' \cdot B + A(B' + B)$$

$$Y = A' \cdot B + A$$

$$Y = A + B$$

# Product of Sum (POS)

**POS → when output is "0"**

| | $A$ | $B$ | $C$ | $Y$ |
|---|---|---|---|---|
| $m_0$ | 0 | 0 | 0 | 0 |
| $m_1$ | 0 | 0 | 1 | 0 |
| $m_2$ | 0 | 1 | 0 | 1 |
| $m_3$ | 0 | 1 | 1 | 0 |
| $m_4$ | 1 | 0 | 0 | 1 |
| $m_5$ | 1 | 0 | 1 | 1 |
| $m_6$ | 1 | 1 | 0 | 1 |
| $m_7$ | 1 | 1 | 1 | 1 |

Standard or canonical POS form

$$Y=(A+B+C)\cdot(A+B+C')\cdot(A+B'+C')$$

$$Y=(A+B+C)\cdot(A+B+C')\cdot(A+B'+C')$$
$$Y=A+B(C.C').(A+B'+C')$$
$$Y=(A+B).(A+B'+C')$$
$$Y=A+B.(B'+C')$$
$$Y=A+BC'$$

Minimal POS form

# Sum of Product (POS)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**For the given Truth Table, Minimize the POS expression**

$$Y = (A + B').(A' + B')$$
$$Y = B' + A.A'$$
$$Y = B'$$

| | $A$ | $B$ | $Y$ |
|---|---|---|---|
| $M_0$ | 0 | 0 | 0 |
| $M_1$ | 0 | 1 | 1 |
| $M_2$ | 1 | 0 | 0 |
| $M_3$ | 1 | 1 | 1 |

**Simplify the expression for Y(A,B)=Πm(0,2)**

$$Y = M_0 + M_2$$
$$Y = (A + B).(A' + B)$$
$$Y = B + A.A'$$
$$Y = B$$

# SOP & POS

| | A | B | C | F |
|---|---|---|---|---|
| $m_0$ | 0 | 0 | 0 | 1 |
| $m_1$ | 0 | 0 | 1 | 0 |
| $m_2$ | 0 | 1 | 0 | 1 |
| $m_3$ | 0 | 1 | 1 | 1 |
| $m_4$ | 1 | 0 | 0 | 0 |
| $m_5$ | 1 | 0 | 1 | 0 |
| $m_6$ | 1 | 1 | 0 | 1 |
| $m_7$ | 1 | 1 | 1 | 1 |

$$Y(A,B,C) = \sum m(0,2,3,6,7)$$

$$Y = A'B'C' + A'BC' + A'BC + ABC' + ABC$$

$$Y = A'B'C' + A'B + AB$$

$$Y = A'B'C' + B$$

$$Y = B + A'C'$$

$$Y(A,B,C) = \prod M(1,4,5)$$

$$Y = (A+B+C').(A'+B+C).(A'+B+C')$$

$$Y = (A+B+C').(A'+B)$$

$$Y = (B+A+C').(B+A')$$

$$Y = B+(A+C').A'$$

$$Y = B + A'C'$$

# SOP: Minimal to Canonical form

$$Y = A + B'C$$

**Step 1:** Check number of variables (A, B, C)

**Step 2:** Check availability of variables in minimal ($m_1$, $m_2$)

| $m_1$ | $m_2$ |
|---|---|
| A: ✓ | A: x |
| B: x | B: ✓ |
| C: x | C: ✓ |

**Step 3:** Write down missing variable → 1 as (x+x')

$$Y = A \cdot 1 \cdot 1 + B' \cdot C \cdot 1$$

$$Y = A \cdot (B + B') \cdot (C + C') + B' \cdot C \cdot (A + A')$$

$$Y = (AB + AB') \cdot (C + C') + B' \cdot C \cdot A + B' \cdot C \cdot A'$$

$$Y = ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$$

$$Y = ABC + ABC' + AB'C + AB'C' + A'B'C$$

# POS: Minimal to Canonical form

$$Y = (A + B + C') \cdot (A' + C)$$

**Step 1:** Check number of variables (A, B, C)

**Step 2:** Check availability of variables in minimal ($m_1$, $m_2$)

| $m_1$ | $m_2$ |
|---|---|
| A: ✓ | A: ✓ |
| B: ✓ | B: x |
| C: ✓ | C: ✓ |

**Step 2:** Write down missing variable → 0 as (x.x')

$$Y = (A + B + C') \cdot (A' + C)$$

$$Y = (A + B + C') \cdot (A' + C + 0)$$

$$Y = (A + B + C') \cdot (A' + C + B \cdot B')$$

$$Y = (A + B + C') \cdot (A' + C + B) \cdot (A' + C + B')$$

# Karnaugh Map (K-Map)

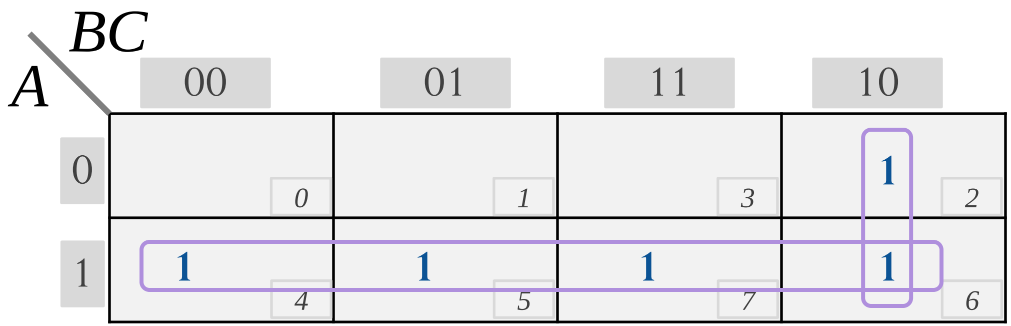
Maurice Karnaugh, developed the K-Map at Bell Labs in 1953 while designing digital logic based telephone switching circuits

- Reduce logic function more quickly and easily than Boolean algebra

# Karnaugh Map (K-Map)

Find minimal form using SOP $A + BC'$

| $A$ | $B$ | $C$ | $Y$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | 0 | $m_1$ |
| 0 | 1 | 0 | 1 | $m_2$ |
| 0 | 1 | 1 | 0 | $m_3$ |
| 1 | 0 | 0 | 1 | $m_4$ |
| 1 | 0 | 1 | 1 | $m_5$ |
| 1 | 1 | 0 | 1 | $m_6$ |
| 1 | 1 | 1 | 1 | $m_7$ |




Pairing: 1, 2, 4, 8 ....

$$Y = I + II \text{ (Implicants)}$$

$$Y = A + BC'$$



KK Dudani

# Karnaugh Map (K-Map)

$$f(A,B,C) = \sum m(1,3,5,7)$$

| $A$ | $B$ | $C$ | $Y$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | 1 | $m_1$ |
| 0 | 1 | 0 | 0 | $m_2$ |
| 0 | 1 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | 1 | $m_5$ |
| 1 | 1 | 0 | 0 | $m_6$ |
| 1 | 1 | 1 | 1 | $m_7$ |

| $A \backslash BC$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 1 | 1 3 | 2 |
| 1 | 4 | 1 5 | 1 7 | 6 |

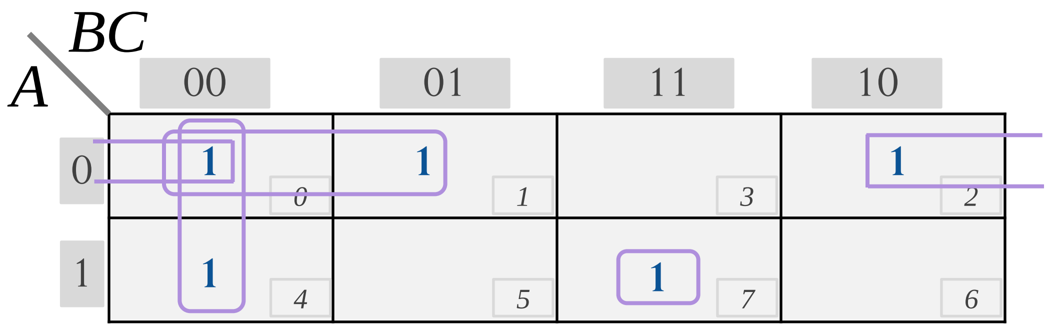
Pairing: 1, 2, 4, 8 ....

$$Y = I \text{ (Implicants)}$$

$$Y = C$$

# Karnaugh Map (K-Map)

$$f(A,B,C) = \sum m(0,1,2,4,7)$$




Pairing: 1, 2, 4, 8 ....

$$Y = I + II + III + IV$$

$$Y = B'C' + A'B' + ABC + A'C'$$

# Karnaugh Map (K-Map)

$$f(A,B,C)=\sum m(1,3,6,7)$$

| $A \backslash BC$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 1 | 1 3 | 2 |
| 1 | 4 | 5 | 1 7 | 1 6 |

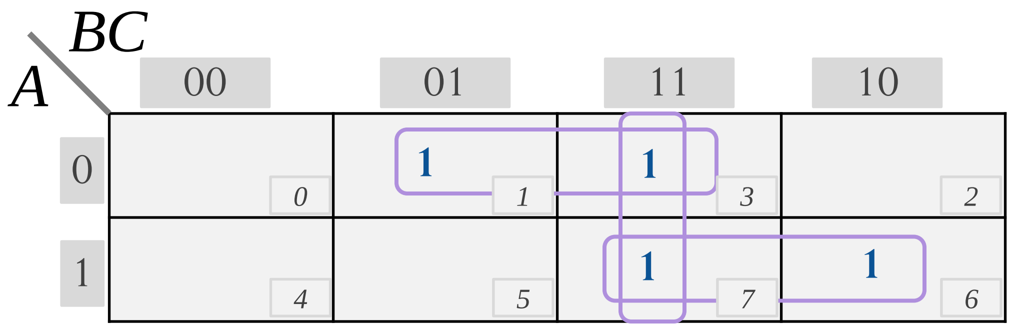
Pairing: 1, 2, 4, 8 ....

$$Y = I + II$$

$$Y = A'C + AB$$

# Karnaugh Map (K-Map)

$$f(A,B,C) = \sum m(1,3,6,7)$$




Pairing: 1, 2, 4, 8 ....

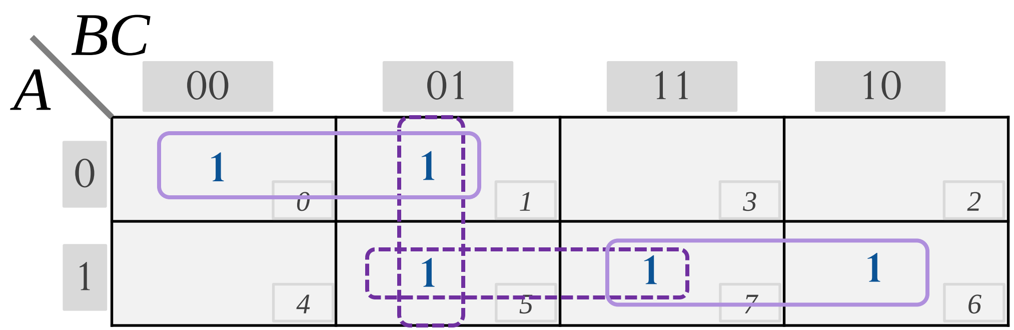
$$Y = I + II$$

$$Y = A'C + AB + BC$$

$$Y = A'C + AB \;\; (\because \text{Redudndancy Theorem})$$

# Karnaugh Map (K-Map)

$$f(A,B,C) = \sum m(0,1,5,6,7)$$




Pairing: 1, 2, 4, 8 ....

$$Y = I + II + III_A$$
$$Y = AB + AB' + AC$$

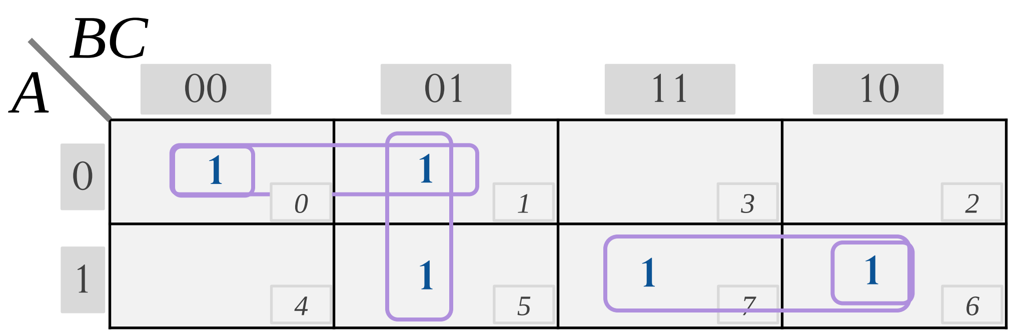
$$Y = I + II + III_B$$
$$Y = AB + AB' + B'C$$

# Karnaugh Map (K-Map) - Implicants

Implicants: Group of 1's

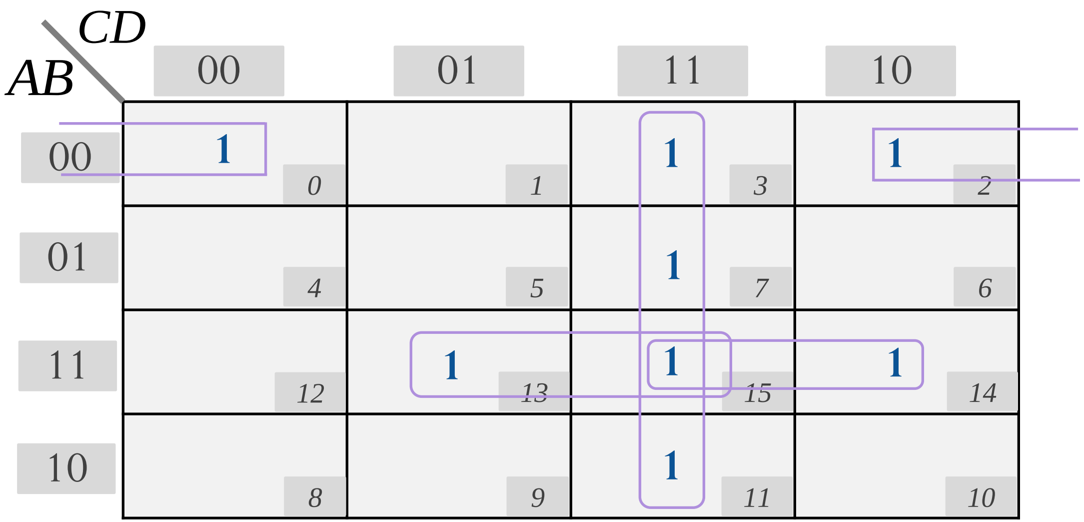
Prime Implicants: Largest possible group of 1's

Essential Prime Implicants: Single 1 which can not be combined in any other way



# Karnaugh Map (K-Map) – 4 variable

$$f(A,B,C,D)=\sum m(0,2,3,7,11,13,14,15)$$

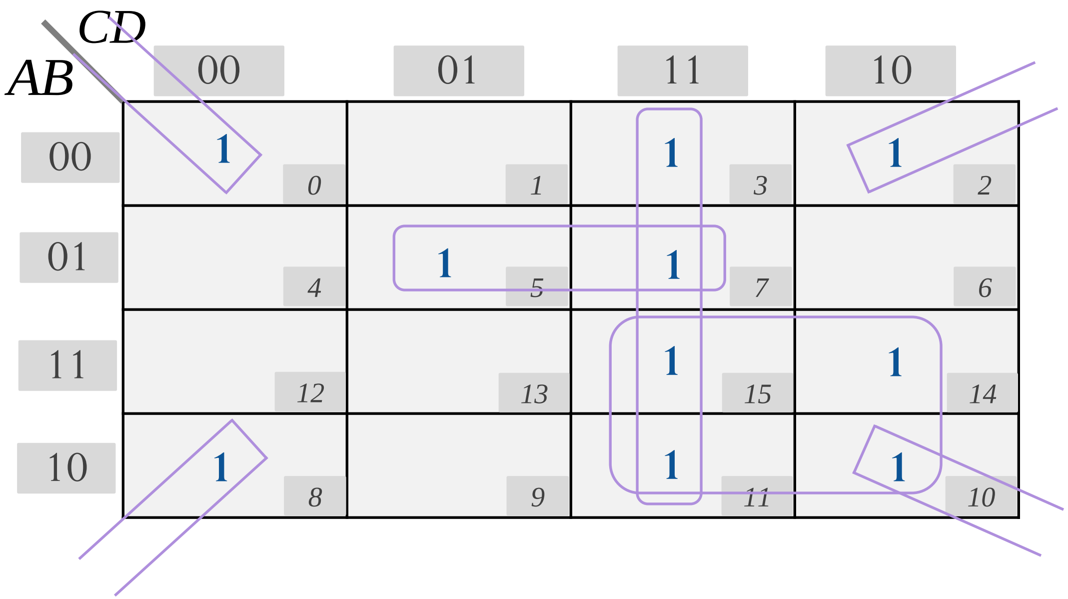



$$Y=I+II+III+IV$$

$$Y=CD+A'B'D'+ABD+ABC$$

# Karnaugh Map (K-Map)

$$f(A,B,C,D)=\sum m(0,2,3,5,7,8,10,11,14,15)$$

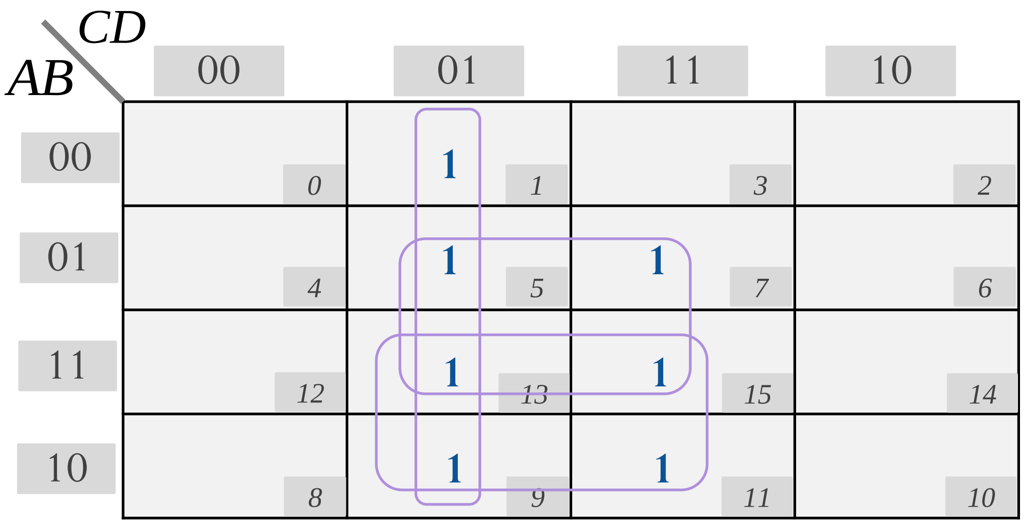



$$Y=I+II+III+IV$$

$$Y=CD+B'D'+AC+A'BD$$

# Karnaugh Map (K-Map)

$$f(A,B,C,D)=\sum m(1,5,7,9,11,13,15)$$




$$Y = I + II + III$$

$$Y = C'D + AD + BD$$

# Karnaugh Map (K-Map) – Using Maxterm

| $A \backslash BC$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 $m_0$ | 1 $m_1$ | 0 $m_3$ | 0 $m_2$ |
| 1 | 1 $m_4$ | 1 $m_5$ | 1 $m_7$ | 1 $m_6$ |

$$Y = I + II$$
$$Y = A + B'$$

$$Y = I$$
$$Y = A + B'$$

$OR$

$$Y = I$$
$$Y' = (A'B)'$$
$$Y' = A + B'$$

# Karnaugh Map (K-Map) – Don't care condition

$$f(A,B,C)=\sum m(2,3,4,5)+\sum d(6,7)$$

| $A \backslash BC$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_1$ | 1 $m_3$ | 1 $m_2$ |
| 1 | 1 $m_4$ | 1 $m_5$ | 1 $m_7$ | 1 $m_6$ |

In Minterm(SOP) replace Don't care by 1

In Maxterm(POS) replace Don't care by 0

$$Y = I + II$$
$$Y = A + B$$

OR gate

$$Y = I + II$$
$$Y = A'B + AB'$$

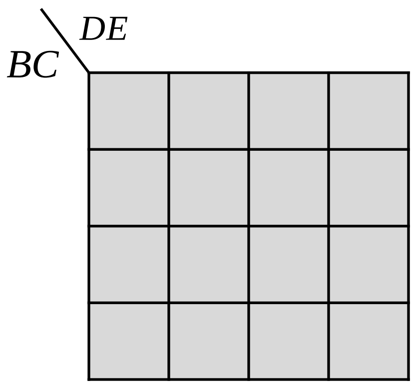
AND, OR, AND gate

# 5 variable Karnaugh Map (K-Map)

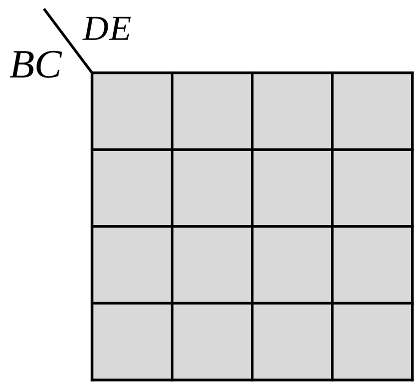
A
0
0
0
0
0
0
0
.
.
.
.
$0_{16}$

| B | C | D | E |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

$A = 0$



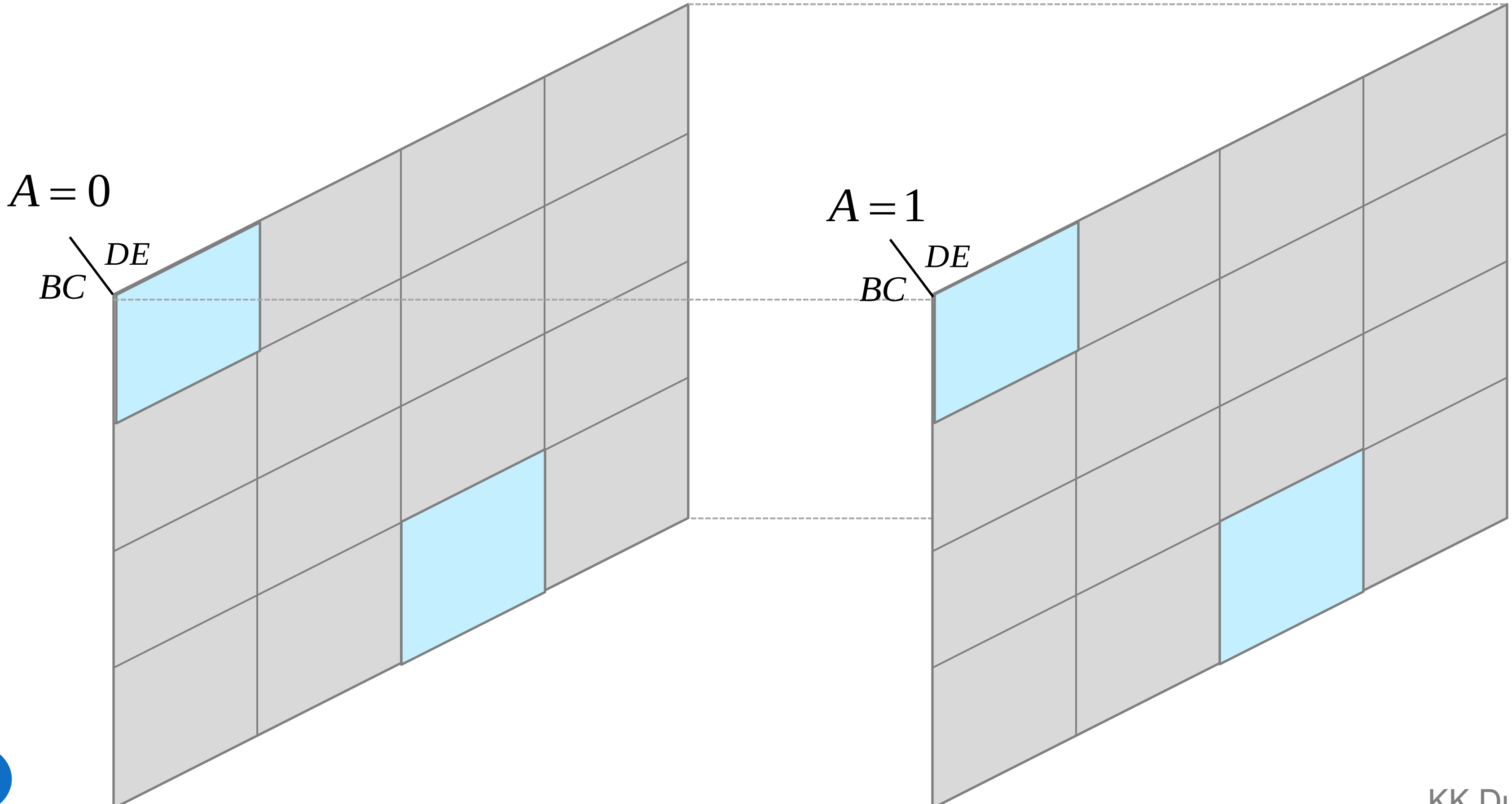

$A = 1$




A
1
1
1
1
1
1
.
.
.
.
$1_{16}$

| B | C | D | E |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

# 5 variable Karnaugh Map (K-Map)



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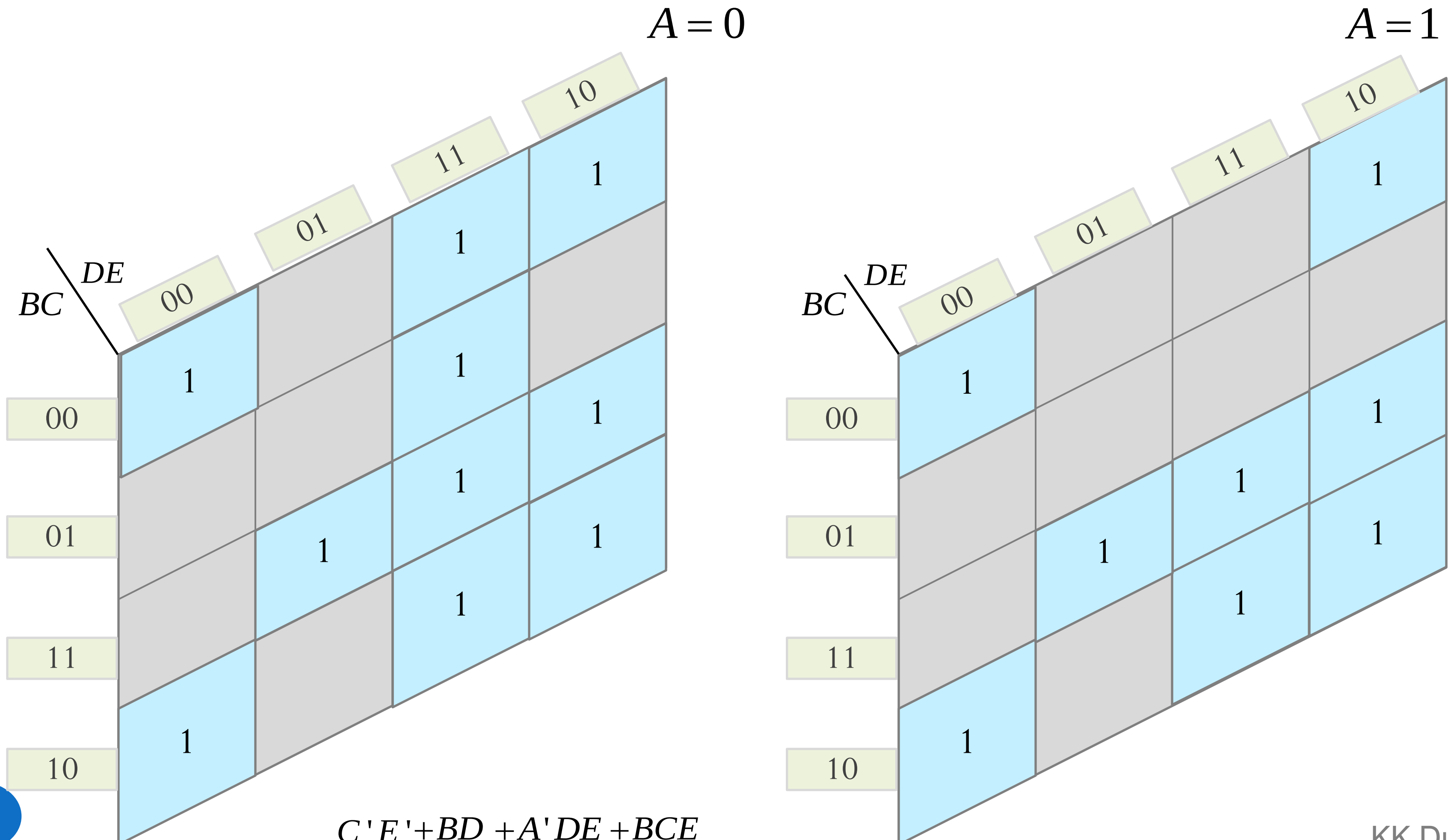
# 5 variable Karnaugh Map (K-Map)




$$C'E' + BD + A'DE + BCE$$



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